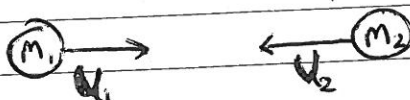


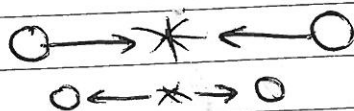
Conservation of Momentum

This states that after a collision between two particles, the ^{total} momentum stays the same.

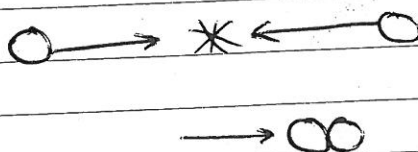


$$\underbrace{m_1 u_1 + m_2 u_2}_{\text{momentum before}} = \underbrace{m_1 v_1 + m_2 v_2}_{\text{momentum after}}$$

'Elastic' collisions - where two particles bounce off each other.

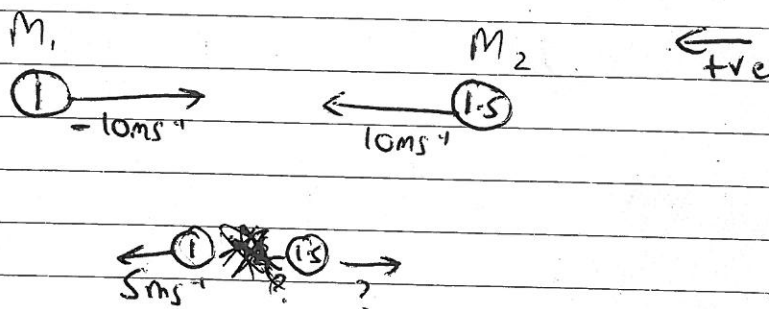


Inelastic collisions - where two particles collide and 'stick' together.



Example

Two balls are thrown at each other at 10ms^{-1} along the ground, one with mass 1kg , the other with mass 1.5kg . IF the ~~first ball~~ ~~first ball~~ First ball moves backwards at 5ms^{-1} after the collision, find the speed of the Second ball.



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$1(-10) + 1.5(10) = 1(5) + 1.5V_2$$

$$-10 + 15 = 5 + 1.5V_2$$

$$5 = 5 + 1.5V_2$$

$$5 - 5 = 1.5V_2$$

$$0 = V_2$$

What if the 1st ball was thrown at 5ms^{-1} ?

$$1(-5) + 1.5(10) = 1(5) + 1.5(V_2)$$

$$-5 + 15 = 5 + 1.5V_2$$

$$10 = 5 + 1.5V_2$$

$$5 = 1.5V_2$$

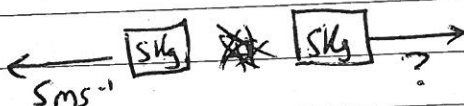
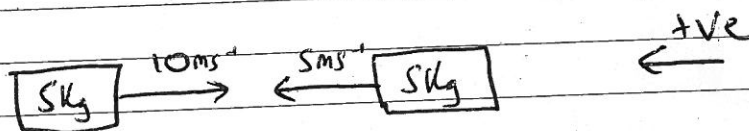
$$3.33 = V_2$$

ms^{-1}

Example

of mass 5kg

Two masses travel towards each other, one at 10ms^{-1} , the other at 5ms^{-1} . If the first ~~mass~~ then changes direction after the collision with speed of 5ms^{-1} , find the speed of the second ~~mass~~.



$$M_1 u_1 + M_2 u_2 = M_1 v_1 + M_2 v_2$$

$$5(-10) + 5(5) = 5(5) + 5(V_2)$$

$$-50 + 25 = 25 + 5V_2$$

$$-25 = 25 + 5V_2$$

$$-25 - 25 = 5V_2$$

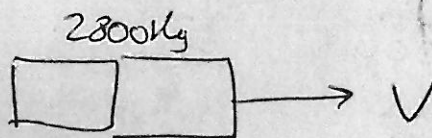
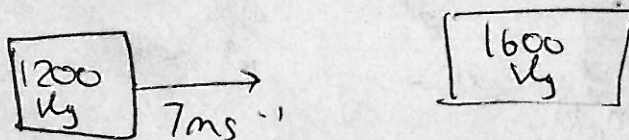
$$-50 = 5V_2$$

$$\frac{-50}{5} = V_2$$

$$\boxed{-10 = V_2 \text{ ms}^{-1}}$$



A truck of mass 1200 kg is moving with speed of 7 ms^{-1} when it collides with a second truck of mass 1600 kg which is stationary. If the two trucks automatically stick together, with what speed do they move on together?



$$(1200 \times 7) + (1600 \times 0) = 2800 v$$

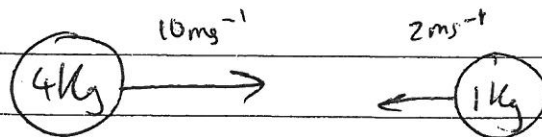
$$1200 \times 7 = 2800 v$$

$$8400 = 2800 v$$

$$3 = v$$

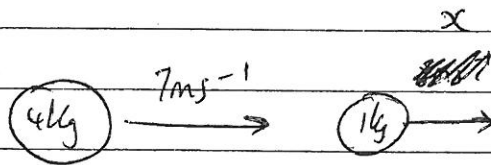
Elastic

Before



+ve
→

After



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$4(10) + 1(-2) = 4(7) + 1(x)$$

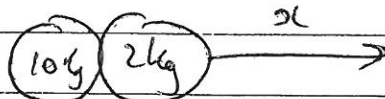
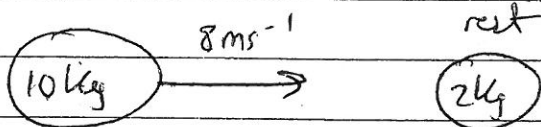
$$40 - 2 = 28 + x$$

$$38 = 28 + x$$

$$38 - 28 = x$$

$$10\text{ms}^{-1} = x$$

Inelastic



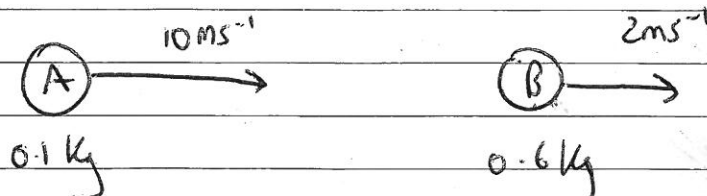
$$10(8) + 2(0) = 12x$$

$$80 = 12x$$

$$6.67\text{ms}^{-1} = x$$

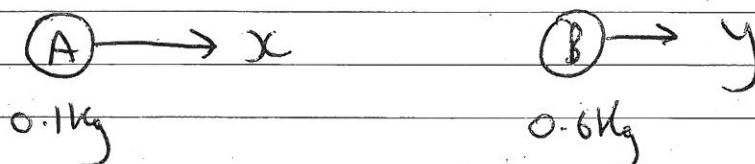
- Q2. A particle 'A' of mass 0.1 kg moving with speed 10 ms^{-1} collides with another particle 'B' of mass 0.6 kg moving with speed 2 ms^{-1} in the same direction. Coef. of restitution $= 3/4$. After collision both move off in the same direction. Calculate their speeds.

Before



$$e = 3/4$$

After



Conservation of momentum

$$10(0.1) + 2(0.6) = 0.1x + 0.6y$$

$$1 + 1.2 = 0.1x + 0.6y$$

$$2.2 = 0.1x + 0.6y \quad \text{--- (1)}$$

Coef. of restitution

$$e = \frac{\text{Vel of sep}}{\text{Vel of app}}$$

$$e = \frac{y - x}{10 - 2}$$

$$\frac{3}{4} = \frac{y-x}{8}$$

$$\frac{24}{4} = y-x$$

$$6 = y-x \quad - (2)$$

Solve ① & ② simultaneously

$$0.1x + 0.6y = 2.2 \quad (1) \times 10$$

$$-x + y = 6 \quad (2)$$

$$x + 6y = 22 \quad (3)$$

$$(3) + (2)$$

$$7y = 28$$

$$y = 4 \text{ ms}^{-1} \rightarrow \text{in } (2)$$

$$-x + 4 = 6$$

$$4 - 6 = x$$

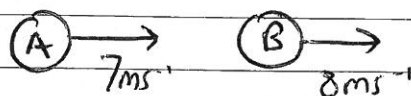
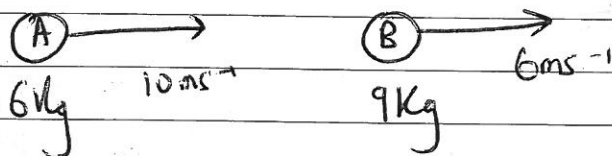
$$-2 = x$$

- 4) A body 'A' of mass 6 kg moving with speed 10 ms^{-1} collides with a body 'B' of mass 9 kg, moving with speed 6 ms^{-1} in the same direction as A. After the collision, 'A' moves with speed 7 ms^{-1} in the original direction of motion and B moves with speed 8 ms^{-1} .

a) Find the coef. of restitution.

$$e = \frac{\text{vel of sep.}}{\text{vel. of app.}}$$

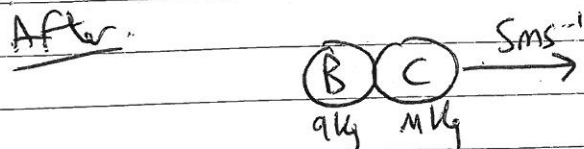
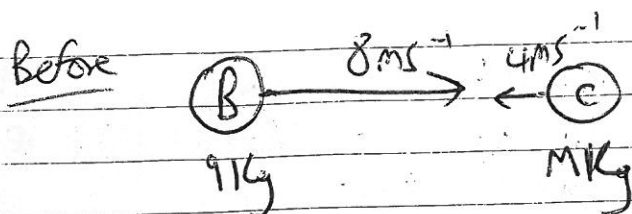
→ +ve



$$e = \frac{8-7}{10-6}$$

$$e = \frac{1}{4}$$

- b) B then collides with another body C moving with speed 4 ms^{-1} , towards B. After the collision, B & C move as one with speed 5 ms^{-1} in the direction of motion of B. Determine the mass of C.



Cons. of momentum :

$$(9 \times 8) + (M \times (-4)) = (9 \times 5) + (M \times 5)$$

$$72 - 4M = 45 + 5M$$

$$72 - 45 = 5M + 4M$$

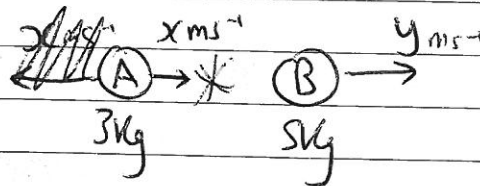
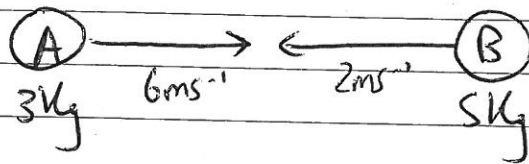
$$27 = 9M$$

$$3 \text{ kg} = M$$

- 7) A Sphere 'A', of mass 3kg moving with Speed 6ms^{-1} Collides directly with another Sphere 'B' of mass 5kg Moving in the opposite direction with Speed 2ms^{-1} . The Coef. of restitution is $\frac{1}{3}$.

• Find the Speed of each sphere after the Collision.

$\Rightarrow$ +ve



$$e = \frac{1}{3}$$

Cons. momentum

$$(3 \times 6) + (5 \times (-2)) = (3 \times x) + (5 \times y)$$

$$18 - 10 = 3x + 5y$$

$$8 = 3x + 5y \quad (1)$$

Coef. restitution

$$V_{\text{sep}} = y - x$$

$$V_{\text{app}} = 6 + 2 = 8$$

$$\frac{1}{3} = \frac{y - x}{8}$$

$$8 = 3y - 3x \quad (2)$$

Sim eqn.

$$8 = 3x + 5y \quad (1) \quad +$$

$$8 = -3x + 3y \quad (2)$$

$$16 = 8y$$

$$\underset{ms^{-1}}{2 = y} \rightarrow \text{in } (1)$$

$$8 = 3x + 5(2)$$

$$8 = 3x + 10$$

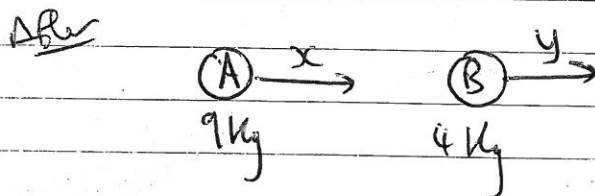
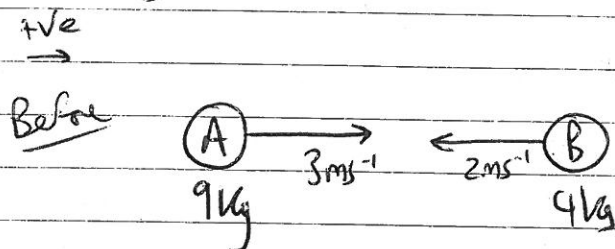
$$-2 = 3x$$

$$\underset{ms^{-1}}{\frac{-2}{3} = x}$$

A's direction reversed.

- 4) A Sphere A of mass 9 kg moving with speed 3 ms^{-1} collides directly with another Sphere B of mass 4 kg moving in the opposite direction with speed 2 ms^{-1} .
Coef restitution = 0.3

- Calculate the speeds of the spheres directly after the collision.



Cons. of Momentum

$$(9 \times 3) + (4 \times (-2)) = 9x + 4y$$

$$27 - 8 = 9x + 4y$$

$$19 = 9x + 4y \quad (1)$$

Coef. restitution

$$e = \frac{\text{vel sep}}{\text{vel app}}$$

$$0.3 = \frac{y - x}{3 + 2}$$

$$0.3 = \frac{y - x}{5}$$

$$1.5 = y - x \quad (2)$$

$$19 = 9x + 4y \quad (1)$$

$$1.5 = -x + y \quad (2) \quad \times 4$$

$$6 = -4x + 4y \quad (3)$$

$$(1) - (3)$$

$$13 = 13x$$

$$\frac{1}{\text{ms}^{-1}} = x \rightarrow \text{is} \quad (2)$$

$$1.5 = -1 + y$$

$$\frac{2.5}{\text{ms}^{-1}} = y$$