Examples of activities using mathematical representations: manipulative and images

The following booklet contains examples of activities that could be used to support children's mathematical understanding using representations in the form of particular manipulatives or images.

Articles and activities have been used from websites such as www.nrich.org.uk which supplies a range of games and activities to support particular resources.

Interactive versions of images and manipulatives can also be found online, for example:

http://www.topmarks.co.uk/Flash.aspx?f=NumberLinev5 for an interactive number line

www.topmarks.co.uk has place value resources such as charts and place value cards (search whiteboard resources, place value mathematics)

http://www.primarygames.co.uk/pg2/splat/splatsq100.html has an interactive 100 square

http://www.iboard.co.uk/iwb/Interactive-Number-Square-Integers-691 has an interactive hundred square which allows numbers to be hidden/revealed and rearranged

http://www.taw.org.uk/lic/itp/itps/number_grid_4_0.swf for interactive hundred square

http://www.taw.org.uk/lic/itp/itps/numberLine1_6.swf for interactive number line

http://www.edu.dudley.gov.uk/numeracy/ITPs/New%20shockwave%20ITPs/multiarray.swf for interactive array

http://exchange.smarttech.com/details?id=bea8c1a3-578e-42ba-aa75-97aaffd3a783 for an interactive ten frame for a SMART board

Copiable spinners, ten frames, number lines, hundred squares can be found online at sites such as: www.sparklebox.co.uk

Examples of activities using mathematical representations: manipulative and images

Manipulatives in mathematics

A manipulative is something which can be physically moved. A mathematical manipulative can be physically handled by a child in order to support mathematical understanding of a concept.

Numicon

Numicon is a resource which is designed to support children's understanding of number structure. Each shape (or tile) represents a particular number from 1 to 10. In order for numicon to be used effectively, children should be encouraged to recognise and visualise the shapes for each number rather than needing to count the holes.

Getting to know numicon tiles: Typical activities for early Foundation Phase

How many 1s?

You will need: a basket of 1 tiles, a baseboard and a set of numicon tiles

Children take a handful of 1 tiles and find out how many there are by counting, they should arrange into the the numicon pattern on the baseboard and then check by finding the equivalent numicon tile

Count and check

You will need: numicon pegs, a numicon number track a baseboard and a set of numicon tiles

Children take a handful of pegs and find out how many there are by matching each along a number track. The pegs should then be moved and arranged on the baseboard into the corresponding numicon pattern. Children should check with the numicon shape by placing the correct shape on top of the pegs.

Hidden Tiles

You will need: a baseboard, a basket of mixed tiles, a feely bag

Children take turns to feel a tile that they cannot see inside the feely bag. They should try to select the matching tile from the basket and then check.

Ordering the numicon tiles

You will need: 1-10 tiles

Arrange the 1-10 tiles in order. Ask children to close their eyes and swap the positions of two tiles. Ask children to identify the tiles that have been swapped and move them back to their correct positions. As a variation, you could ask the children to select the correct tile to fill a gap from a random selection.

Using numicon to introduce mathematical concepts: Typical activities for early Foundation Phase

One more

You will need: numicon tiles, paper

Start by asking the children to arrange the 1-10 numicon tiles in order. Begin with a 1 tile and add a a 1 tile, matching it to the 2 tile. Children to record 1 + 1 = 2.

Then take a 2 tile and add a 1 tile, matching it to a 3 tile. Children to record 2 = 1 = 3.

Continue recording for tiles up to 10.

You could use a pan balance to show the tiles equal each other.

One less

You will need: numicon tiles, numeral cards 0-10

Cover one hole of a 10 tile so that the 9 pattern can be seen. Ask children 'What can you see?'. When children have answered, model the language of subtraction by saying 'Yes, ten take away one equals nine'. Children can check with the 9 tile.

Then have the 1-10 tiles arranged in order. Ask children to pick a numeral card from a shuffled pile and to identify that number. The children must then identify the tile that is one less than that number.

Addition

You will need: a basket of tiles, a spinner labelled 1-10

Children take it in turns to spin a spinner and read out the number. The children should make that number using two or more tiles and say what has been made.

You could use a pan balance to show that the tiles equal each other.

Doubles

You will need: a basket of tiles

Choose the 8 tile and invite the children to find out how many combinations of two tiles make 8. When they have all been found, then suggest keeping the 4 and 4 on top of the 8 tile and clear the rest away.

Ask how 10 could be made and when all the combinations have been made, suggest keeping the 5 and 5 on top of the 10 tile.

Do the same for 6, 4 and 2. Ask the children to describe what they say and introduce the word 'double'.

You could use a pan balance to show the doubles relationship.

Subtraction as take away

You will need: a basket of tiles

Choose the 7 tile and ask children to name it. Say 'we want to take 3 away from the 7 shape. We can pretend to take 3 away by hiding part of the 7 shape'. Show how the 3 hidden would result in 4. Model the mathematical language by saying 'Seven take away 3 equals 4'. Repeat and ask children to choose their own shape then 'take away'.

Subtraction as difference

You will need: a set of numicon tiles, numeral cards, feely bag containing 1-10 tiles

Together with the children, arrange the numicon tiles in order, placing a numeral card in order below them.

Pick up two shapes, putting the smaller shape on top of the larger one and ask children what difference they can see. You will need to draw the children's attention to the pattern. Use a numicon tile to show the difference and reinforce the language, for example ' The difference between two and three is five'.

Children can then take it in turns to take two different shapes from a feely bag and compare them, explaining what the difference is between them.

Using numicon to reinforce mathematical concepts: Typical activities for mid to late Foundation Phase

Inverse relationship between addition and subtraction

You will need: numicon tiles

Ask children to make facts for 11. Using two numicon tiles for each number. Children close their eyes. Hide a pair of tiles that total 11 behind your back. Now say 'I have 11 behind my back, so 11 take away 8 equals…, while showing the children the 8 tile'. When children have answered and explained their thinking, use the 3 tile to check. Record as 11-8=3. Now show the inverse by putting

the 8 and 3 together and the 3 and 8 together, recording as 8 + 3= 11 and 3 + 8 = 11. Then take apart, to show 11 – 3 = 8.

Children can then so this activity in pairs, taking it in turns for one child to close eyes whilst the other takes a pair of tiles totalling 11. The child with the tiles must then show one tile from the pair. The other child must then identify what the other tile is and together they could record the four facts.

Snap and Bond

You will need: numicon tiles

Give each child a numicon tile. Hold up a numicon tile. Children to look at the teacher's shape and their own shape – if the shape is the same they jump up and shout 'snap', if the two tiles would make 10 they shout 'bond'.

Introducing multiplication as repeated addition

You will need: numicon tiles

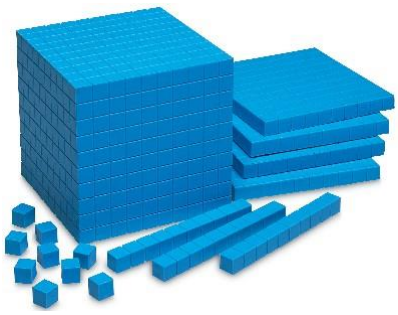
Ask child to get one 5 tile. Ask the same child to get another 5 tile. Then ask for another 5 tile. Ask how many times the child got a 5 tile. Model the language of multiplication ' I have three times a 5' or 'I have three 5s'. Repeat using different examples. Children must use the correct language and must understand that, structurally, three times a four is different to four times a three. At this point it is not important to focus on the answer to the calculation but more important to focus on what the language of 'times' means, relating to the physical action of getting another tile.

Introducing division as 'how many in …'

You will need: lots of 10 numicon tiles

Lay out the 10 tiles in a long line. Choose a number that is a multiple of 10 and identify where this would be along the line (e.g. 50). Ask children to identify 'how many tens in 50'? Get children to count the tens in 50. Show the children that 50 ÷ 10 can be read as 'how many tens in 50'. Reinforce using other examples.

Base 10 Blocks



Base 10 blocks (also known as Dienes blocks) are commonly used in late Foundation Phase and Key Stage 2 to support children's understanding of place value and the base 10 number system. Most of the activities below can be varied in terms of the size of number being considered.

Race to a number

You will need: base 10 blocks, place value mats, dice

Choose a number to 'race' to (e.g. 50).

Players take it in turns to roll a dice and take that number of ones. Once they have ten ones they must exchange them for a ten. As part of the player's turn, he/she must state what they have in total (e.g. I have one ten and one unit which is 11). The winner is the person closest to the target number at the end. This activity could be timed. You could use a number line or hundred square to support the players in seeing the target number.

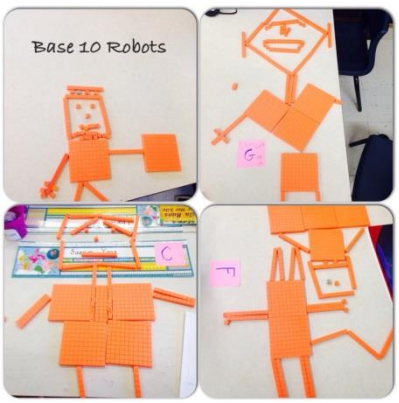
Variations: use a tens dice and a ones dice for larger target numbers, start at a large number and subtract to reach 0

Describe my number

You will need: base 10 blocks, a feely bag

Place a combination of base 10 apparatus in a feely bag. Children take it in turns to feel and describe what is in the bag in terms of 'tens' and 'units' (or hundreds depending on number). The rest of the children have to either say or write the number or could make the number using place value cards. Check each time, reinforcing the language of 'hundreds', 'tens' or 'ones'.

Place Value Pictures



You will need: base 10 blocks

Ask children to make a picture/recognisable object using their base 10 apparatus. What is the total? Alternatively the children could be given a number and asked to make a picture using base 10 apparatus in a way that represents that number.

Play a number

You will need: a drum, base 10 blocks

Use a very loud beat for hundreds, a medium sound beat for tens and a very quiet beat for ones. Play a number and ask children to represent it using base 10 apparatus.

Silent Bingo

You will need: base 10 blocks, blank bingo boards

Ask children to write some numbers (e.g. between 1 and 20) on a blank bingo board. Instead of calling out a number the teacher displays a number using base 10 apparatus.

Get to 100

You will need: a dice, base 10 blocks, place value mat (tens and ones)

Children take it in turns to roll the dice. When they have rolled they can choose to take that number of tens or ones. However they must not go over a hundred or they are out. The winner is the person who gets to exactly a hundred or is the nearest without being above a hundred.

Base 10 to represent decimal fractions

Use a hundred block to represent 1 whole. Demonstrate how this is divided into a hundred ones and then establish that the ten block can represent one tenth and the one block can represent one hundredth. Any of the above activities can be adapted to support children in understanding decimal fractions.

Multilink interlocking cubes

Multilink is a very adaptable manipulative that can used to support children's understanding of number, pattern, shape and size.

Examples of Foundation Phase multilink activities

Comparison

You will need: multilink cubes

Each pupil is to take a handful of cubes from a bag, closing their eyes as they do so. Count the cubes. How many do they have? How many of each colour? Who has the most? Who has the most yellow/green/blue etc?

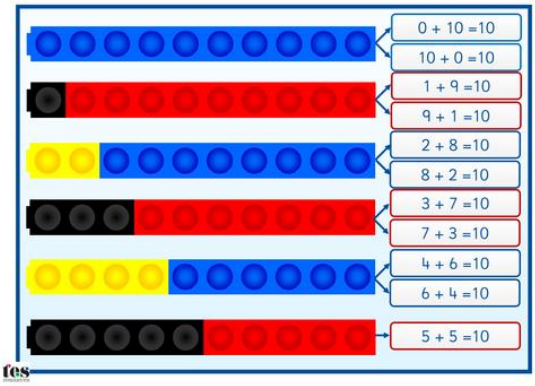
Build something

You will need: multilink cubes

Build something with a given number of cubes. Compare lengths/heights/colours.

Number Bonds

You will need: multilink cubes



Ask children to show the number bonds for ten using multilink (as above). Children could then be asked to hide the 5 + 5 stick behind their back and remove a section of it. The other children need to guess what has been removed.

Make a pattern

You will need: multilink cubes

Describe a pattern e.g. 1 red, 1 yellow, 1 blue and ask children to repeat this so that it becomes a sequence. Alternatively ask the children to make their own pattern for other children to copy.

Using cubes to measure

You will need: multilink cubes, pan balance

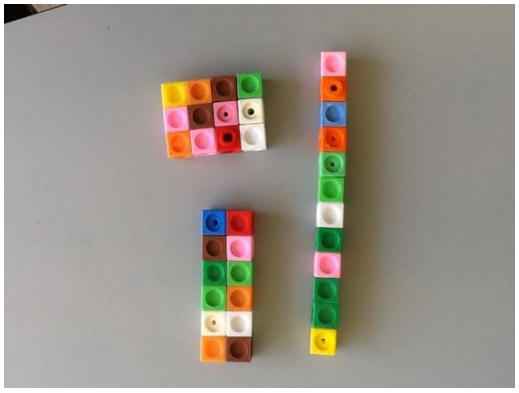
Use cubes as a non-standard unit of measure. For example: how many cubes high is the robot? How many cubes weigh the same as the teddy?

Key Stage 2

Arrays

You will need: multilink cubes

Give the children enough cubes and ask them to make rectangular arrays to represent 12 as below:



4 x 3 =12, 3 x 4 = 12

6 x 2 = 12, 2 x 6 = 12

1 x 12 = 12, 12 x 1 = 12

The children should then be encouraged to relate this to multiplication and division facts. Investigate arrays for different numbers.

Fraction sticks

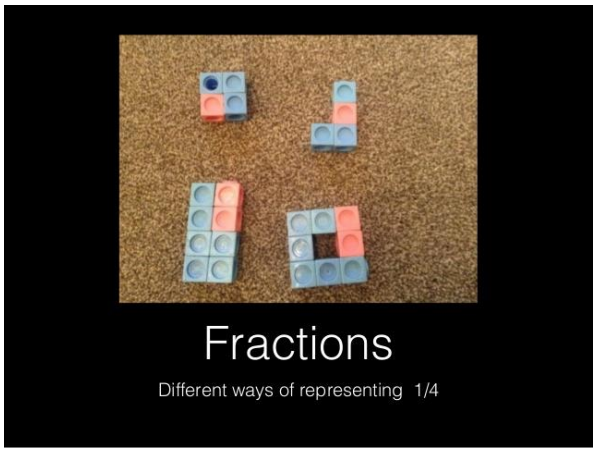
You will need: multilink cubes

Make a 'fraction stick' with a number of cubes and two colours (e.g. 5 cubes, 2 colours). Demonstrate how this fraction stick can show fractions (e.g. a stick with 5 cubes, 2 red and 3 green could show 2/5 of the stick is red and 3/5 of the stick is green). Children to make their own fraction sticks. You could specify what fractions the stick is to show or let them choose.

Equivalent fractions

You will need: multilink cubes

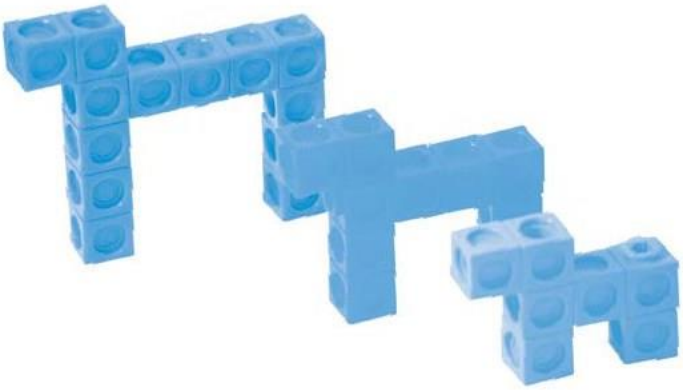
Explore different ways of showing a particular fraction using multilink cubes. This might start with 4 cubes then progress onto different numbers of cubes (e.g. 8, 16).



Making dogs – to develop algebraic thinking

You will need: multilink cubes – it may help to have these arranged in colours

Encourage children to make dogs as in the picture below:

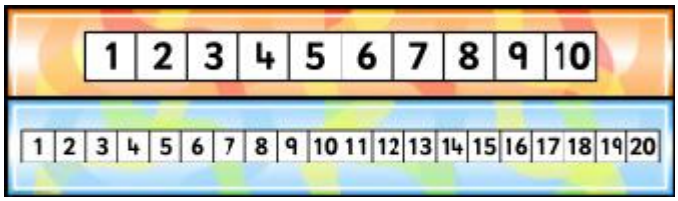


Ask the children to explain the patterns they see. For example how do the head/body/legs change as the dog family grows? Can they predict what the tenth dog would look like? What about the 100th dog?

Images in mathematics

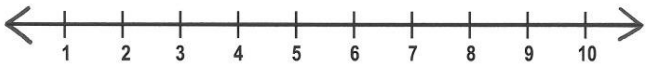
A mathematical image is a particular representation used to support mathematical understanding. Mathematical images related to number usually involve particular ways of organising number to emphasize particular features. Images can be physical in nature but are designed to encourage the user to be able to visualise number arrangements in a particular way.

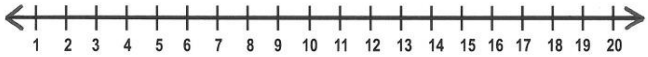
<u>Number tracks and number lines in the early Foundation Phase</u>



A number track can be found on many board games. Typically a number track has the number placed in the middle of squares or blocks and can serve as an introduction to a number line.

A number line is any line with numbers placed at intervals along it. Number lines can be used to represent positive and negative numbers, fractional and decimal numbers.





One key thing to remember is that children must understand that they will be counting the intervals (or jumps) on a number line or track and not the actual markings. Therefore lots of experience of jumping or hopping along number lines and tracks is necessary in early Foundation Phase. This point should be reinforced and modelled by the teacher.

Number recognition using a number track

You will need: number tracks 1-6, dice, a bowl of counters

The children take turns to roll a spot dice, count the spots if necessary and then place that number of counters on the matching numeral on the number track. The winner is the first person to match every number on their track. As the game progresses, ask the children which numbers they still need and how many spaces they have left. Extend by using a 1 to 9 track and two dice (1 to 6 and 1 to 3) and allowing the children to choose whether to use one or both dice during each turn.

Spot snap

You will need: cards with dots (as on a dice) displaying numbers 1 – 6, number tracks 1-6

Use the spot cards and number tracks to practice instant recognition of spot patterns. Each child needs a track and a matching set of spot cards. Shuffle the cards and turn them over one at a time and match with the appropriate numeral. When the children are familiar with the activity, they could race against a one minute timer. Who can match all the cards before the sand runs out? Who is the fastest? Reinforce recognition by chalking or sticking dot patterns on the walls and floor outside. The children will invent their own games, but you can also give the children fun challenges such as 'Run to number 3 then number 5,' 'Hop to number 2,' Stand by number 4 and clap four times'.

Race to a number

You will need: giant number line or number track, giant spinner

Split the children into two teams. Each team takes it in turns to spin the spinner and race to a number (e.g. 10/20). The winning team is the first to get to that number.

Incey Wincy Spider

You will need: a track made to look like a drainpipe, a spinner or dice, a spinner labelled with rain or sunshine, spider counters

This activity links to the rhyme 'Incey Wincey Spider'. The aim of the game is to climb up the water spout without the rain causing you to slide back down. Each player places their spider counter at the start and takes it in turn to spin or roll a number and then spin the weather spinner. If the spinner displays sunshine then the player must move that number of spaces up the spout but if the spinner shows rain then the player must move that number of spaces back down the spout. The winner is the first person to reach the end.

Jigsaw

You will need: a cut up number track or number line

Challenge the children to put the number track together again. You could also blank out some numbers and ask the children to identify which numbers are missing.

Secret steps

You will need: number cards, a large number line for the playground or hall

Children take it in turns to pick a card without the other children seeing. They should whisper the number to the teacher but not tell the children. The child should then take that number of steps along the number line while the others count the steps. The observers hold up their fingers to show how many steps were taken. This could be adapted to start at different numbers to develop the skill of counting on.

Guess

You will need: a jar with objects, post it notes, a number line or track

Children should be shown a jar of objects and asked to estimate how many objects are in the jar. Their estimate should be placed on a post it note along the number line with lots of discussion about relative size. Empty the jar and encourage the children to count the objects with you. Children could then play in pairs with a number track or line, an empty plastic bag and cubes or other objects. One child closes their eyes whilst the other counts some cubes into the bag and rings the number on the track or line and then turns the line or track upside down. The partner then has to estimate how many in the bag and compare with the actual ringed number.

Number lines for late Foundation Phase and Key Stage 2

Round it

You will need: 0-100 number lines, two 0-9 dice or spinners, two pens in different colours

Model with the children how two digits could be written to make a number (e.g. if a 3 and 6 are spun that could make either 36 or 63). Model where these numbers are placed on the number line, drawing attention to the nearest multiple of 10, including the convention of rounding up for a number ending in 5. Children can then play in pairs; each player takes it in turn to spin or roll two digits which he/she must arrange to make a two digit number number. That number must then be rounded to the nearest multiple of ten – the player circles and initials that multiple of 10 on the number line. Only two initials can be written next to any multiple of 10 on the number line (this could be the same or different initials but each multiple of 10 can only be used twice). If a player rounds to a multiple of 10 that already has two initials next to it he/she misses a turn. The winner is the first person to write his/her initials six times.

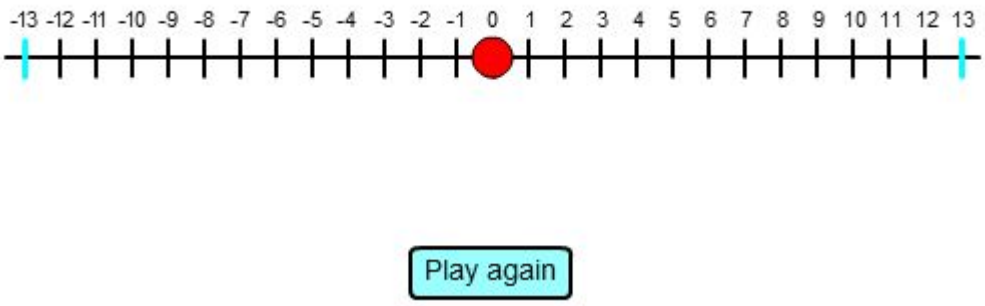
Thumbs up/thumbs down

You will need: spinner or random number generator, number line

Choose a number to jump in steps of, each time you will be starting from 0. Randomly generate a number. Children must show thumbs up if they think you will land on that number or thumbs down if they think you will not land on that number. Model the jumping to get to the answer. To work independently the children could be given a set of numbers and could investigate what jumps they would take to land on that number.

Tug of war

You will need: a tug of war board line (below), two dice, one counter



On player is positive and goes from left to right, the other is negative and goes from right to left. The players take it in turns to throw two dice which should be added. The player then moves the counter that number in his/her direction. If the counter reaches -13 negative wins and if the counter reaches 13 positive wins.

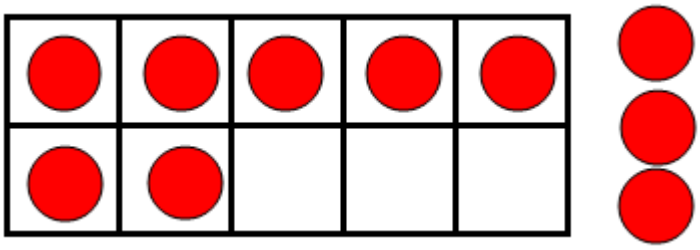
Battleship number lines

You will need: number lines marked appropriately between 0 and 1, a 'battleship' counter

Players each have a number line and choose a position (either fraction or decimal fraction) to hide their battleship from their partner. The partner has to guess a position on the number line where

the battleship may be hidden. The partner responds with either 'hit' or 'miss'. The winner is the first to hit 5 battleships.

Ten frame



A ten frame is an image which can support children's understanding of number structure and place value.

Ten frame flash

You will need*:* A dozen ten-frames with dot arrangements on them, a blank ten-frame for each child, counters.

One child shows a ten-frame for a count of three, then hides it while the other children place counters in the same positions on their frames from memory. The 'flasher' shows the card again and helps each child check his/her display. After three cards the next child becomes the 'flasher' and so on, until everyone has had a turn. Points can be awarded for each correct response. The child with the most points wins.

Twenty

You will need: Blank ten-frames (2 per child), counters, dice

Each child takes a turn to roll a die, places that number of counters onto his/her ten-frames, then announces the total number of counters on the frames. The winner is the first player to fill all twenty spaces. This could be varied by including placing the correct numeral cards under the frames or asking each player to announce the number of counters needed to reach twenty. The exact number must be rolled to win the game.

Guess What

You will need*:* Blank ten-frames, counters, large hard-cover books to form a barrier between pairs of children.

One player secretly arranges some counters on a ten-frame. The other player asks questions that can be answered yes or no, trying to gain enough clues to work out and replicate the arrangement of counters. For example: Is the top row full? Are there 8 counters? Is there an empty box in the bottom row? As players become more skilled, the number of questions can be counted. The player asking fewer questions wins.

The hundred square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The hundred square is a visual representation of the numbers from 1 to 100 which can support understanding of properties of numbers. Typically it is arranged as above but other arrangements can be used. It can help children to see that a hundred square and a number track are simply different arrangements of the same thing.

Jigsaw

You will need: cut up into pieces of a hundred square, cut along lines not through numbers

Children to arrange the pieces into the hundred square.

Missing numbers

You will need: a hundred square with only some numbers showing (this could be on the Interactive Whiteboard).

Children have to identify and explain how they worked out a particular missing number.

Four in a row

You will need: three 0-9 dice or spinners, hundred square, counters

Throw three 0 – 9 dice. Use the numbers and any operation to make one of the numbers on the hundred square. If all the players agree that the calculation is correct, cover the number with one of your counters. The aim is to get four of your counters in a row.

Squares

You will need: a hundred square

Draw a 2 by 2 square on the hundred square, for example:

3	4
13	14

Add the numbers in opposite corners.
What do you notice? Is it the same for different 2 by 2 squares?
Now multiply the numbers in opposite corners. What do you notice this time? Is it always true?

Crosses

You will need: a hundred square

Draw a 3 by 3 cross on the hundred square, for example:

	4	
13	14	15
	24	

Add the four numbers around the edge. Can you see a relationship to the middle number? Repeat for other crosses. Explain what you have noticed to a friend.

Race to 100

You will need: a hundred square, counters, a dice or spinner labelled with: +1, -1, +10, -10, +10, miss a turn

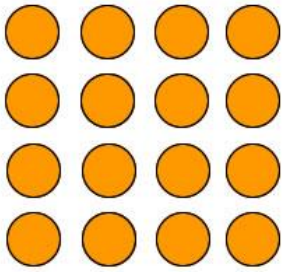
Each player starts at 1 and takes it in turn to roll or spin. The player takes the move by adding 1, subtracting 1, adding 10 etc. You could vary the game by adding further multiples of 10 to the options on the dice or spinner. Another option is to use +9, -9 and/or +11, -11 to encourage children to use adding or subtracting 10 and then adjusting.

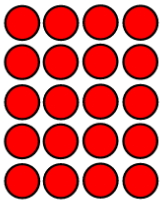
Multiples

You will need: a hundred square, coloured pencils

Investigate the different patterns made on the hundred square by multiples of different numbers (e.g. 2, 3, 4, 5).

Arrays



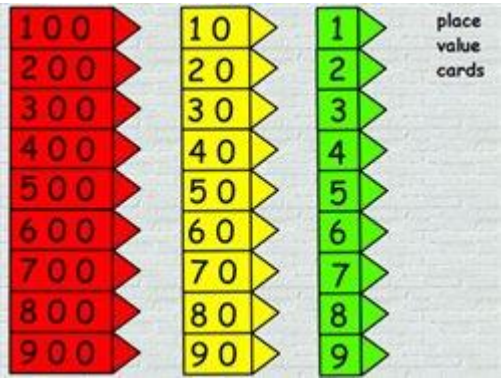


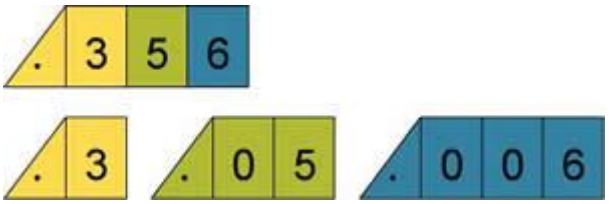
Arrays are arrangements of objects in a rectangle or square to visually represent repeated addition, multiplication, repeated subtraction and division.

How many arrays?

Investigate how many arrays can be made for different numbers. For example 9 can have two arrays (1 x 9 or 9 x 1 or 3 x 3 – a square array). Children chould be encouraged to consider the number facts that the number array illustrates.

Place value cards





Place value cards are very useful in helping children understand the value of digits in numbers. They can be used alongside base 10 blocks to support children's understanding of place value.

Make a number

You will need: place value cards for each child

Call out numbers asking children to make these using their place value cards. Reinforce the value of the digits by asking children 'What is the value of __ in your number'?

Guess my number

You will need: place value cards for each child

Children take it in turns to make a number which is hidden from view. Other children take it in turns to ask questions about the value of the digits.

Biggest and smallest

You will need: a 0-9 dice or spinner, place value cards

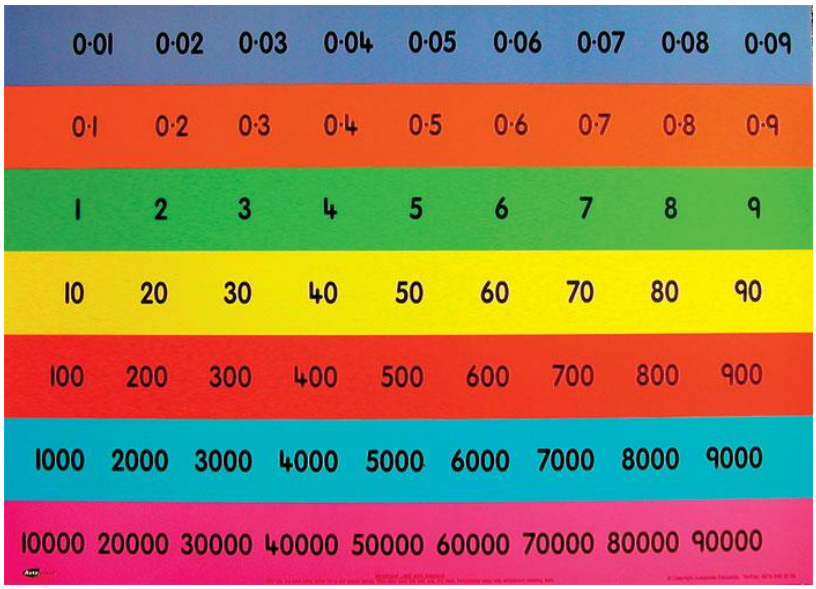
Generate three digits using the dice or spinner and ask children to identify and make the different numbers that could be made using those digits. They should then order the digits, justifying their choices.

100/10/1 more or less

You will need: place value cards

Ask the children to make a number using place value cards. Ask them to add 100 or 10 or 1 to that number and ask them to change the relevant plave value card. You could ask children to start with a number of their choice and then ask the other children the work out what number they must have started with.

Place value charts



Place value charts (or Gattegno charts) can support understanding of place value. They are particularly useful when used alongside place value cards and/or base 10 apparatus. They are also useful for reinforcing the relationship between multiplying and dividing by digits by powers of ten.

Write the number

You will need: a place value chart (this could be on the IWB)

Circle a number on each row and ask children to write the number that will result from adding these. Reinforce the value of the digits.

Journey to a number

You will need: a place value chart (this could be on IWB)

Reinforce how numbers change along rows (e.g. counting on or back in steps of 1, 10, 100, 0.1, 0.01 etc) and along columns (multiplying or dividing by 10). Then circle two numbers on the chart and ask children the journey from one number to the other number by travelling vertically or horizontally along the Gattegno chart. For example 400 to 5 could be 400 ÷ 10 then 40 ÷ 10 to get to 4 then 4 + 1 to get to 5, or it could be 400 + 100 then 500 ÷ 10 then 50 ÷ 10. Children could be asked to investigate different ways of getting from one number to another number.