

Advanced Physics – Shaped Charge FEA

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Figure 1 – Cross-section of a Shaped Charge Munition [1]

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List of Acronyms

ALE	Arbitrary Lagrangian-Eulerian
CAD	Computer Aided Design
CAE	Computer Aided Engineering
EOS	Equation of State
FEA	Finite Element Analysis
HEAT	High-Explosive Anti-Tank
JC	Johnson-Cook
JWL	Jones Wilkins Lee
OFHC	Oxygen-Free High Conductivity
SCJ	Shaped Charge Jet
SPH	Smooth Particle Hydrodynamics
TNT	Trinitrotoluene

Scope of assignment

This assignment aims to perform Finite Element Analysis (FEA) of 2 shaped charge designs to evaluate the impact of the cone liner angle on performance. This includes a review on core FEA principles and equations which drive the workings of the software used on the test cases. The assignment covers a literature review of shaped charges and the process of FEA simulation to include influence of boundary conditions and meshing on the efficiency and accuracy of FEA solves. Both 2D and 3D solves will be conducted and an analysis on the performance of the software and used settings will be concluded with recommendations based on the findings.

1 Introduction

1.1 Introduction to Finite Element Analysis

Finite Element Analysis (FEA) is a process of simulating stimuli applied to a digital model of a part or prototype. Within this assignment, Dassault ABAQUS will be used as the Computer Aided Design (CAD) and FEA packages. FEA work by constructing a mathematical representation of the part through the conversion of its geometry into elements consisting of nodes.

1.2 Introduction to Shaped Charges

Shaped charges are explosive devices designed to explosively drive a liner into a high velocity projectile or penetrating jet. Whereas typical explosive devices rely on omnidirectional blast, a shaped charge uses a directed blast to collapse a high ductility liner, typically OHFC Copper, using the detonation wave in the direction of a target. This principle was initially described by Birkhoff [2] who produced a hydrodynamic theory that the liner behaves as an incompressible fluid under the extreme pressure generated by the detonation.

The formation process of the SCJ begins with the detonation being initiated at the tip of the liner cone, which generates a detonation wave which progresses along the liner. The detonation wavefront then pushes the liner, generating extreme pressure which exceeds the yield strength of the liner material. This then transforms the material into a hydrodynamic state and the liner rapidly collapses inward in the direction of the target [3].

The liner converges and a portion of the material forms a thin jet travelling at an extreme velocity and the remainder forms a larger, slower slug of material.

2. Key equations within FEA

This section contains a technical review of critical equations and concepts used within FEA as well as specific equations used within the FEA of explosives.

2.1 Johnson-Cook

The Johnson-Cook constitutive model was derived in 1985 by Gordon R Johnson and William H Cook. This model is a constitutive flow-stress model which expresses stress as a function of strain, strain rate, and temperature [4].

The mathematical expression of the model is $\sigma = (A + B\epsilon^n) (1 + C \ln(\dot{\epsilon}^*)) (1 - T^m)$ where [4]:

- σ : Flow stress.
- ϵ : Equivalent plastic strain.
- $\dot{\epsilon}^* = \dot{\epsilon} / \dot{\epsilon}_0$ is the dimensionless plastic strain rate normalized by a reference strain rate.
- $(A + B\epsilon^n)$: Represents strain hardening where A is the yield stress, and B and n are the hardening modulus and exponent.
- $(1 + C \ln \dot{\epsilon})$: captures strain rate effects through constant C.
- $(1 - T^m)$: Accounts for thermal softening, where $T^* = (T - T_r)/(T_m - T_r)$ is the temperature normalized between reference temperature T_r and melting temperature T_m , with m as the thermal softening exponent.

Constants A, B, n, C and m are determined through a set of sequential calibration tests using experimental data. This approach entails isolating the contribution to separate the effects of strain, strain rate, and temperature on flow stress.

The Johnson-Cook damage model calculates the equivalent plastic failure strain as a product of three multiplicative terms. This model assumes the effects of strain rate, temperature and the stress triaxiality affect material ductility are decoupled.

The equation for the model is $\epsilon_f = [D_1 + D_2 \exp(D_3 \sigma^*)] [1 + D_4 \ln(\dot{\epsilon}^*)] [1 + D_5 T^*]$ where [4]:

- σ^* : Stress triaxiality ratio, this is mean stress divided by the von Mises stress.
- $\dot{\epsilon}^*$: $\dot{\epsilon} / \dot{\epsilon}_0$ is the dimensionless plastic strain rate normalized by a reference strain rate.
- T^* : Homologous temperature.

The Johnson-Cook damage model uses calculated D values for a material which are determined through experimental data.

2.2 Equation of State – Mie-Grüneisen

The Mie-Grüneisen equation of state (EOS) model is used within the modelling of high-pressure physics, like that of a shaped charge. The model details the thermodynamic behaviour of solid materials under a compressive shock load. This model has the core assumption that the pressure is not linearly dependent on the density but is related to the specific inertial energy of relative to a known reference curve [5].

The equation for the model is $P(\eta, E) = P_R(\eta) + \alpha[E - E_R(\eta)]$ where:

- η : Volumetric strain
- P : Pressure
- E : Specific internal energy
- R : Reference curve value#

The reference curve for use in this model is derived from the shock Hugoniot. The Hugoniot conservation equations are used to determine the final shocked state of a material when influenced by an intense shock wave [6]. The shock Hugoniot in use with the Mie- Grüneisen EOS is experimentally measured and plots the pressure response of the material under load.

2.3 Jones-Wilkins-Lee

The Jones-Wilkins-Lee is another EOS model used within FEA which describes the hydrodynamic pressure produced by the detonation products of an explosive during the expansion phase. It provides the critical driving force to produce the shaped charge jet through liner collapse. The model deals with the pressure at each integration point of an element as a function of its volume whilst also considering the energy density [7].

The equation for the model is:

$$P_S(v) = A \exp\left(-R_1 \frac{v}{v_0}\right) + B \exp\left(-R_2 \frac{v}{v_0}\right) + C \left(\frac{v}{v_0}\right)^{-(\omega+1)}$$

Figure 1 - Jones-Wilkins-Lee equation [8]

Where:

- A : Dominant coefficient, pressure at the Chapman-Jouget (CJ) detonation state. This is the initial pressure behind the detonation front.
- B : Secondary pressure coefficient. Used to fine tune the pressure-volume curve to ensure it still fits where the influence of the A coefficient starts to fade.
- R_1 : Decay rate constant 1, this value decays with the expansion of the products. It is used to provide the sharp pressure drop after the first expansion of the products.
- R_2 : Decay rate constant 2, this value is used to govern the longer-term pressure decay found after the initial expansion.

These constants and coefficients are determined using the explosive cylinder test and fitting the gathered results to the equation.

3 Literature Review

3.1 Lagrangian Modelling

Lagrangian FEA modelling uses meshes consisting of elements and nodes to model the geometry of the parts within the solve. Each node within the mesh represents a specific point within a predetermined material and distorts along with stress/strain on the part. This approach is used as a default within FEA due to its effectiveness in solid mechanical problems [9].

Within this style, the mass within each element remains constant throughout the solve of the simulation and the solver calculates the motion of the nodes through applying Newton's second law of motion. The displacement and velocity of each node is calculated from the internal forces caused by stresses to the elements, external forces from loads and set boundary conditions. The mesh within these models tracks the geometry of the parts allowing for a clear visual representation of the parts during the result. The Johnson-Cook model is also history dependent and Lagrangian elements naturally retain this history.

The main limitation of using a Lagrangian model is the severe mesh distortion. Within a shaped charge, the liner experiences an extreme amount of displacement and distortion which leads elements to extreme aspect ratios. The recommended maximum aspect ratio for Lagrangian models is 5:1 however the shaped charge jet distortion is multiple times this recommendation. This will cause the stable time increment to tend to zero potentially causing crashing. For 3D SCJ simulations, Lagrangian models should be paired with other FEA methods to overcome this limitation. Element deletion within the model will improve the stability and runtime of the simulation however it will reduce the accuracy of the result due to missing smaller fragments within and around the jet.

3.2 Smooth Particle Hydrodynamics Modelling

Smooth Particle Hydrodynamics (SPH) is a meshless Lagrangian computational method that is used within explicit FEA to simulate extreme deformation of materials. This method converts the geometry into spherical particles which each represent a material volume which has a set mass, position, velocity and state [10] using kernel approximation. This derives the location of a particle using a weighted summation over neighbouring particles within a smoothing length. Within this mode, the particles move physically and their position is updated by integrating their velocities. The advantage of this is that the particle modelling is able to handle extreme deformation where element aspect ratios would normally become an issue. Element deletion is not present within SPH either which further increases the fidelity of the results by showing spall and other small elements.

The main limitation of SPH is the tensile instability. The particles will form clusters unnaturally or produce voids within the material during tensile stress. This is due to the interaction between the kernel's second derivative and equation of state under negative pressure [11]. This discovered interaction demonstrated an exponential growth of the gaps between particles during penetration.

SPH is also more computationally expensive than the equivalent mesh-based methods due to the neighbour searching when solving each particle. This means all particles within the smoothing length must be identified before any calculation is done. This limits the accuracy of the solution due to the requirement of larger particles where time constraints and limited resource apply. SPH also suffers from a lack of convergence which can influence the validation of the modelling which is shown by the model failing the patch test [12]. This is where the first-order consistency of a model is tested using irregular particle distributions proving that it is sensitive to particle distribution.

3.3 Eulerian Modelling

Eulerian modelling relies on a fixed computational grid in a space where the material is able to flow through the stationary elements of that grid. This is fundamentally different to the Lagrangian method where the mesh represents the part. The difference between these methods makes Eulerian modelling well suited to solves which experience extreme deformation. A review by Benson described the Eulerian hydrocodes showing that each element represents a set volume which the simulation runs through [13].

Within a Eulerian solve, there are two phases within each of the time increments. The first step is to perform a standard Lagrangian solve with the mesh temporarily moving with the material. The conservation equations update the internal energy and the velocity of the nodes due to gradients in pressure and stress. The deformed mesh is then removed and the solve is matched back to the original undistorted Eulerian grid. This method prevents element tangling seen in pure Lagrangian solves with high element deformation.

Within a Eulerian solve, a SCJ can naturally form from the OFHC liner and stretch to extreme length without the typical crashing of the software or errors. This is especially beneficial where there are multiple materials within the simulation in the same area since the materials are represented as a portion of that space. The drawback of a Eulerian approach is the loss of the precise material interface [13].

3.4 Modern Adaptive Meshing

Adaptive meshing, or Arbitrary Lagrangian-Eulerian (ALE), is a hybrid FEA model which is designed to mesh the benefits of both styles of solver. The aim in this hybrid is to combine the boundary-tracking of the Lagrangian model with the distortion handling ability of the Eulerian model. The main drawback of the Lagrangian model is failure due to excessive deformation of the elements within the mesh. The ALE model solves this issue by allowing the independent movement of the mesh from the material. This allows for increased mesh quality whilst maintaining the history of the material.

4. Main Test Case

This section covers the critical evaluation and assessment of qualitative and quantitative data generated by an FEA simulation, the setup of the solution and the generation of the models for the parts.

4.1. Variables of Interest

The variables of interest within this test case include:

- Qualitative assessment on liner formation.
- Velocity of liner tip.
- S22 shear stress.
- Maximum pressure.
- Maximum principal stress.

4.2. Methodology

4.2.1. Generation of models

Before a simulation can be ran on a part, one must first be created using the built-in CAD package. First the approximate sizing of the part is determined and a build plane is provided. The separate parts were individually modelled using each other as reference geometry to produce models for the liner, case and explosive fill.

4.2.2. ABAQUS PARAMETERS

In order to apply materials to the sets/meshes, the necessary parameters/information must be provided within the material database. This information will consist of values achieved through the testing of the materials or via verifiable research papers. In the case of this assignment, the material information has been collected from reputable online resources and peer-reviewed journals.

Once the material information has been collected, sets must be created and the materials applied to them. This will allow for the meshed parts to be assigned the material set that is appropriate for that part, i.e. OFHC copper for the liner.

Material Properties					
Density kg/m ³	Y. Mod. (GPa)	ν	γ_0	C (m/s)	S
2700	68.9	0.33	1	5350	1.4
Johnson-Cook Plastic Parameters					
A (MPa)	B (MPa)	η	C	m	T _{melt} (°K)
324	114	0.42	0.002	1.499	652
Johnson-Cook Damage Parameters					
D1	D2	D3	D4	D5	
-0.77	1.45	-0.47	0.00	1.60	

Table 1 - FEA Properties for 6061-T6 Aluminium [14] [15] [16]

Material Properties					
Density kg/m ³	S. Mod. (GPa)	ν	γ_0	C (m/s)	S
8930	46	0.343	1	5240	1.4
Johnson-Cook Plastic Parameters					
A (MPa)	B (MPa)	η	C	m	T _{melt} (°K)
90	292	0.31	0.025	1.09	1360
Johnson-Cook Damage Parameters					
D1	D2	D3	D4	D5	
0.54	4.89	-3.03	0.014	1.12	

Table 2 - OFHC Copper Johnson-Cook Material Information [17]

Material Properties					
Density kg/m³		Pre Det (GPa)		Det. E Density	
1630		9		3680000	
Jones-Wilkins-Lee Parameters					
A (GPa)	B (GPa)	R1	R2	ω	VOD (m/s)
371.21	3.23	4.15	0.95	0.30	6930

Table 3 - TNT JWL Properties [15]

The use of reputable, peer-reviewed sources for information is vital when trying to maintain accuracy when generating a simulated event. This is due to small changes in the material properties having a compounding effect on the overall result of the test, potentially providing a very different picture during interpretation. Especially with metals, using information for the correct grade/treatment of the material is also critical to ensure the output can converge towards its real-life equivalent.

The analysis of the shaped charge designs will begin with a qualitative assessment on the SCJ formation as well as the quality of the displayed material deformation. A review of the velocity, pressure, displacement and max principal strain of each design will also be compared.

4.2.3. Simulation of 45-degree OFHC Liner in 2D

The simulation of the 45-degree liner charge resulted in a successful generation of the SCJ through the use of a symmetry boundary condition along the Y axis, reducing the necessary solve time as well as increasing the stability of the simulation avoiding crashing. A symmetry boundary condition was placed on the Y axis to reduce computational cost; a similar reaction is expected on the other half of the model due to the symmetric nature of the charge.

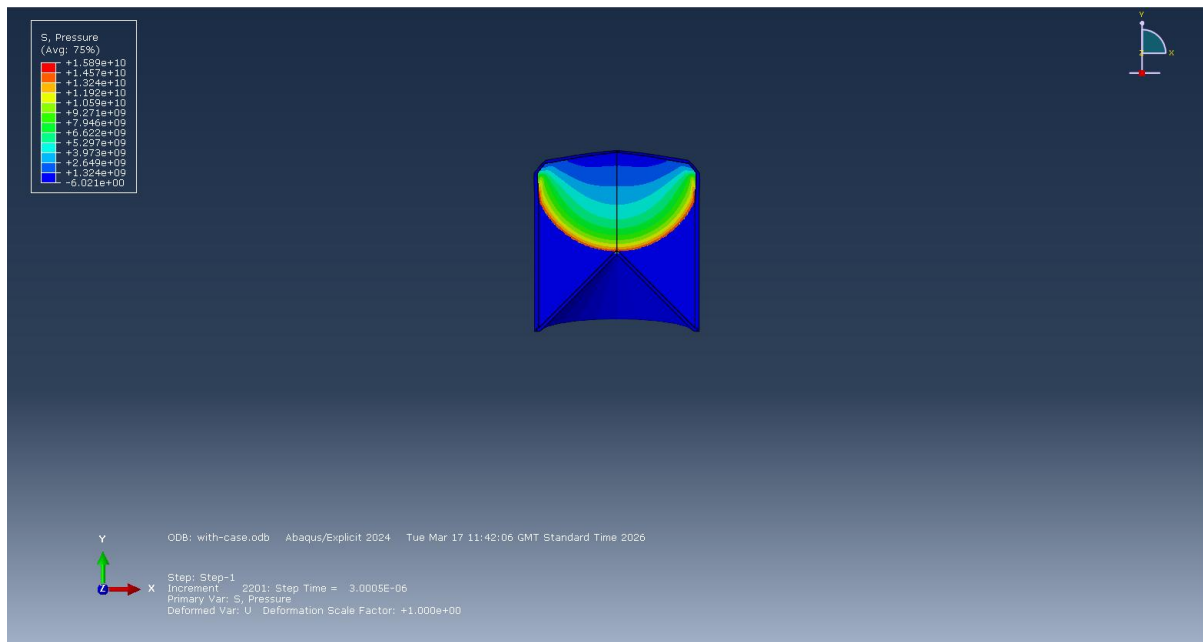


Figure 2 - 45-degree hits liner, pressure

The progression of the explosive wavefront throughout the charge is typical of similar charges. A zone of high pressure at the edge of the wave followed with a lowering gradient, arcing from the point of initiation. The shock front has hit the apex of the liner and a buildup of pressure at that interface can be observed. The peak shock front pressure observed is 15.9 GPa.

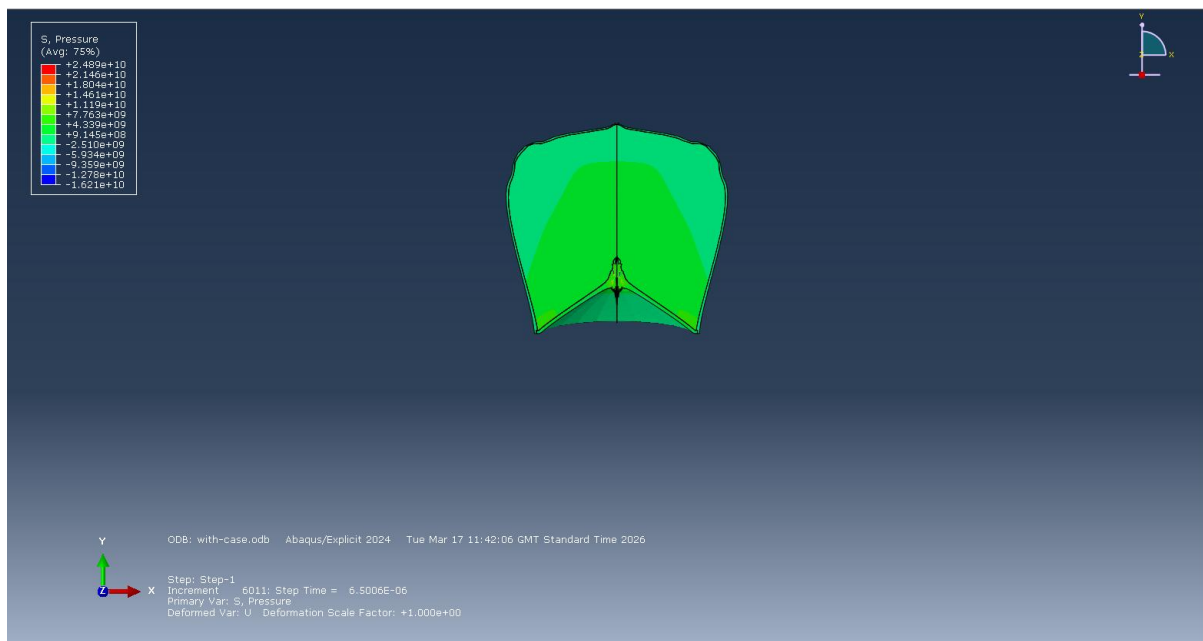


Figure 2 - 45-degree liner start of SCJ formation

The start of the SCJ tip can be seen where the explosive shock front has reached the apex of the conical liner within the charge. A large amount of deformation is already present in the case suggesting the thickness was not appropriate for the quantity of explosive used within this example.

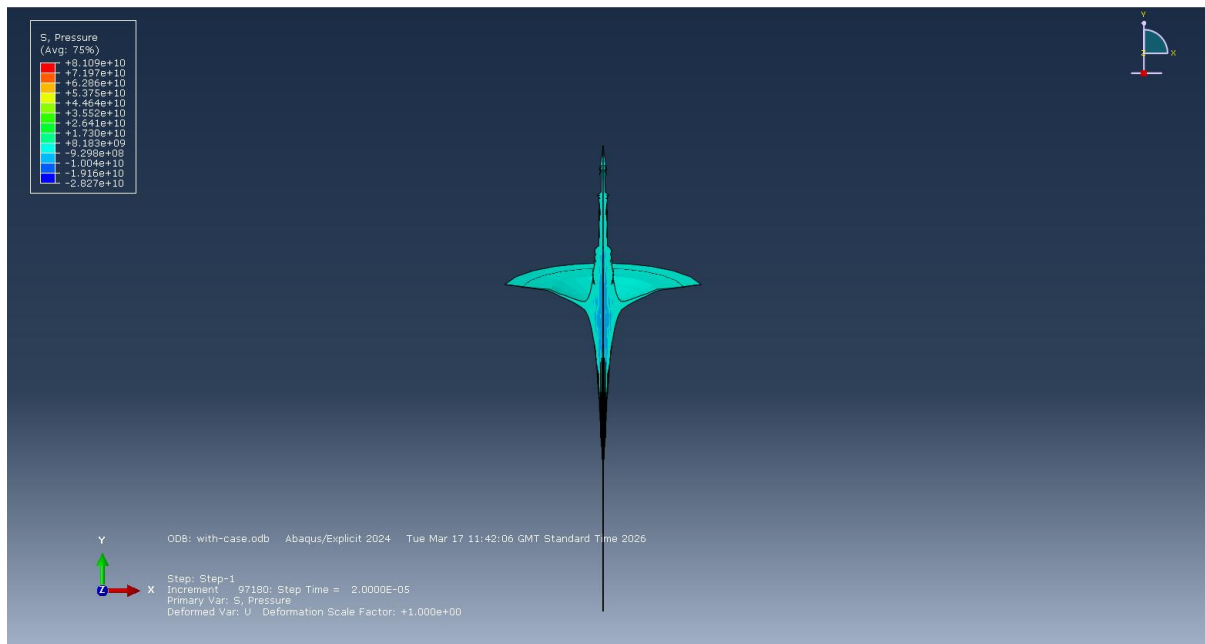


Figure 3 - 45-degree liner completed SCJ formation

The shape of the SCJ is visually consistent with successful formations of SCJ's. The liner has successfully deformed and stretched the material in the -Y axis. The signature wider slug is still in formation at this time step with the expectation that with a longer time step, the wider part of the liner would also fold towards the central Y axis line. The jet is continuous and the distribution of the nodes displays a smooth gradient from the tip to the main slug.

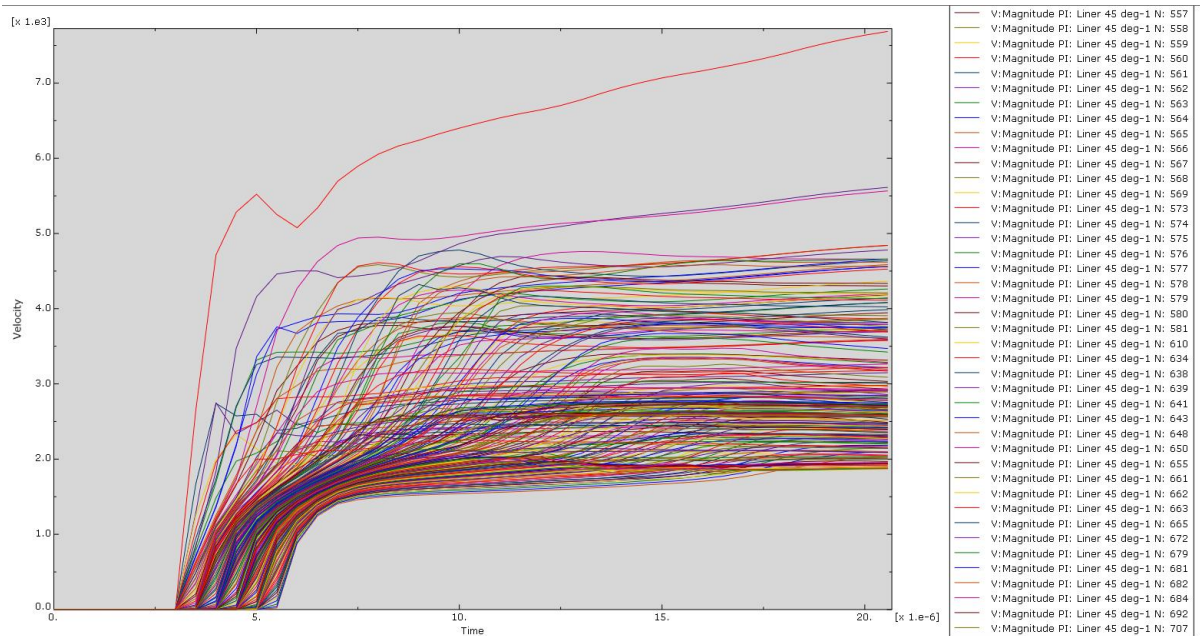


Figure 4 - 45-degree liner, velocity

The velocity of all the nodes of the SCJ which had significant deformation in the -Y direction. The tip of the jet was still accelerating at the end of the runtime for the simulation and had reached approximately 7700m/s. The continued acceleration of the tip suggests the driving force of the explosive is still ongoing and additional performance may be seen with the longer

simulation time. This would include the steady state penetration velocity that is required for a practical performance assessment. However, the velocity gradient of the SCJ is linear with a fast tip and a slower following mass.

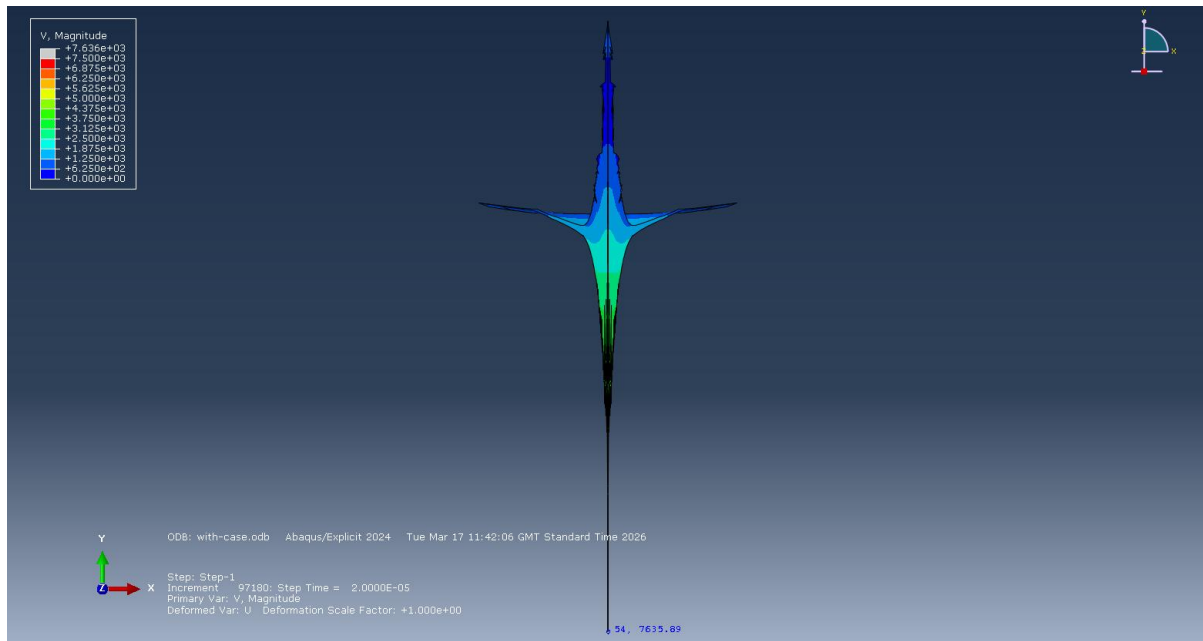


Figure 5 - 45-degree liner, maximum velocity

The SCJ tip node was probed at the end of the runtime and displayed a velocity of 7635 m/s. The main jet can be seen to have a velocity gradient from its tip to the main body of material. No breakup of the jet is visible in this simulation.

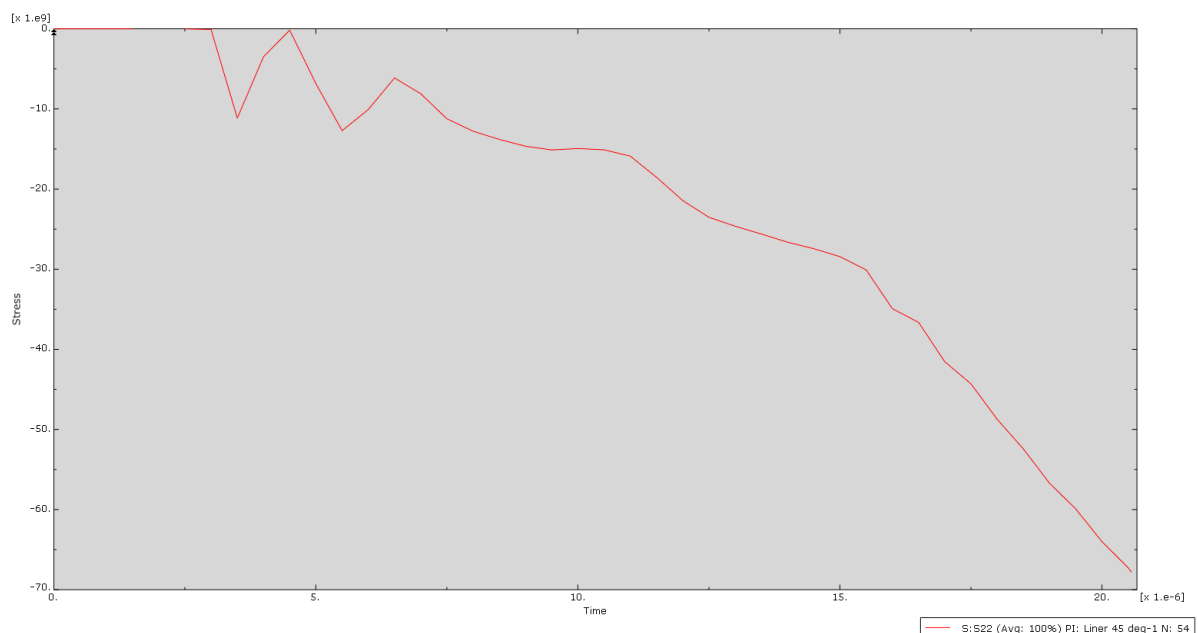


Figure 6 - 45-degree liner, S22 stress

One of the key performance metrics of an SCJ tip is the S22 stress experienced over time. In the first 5 μ s it can be observed that the tip is starting to be influenced by the shock front. The increasing rate of strain is observable from 5 μ s to 15 μ s whereby the main SCJ is completing

its formation. The time after 15 μ s is the continued deformation of the jet tip which is approaching a point of being fully formed.

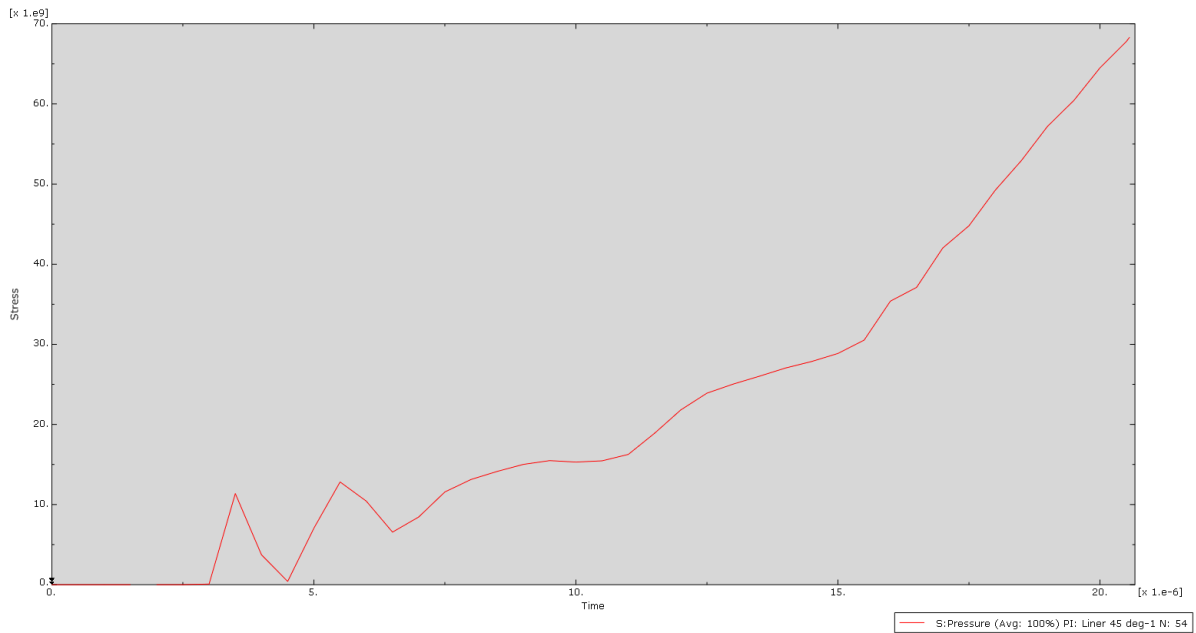


Figure 7 - 45-degree liner, pressure

The driving pressure of the hydrostatic formation of the SCJ from the detonation wave. The S22 stress in Figure 7 is the direct result of this driving pressure with a noticeable period of the liners initial deformation either side of 5 μ s.

4.2.4. Simulation of 60-degree OFHC Liner

The OFHC liner was adjusted along with the explosive fill to maintain the same overall size of charge but with a 60-degree liner angle. The assumption would be that the jet would travel slower with a flatter angle but with a thicker SCJ.

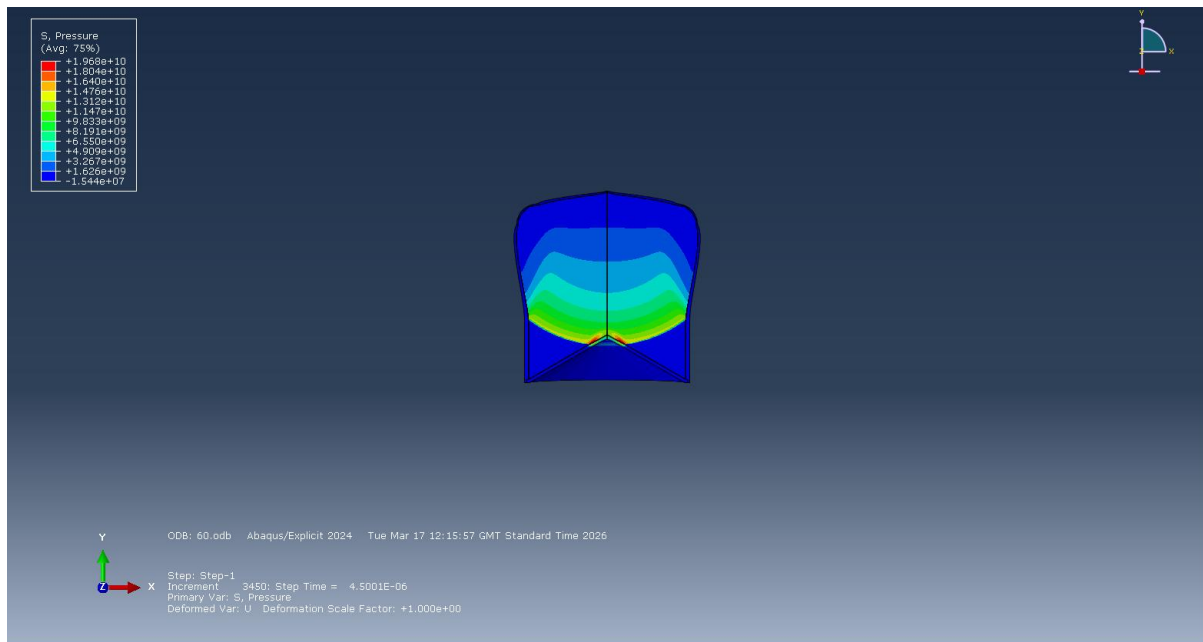


Figure 8 - 60-degree hits liner, pressure

Like the 45-degree liner, the explosive produced the expected arcing shock front. The shock front is less visible than the 45-degree example however the run up required to hit the liner from the detonation point is longer in this design.

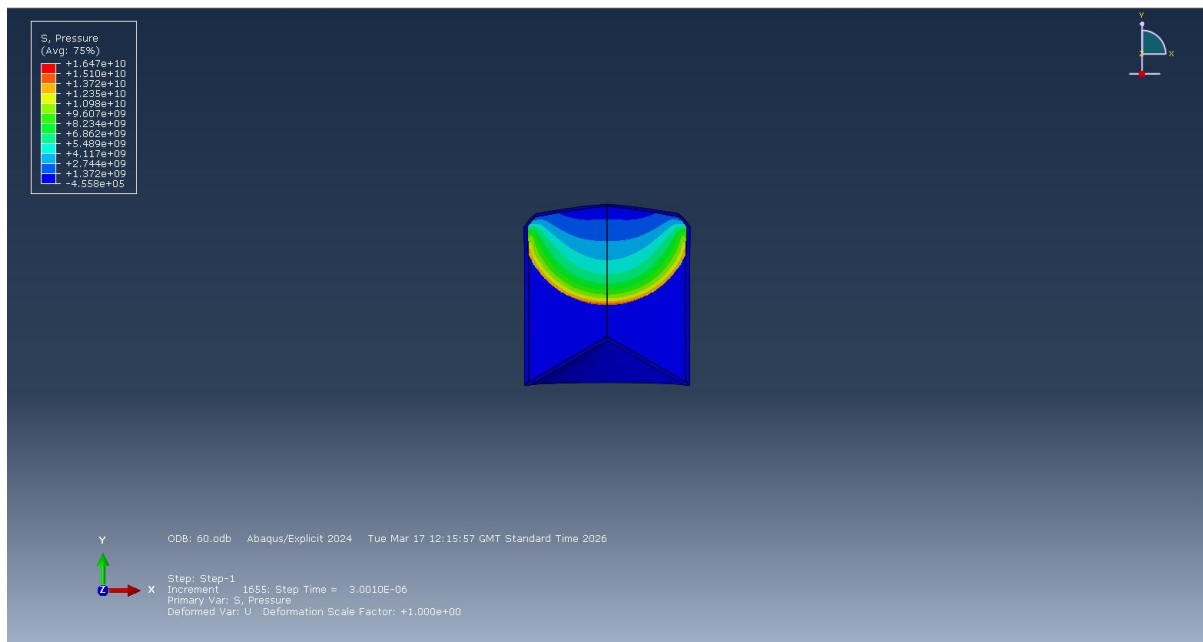


Figure 9 - 60-degree at 45-degree liner impact time, pressure

Figure 10 shows the pressure wave on the 60-degree simulation at the same time step in the 45-degree simulation that hit the liner. The shock front is more noticeable and matches the behaviour of the other design. The peak shock front pressure observed is 16.5 GPa, an increase of 0.6GPa from the other designs simulation, resulting in a difference of 3.8%.

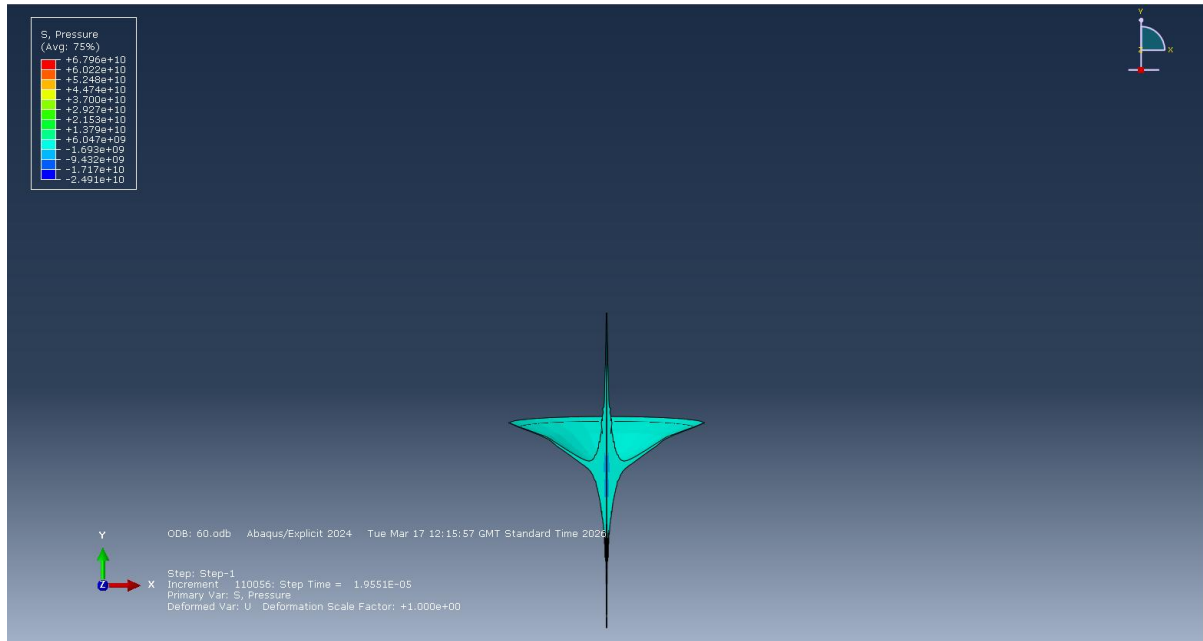


Figure 10 - 60-degree liner formation starts, pressure

Figure 11 shows the formation of the 60-degree SCJ. It is clear in this figure that the SCJ is significantly shorter than the 45-degree SCJ at this time step. This is likely due to the increased run up time and delay in the formation of the SCJ.

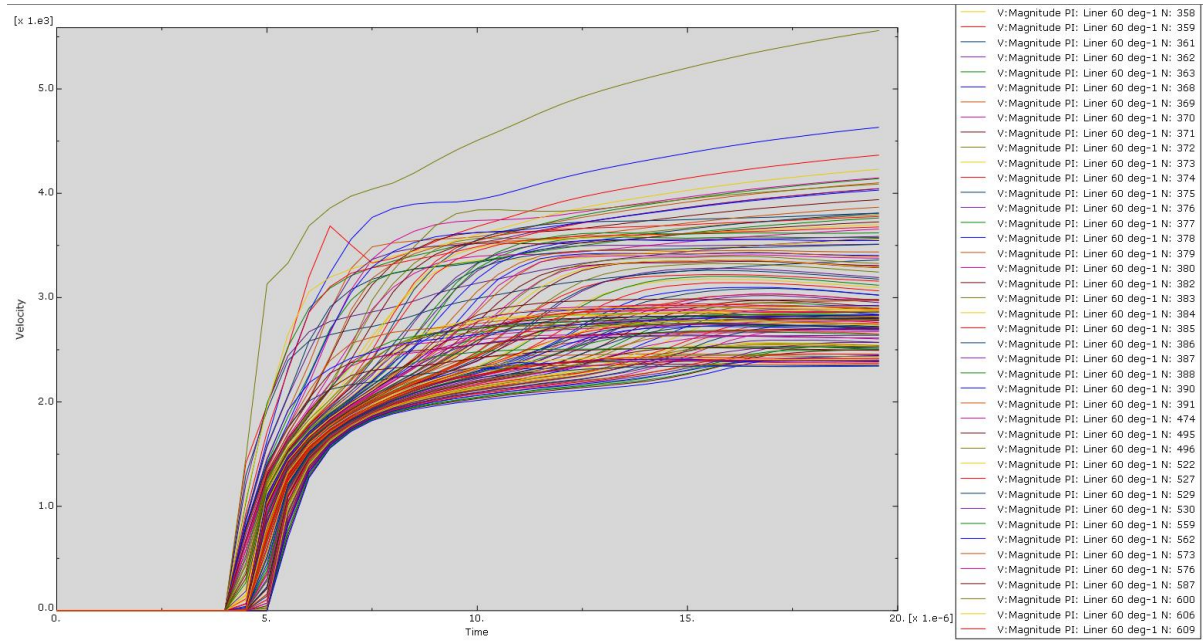


Figure 11 - 60-degree liner, velocity

As seen in Figure 12, the velocity at this time step for the SCJ tip is significantly lower than the charge with the 45-degree liner. There is still a smooth gradient of velocities within the liner showing a successful SCJ formation.

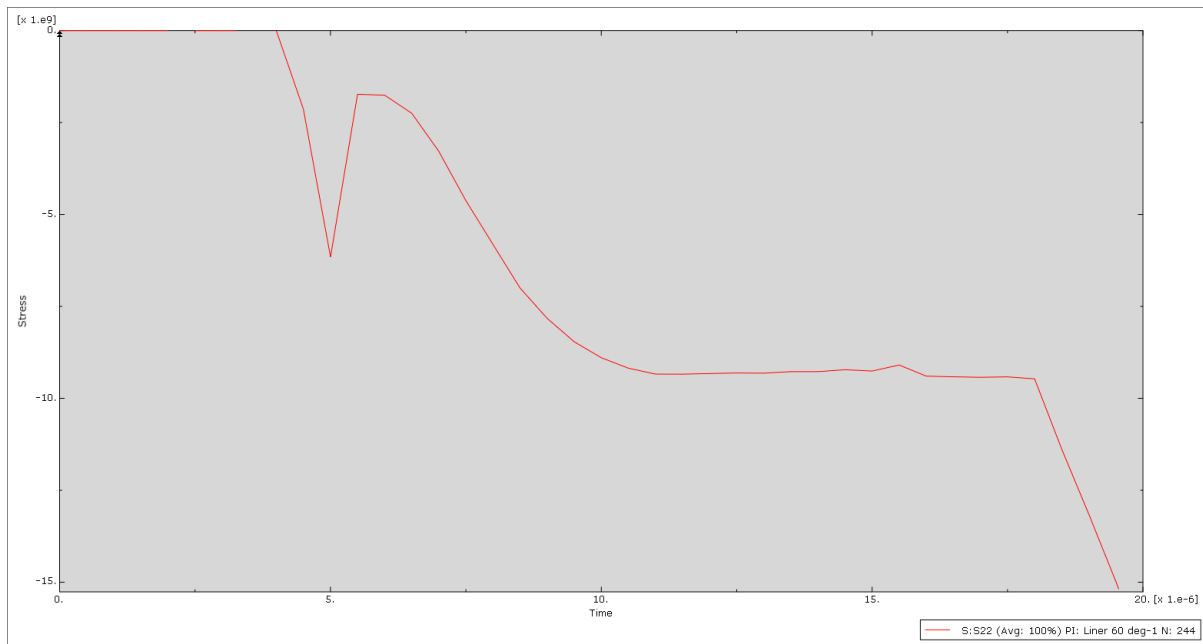


Figure 12 - 60-degree liner, S22

Figure 13 shows the typical S22 stress fluctuation around 5us where the formation of the SCJ begins. However, in this model it is much lower than the 45-degree liner suggesting the SCJ is stretching less aggressively and will result in a thicker, slower stream. This will lower its penetration depth on armour but would increase its effectiveness against lighter armoured targets.

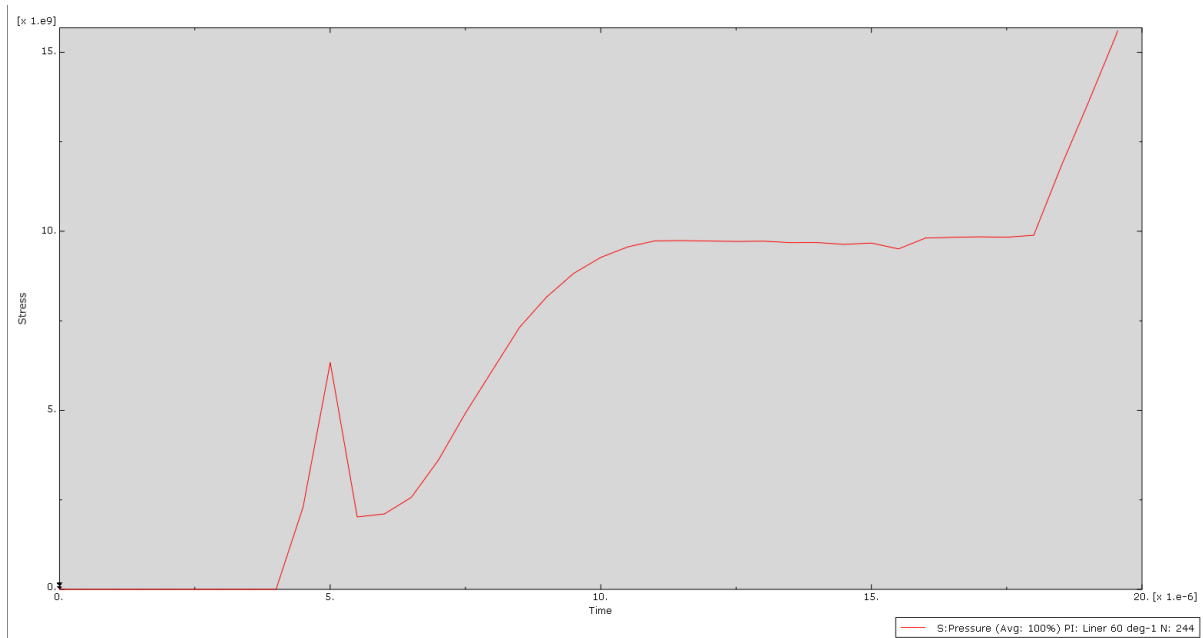


Figure 13 - 60-degree liner, pressure

The liner for the 60-degree design, the deformation of the liner takes approximately 5us.

4.2.5. Analysis of liner performance

In the 2D simulations, a clear difference in performance can be seen. The 45-degree liner is seen to produce a higher velocity than the 60-degree liner, ~36% increase in peak velocity. The final penetrating velocity was not able to be recorded due to the computational requirements for a 40 μ s total step time. A paper discussing FEA simulation of a shaped charge, using similar materials and explosive, achieved an SCJ formation velocity of 7189 m/s [18]. This is within 6.2% of the 45-degree simulated charge within this report.

The assumptions made within the Lagrangian model will have affected the results of the 2D simulations. Within this model, materials are considered completely homogenous, which for the liner has less impact than the case but will have affected the accuracy due to surface hardening of manufactured parts/cases. The materials are also considered isotropic; the materials have the same properties in all directions. One of the main limitations of the used model is the element method of representing the interactions. Where an element experiences extreme distortion, it can cause the simulation to fail 'excessive distortion'. With the high-speed nature of the shaped charge deformation, excessive element deformation is typical within these simulations.

One consideration for FEA is the density of the mesh. This is the number of elements the model has within its volume. A higher mesh density will increase the accuracy up to a point of convergence. This is due to the smaller elements better representing the deformation. A journal article published to Materials (Basel) discussed the influence of mesh density on the results obtained by FEA [19]. Within this journal, the solution accuracy improved with the introduction of a finer mesh. The tested range was approximately 1000 – 100,429 elements, increasing the mesh density with each iteration per example. The deviation of the solution compared to real life testing tended to zero towards the 100,000-element example across all of the simulations. The largest deviation showed up to a 29.4% deviation in deformation shown, giving example as to the dangers of a coarse mesh. When refining the mesh, there are two types of refinement. H-Refinement is the process of reducing the element size. This is useful where there are stress gradients, complexity of the geometry and zones of contact [20]. P-Refinement is where the mesh style is changed to one with more nodes per element. The use case of P-Refinement is where there is a lower mesh density or an excess in processing capacity for the solution. Figure 15 below shows the difference between a coarse and a refined mesh.

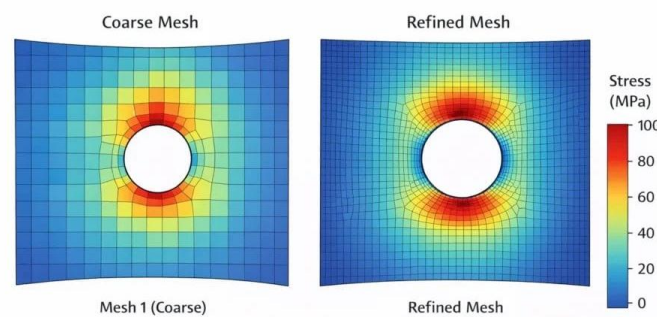


Figure 14 - Comparison of a coarse and refined mesh on a plate [20]

4.2.6. Simulation of 3D Shaped Charge

The 3D simulation introduced significant issues with the deformation of the mesh. To fix this problem, SPH modelling was introduced to the simulation at the point where it crashed with the pure Lagrangian model.

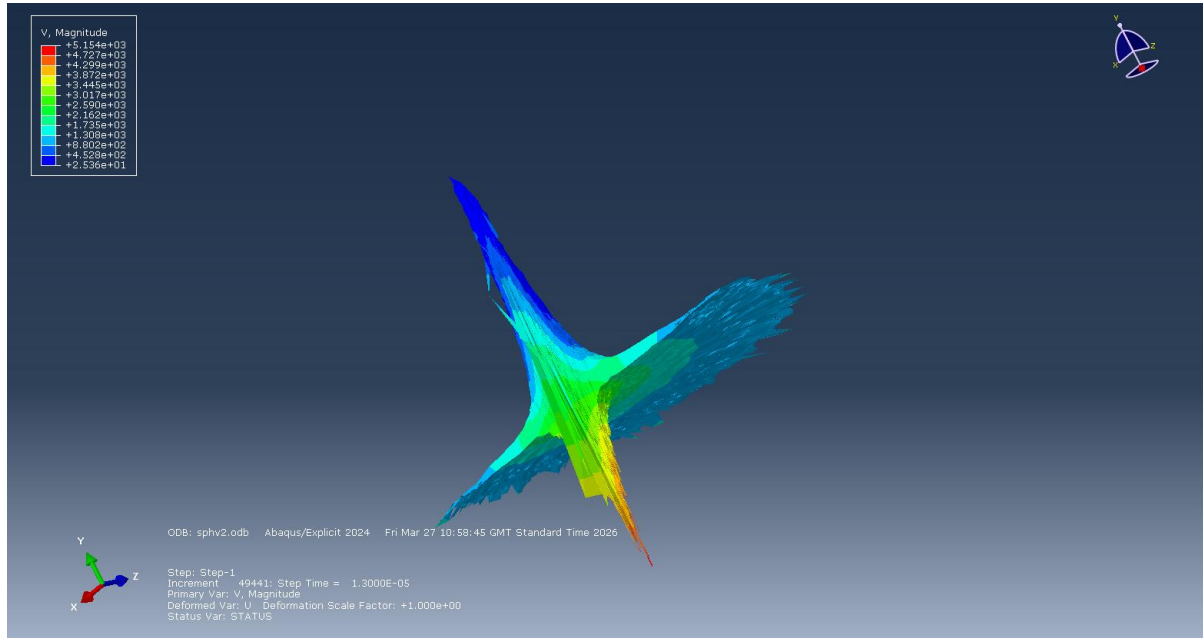


Figure 15 - 3D generation of 45-degree Shaped Charge

The simulation for the 3D SCJ managed to solve for 13us of runtime.

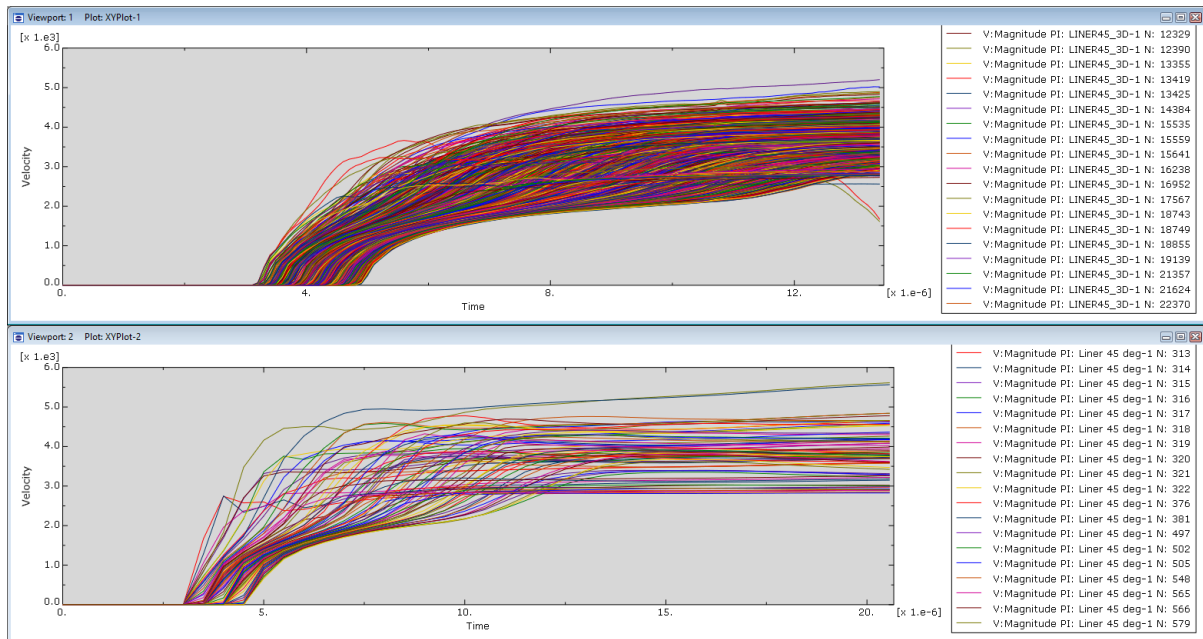


Figure 16 - Comparison of 2D and 3D 45-degree liner, velocity

The velocity of the 2D and 3D simulations follow the same trend in velocity which validates the approximation of this simulation. The key features of the plot flow in the graphs are also

identical, with the initial curve with the acceleration of the SCJ tip into a more stable, flatlined velocity curve. Deviations in the flow occur around 5 μ s however this could be due to more elements being present in the 3D model, overlapping the more noticeable deviations in the 2D graph.

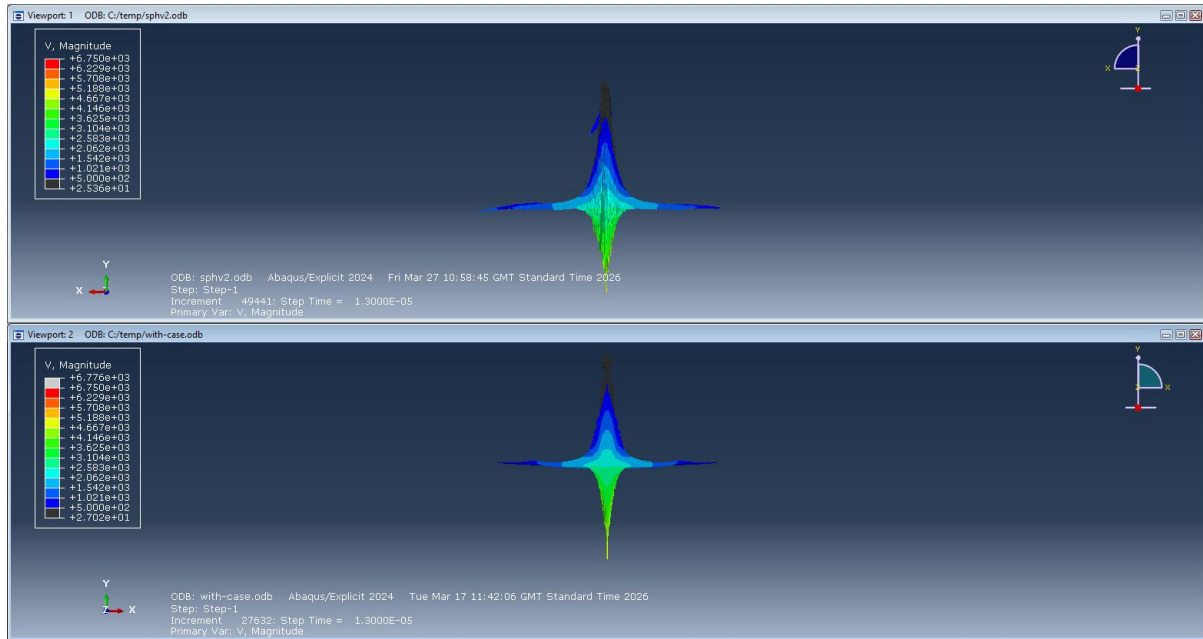


Figure 17 - Direct comparison of 2D and 3D 45-degree liner simulation, velocity

The results of the 2D and 3D simulations were captured at the final timestep the 3D simulation was able to achieve. Both of the velocity mapped solutions share a similar gradient of colouring with the maximum tip velocity of the 2D being 6780 m/s and the 3D being 5180 m/s. This is a 30% difference in the velocity of the SCJ tip. The qualitative similarity in jet shape and the trend in velocity validates the 2D approximation for comparison. However, the quantitative difference in results show that alternate methods like ALE should be explored for further validation of the simulation models. The SCJ tip in the 3D example consists of SPH particles instead of distorted mesh which may have introduced some of the variance since Lagrangian methods overpredict ductility of materials.

5. Technical Conclusions

Within the run simulations, the 45-degree design had increased SCJ tip velocity by 36%. This is a significant increase in velocity from the 60-degree design and is critical to the penetration potential of the charge. The results gained from the 3D experiment whilst beneficial as an initial indicator, would not be appropriate to base a design from. This is because the simulation hardware was unable to run the simulation to an appropriate timestep where the SCJ had completely formed without unnatural mesh deformation. The lack of respect to the boundary conditions placed on the 3D model suggest SPH was not the appropriate method to represent this scenario.

To increase the benefits seen by the simulation, a longer total runtime should be used in the next simulation to visualise the full formation of the SCJ and to determine a peak velocity in flight. The runtimes used in the current simulation are useful to infer future deformation and velocity but are not definite. An experimentation with the ALE method to benefit from a more accurate simulation of the hydrodynamic cone formation process would likely provide increased fidelity of the geometry without crashing the simulation. Whilst the SPH approach prevented this crash from occurring, the result showed an unnaturally deformed jet which also did not respect the boundary conditions provided by the simulation.

The design could also be improved by increasing the thickness of the casing, significant deformation and breakaway of the casing was observed meaning some of the energy is being lost instead of reflected back into the driving of the SCJ. Further experimentation with the effects of liner thickness and apex radii would also provide important information to increase the performance of the product. This could be accompanied with a penetration test whereby a plate of rolled homogeneous armour (RHA) steel of a known thickness is struck by the resulting SCJ's providing another critical measure of performance for that application.

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