

Exercise 1:

A 6-sided die is rolled 60 times and the following frequencies are recorded:

Roll	1	2	3	4	5	6	Total
Observed frequency	8	8	7	20	9	8	60
Expected frequency	10	10	10	10	10	10	60

Determine whether or not the die is fair

1. State your hypotheses
2. Select α , your significance level
3. Calculate χ^2
4. Find χ^2_{crit}
5. Decide whether or not to reject the null hypothesis

Chi-squared test

$$\chi^2 = \sum \left[\frac{(O_i - E_i)^2}{E_i} \right]$$

Degrees of freedom $df = N - 1$

H_0 : The expected frequencies are the same as the observed frequencies, i.e. the die is fair.

H_1 : The expected frequencies are different to the observed frequencies, i.e. the die is unfair.

$$\alpha = 0.05$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} \\ &= \frac{(8-10)^2}{10} + \frac{(8-10)^2}{10} + \frac{(7-10)^2}{10} + \frac{(20-10)^2}{10} + \frac{(9-10)^2}{10} + \frac{(8-10)^2}{10} \\ &= \frac{(-2)^2}{10} + \frac{(-2)^2}{10} + \frac{(-3)^2}{10} + \frac{(10)^2}{10} + \frac{(-1)^2}{10} + \frac{(-2)^2}{10} \\ &= \frac{4}{10} + \frac{4}{10} + \frac{9}{10} + \frac{100}{10} + \frac{1}{10} + \frac{4}{10} = \frac{122}{10} = 12.2 \end{aligned}$$

Degrees of freedom = number of categories - 1 = 6 - 1 = 5

Look up χ^2_{crit} for $\alpha = 0.05$ and $df = 5 \rightarrow \chi^2_{crit} = 11.07$

$\chi^2 > \chi^2_{crit}$ so we can reject the null hypothesis

Exercise 2

A researcher investigates which networking hardware a group of companies use and gets the following results

Hardware	Netstuff	Ciscomp	Total
Observed f	26	49	75
Expected f	37.5	37.5	75

Determine whether or not each company is likely to have an equal market share

1. State your hypotheses
2. Select α , your significance level
3. Calculate χ^2
4. Find χ^2_{crit}
5. Decide whether or not to reject the null hypothesis

H_0 : The market share of each company is the same

H_1 : The market share of each company is different

$\alpha = 0.05$ Use Yates's correction as there are only two categories

$$\chi^2 = \sum \frac{(O_i - E_i - 0.5)^2}{E_i}$$

$$= \frac{(26 - 37.5 - 0.5)^2}{37.5} + \frac{(49 - 37.5 - 0.5)^2}{37.5}$$

$$= \frac{(-11.5 - 0.5)^2}{37.5} + \frac{(11.5 - 0.5)^2}{37.5} = \frac{(11.5 - 0.5)^2}{37.5} + \frac{(11.5 - 0.5)^2}{37.5}$$

$$\chi^2 = \frac{11^2}{37.5} + \frac{11^2}{37.5} = \frac{121 + 121}{37.5} = \frac{242}{37.5} = 6.45$$

$$\text{Degrees of freedom} = k - 1 = 2 - 1$$

Looking up χ^2_{crit} for df = 1 and $\alpha = 0.05$, $\chi^2_{crit} = 3.84$

$\chi^2 > \chi^2_{crit}$ so the null hypothesis can be rejected.

If Ciscamp has a market share twice that of Netstuff, are these results to be expected?

Hardware	Netstuff	Ciscamp	Total
Observed f	26	49	75
Expected f	25	50	75

1. State your hypotheses
2. Select α , your significance level
3. Calculate χ^2
4. Find χ^2_{crit}
5. Decide whether or not to reject the null hypothesis

H_0 : The results are as expected for a 2:1 market share

H_1 : The results are not as expected for a 2:1 market share

$\alpha = 0.05$ Use Yate's correction as there are only two categories

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(26 - 25)^2}{25} + \frac{(49 - 50)^2}{50}$$

$$= \frac{(1 - 0.5)^2}{25} + \frac{(1 - 0.5)^2}{50} = \frac{0.5^2}{25} + \frac{0.5^2}{50} = \frac{0.25}{25} + \frac{0.25}{50}$$

$$\chi^2 = 0.01 + 0.005 = 0.015$$

Degrees of freedom $df = k - 1 = 1$ and $\alpha = 0.05$ $\chi^2_{crit} = 3.84$

$\chi^2 < \chi^2_{crit}$ so we cannot reject the null hypothesis

Exercise 3

20 patients were split into two groups at random. 10 were given medicine M and the other were given placebo P. The extent to which they said their symptoms improved was noted below

Smaller Effect M P P P M P P P M M M P M P P P M M M P M Larger Effect

Determine if the medicine M had a significant effect

1. State your hypotheses
2. Select α , your significance level
3. Calculate U
4. Find U_{crit}
5. Decide whether or not to reject the null hypothesis

H_0 : The order of effect is due to chance

H_1 : One treatment is more effective than the other

$$\alpha = 0.05$$

For each M, starting from the larger effect, count the number of Ps in front of it

$$U_M = 10 + 9 + 9 + 9 + 7 + 6 + 6 + 6 + 3 + 0 = 65$$

$$N_M = 10$$

For each P, starting with the largest effect, count the number of Ms in front of it

$$U_P = 9 + 6 + 6 + 5 + 2 + 2 + 2 + 1 + 1 + 1 = 35$$

$$N_P = 10$$

$$U = 65 \text{ (larger of } U_M, U_P \text{)}$$

$$N_1 = 10$$

$$N_2 = 10$$

Looking up on the table, $U_{crit} = 23$

$U > U_{crit}$ so we can reject the null hypothesis