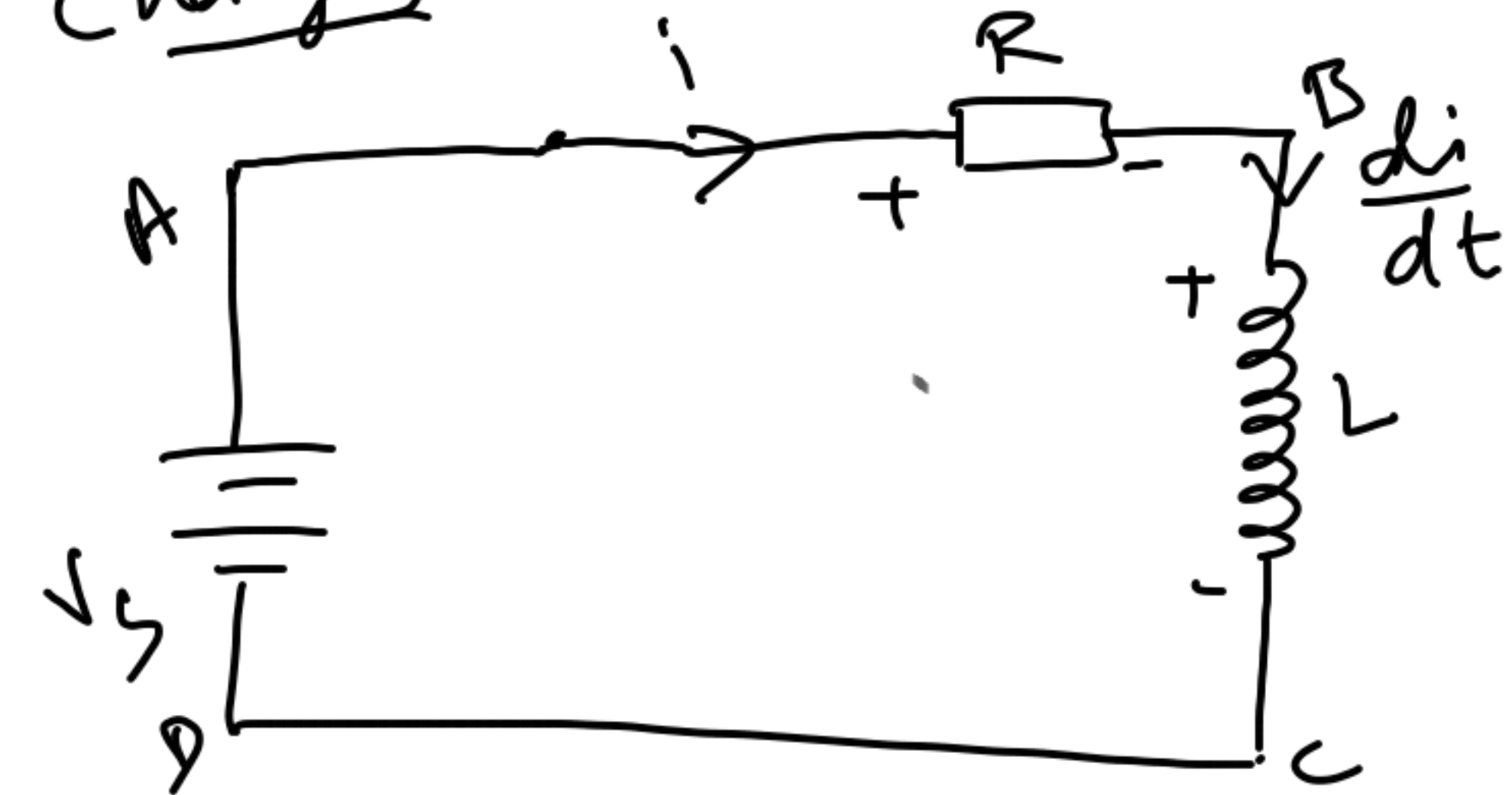


RL transient circuit charging



Apply KVL in the above circ.

$$-iR - L \frac{di}{dt} + V_s = 0$$

$$V_s - iR = L \frac{di}{dt}$$

$$\frac{dt}{L} = \frac{di}{V_s - iR}$$

Apply integration on both the side

$$\int \frac{dt}{L} = \int \frac{di}{V_s - iR}$$

t varies from '0' to ' t '

i varies from 0 to ' i '

$$\frac{1}{L} \int_0^t dt = \int_0^i \frac{di}{V_s - iR} \quad \text{--- (1)}$$

Integration by Substitution.

$$u = V_s - iR \quad \checkmark$$

differentiate the above equation
with respect to i

$$\frac{du}{di} = 0 - R$$

$$\frac{du}{di} = -R$$

$$di = -\frac{du}{R} \quad \checkmark$$

Substitute di and u in eq ①

limits
 u , ranges from V_s to $V_s - Ri$

$$\text{①} \Rightarrow \int_0^t \frac{1}{L} dt = \int_{V_s}^{V_s - Ri} \frac{-1}{R} \frac{du}{u}$$

$$\frac{1}{L} [t]_0^t = -\frac{1}{R} \left[\ln(u) \right]_{V_s}^{V_s - Ri}$$

$$\int \frac{du}{u} = \ln[u]$$

Sub. $u = V_s - iR$

$$\frac{1}{L} [t - 0] = -\frac{1}{R} \left[\ln(V_s - Ri) - \ln(V_s) \right]$$

$$\frac{t}{L} = \frac{-1}{R} [\ln(V_s - iR) - \ln(V_s)]$$

$$\left[\ln(A) - \ln(B) = \ln\left(\frac{A}{B}\right) \right]$$

$$\frac{-tR}{L} = \ln\left[\frac{V_s - iR}{V_s}\right]$$

Taking exponential on both the side.

$$e^{\frac{-tR}{L}} = \frac{V_s - iR}{V_s}$$

$$V_s e^{-tR/L} = V_s - iR$$

$$iR = V_s - V_s e^{-tR/L}$$

$$iR = V_s (1 - e^{-tR/L})$$

$$i = \frac{V_s}{R} (1 - e^{-tR/L})$$

Steady state current $\hat{I}_0$

$$\hat{I}_0 = \frac{V_s}{R}$$

$$i = \hat{I}_0 (1 - e^{-tR/L})$$

Time constant.

$$\tau = \frac{L}{R}$$

$$i = I_0 (1 - e^{-t/\tau})$$

when $t = \tau$

$$i = I_0 (1 - e^{-\tau/\tau})$$

$$i = I_0 (1 - e^{-1})$$

$$i = I_0 (1 - 0.37)$$

$$i = 0.63 I_0$$

when $t = 2\tau$

$$i = I_0 (1 - e^{-2\tau/\tau})$$

$$i = I_0 (1 - e^{-2})$$

$$i = 0.86 I_0$$

when $t = 3\tau$

$$i = I_0 (1 - e^{-3\tau/\tau})$$

$$i = I_0 (1 - e^{-3})$$

$$i = 0.95 I_0$$

when $t = 4\tau$

$$i = I_0 (1 - e^{-4\tau/\tau})$$

$$i = I_0 (1 - e^{-4})$$

$$i = 0.98 I_0$$

when $t = 5\tau$

$$i = I_0 (1 - e^{-5\tau/\tau})$$

$$i = I_0 (1 - e^{-5})$$

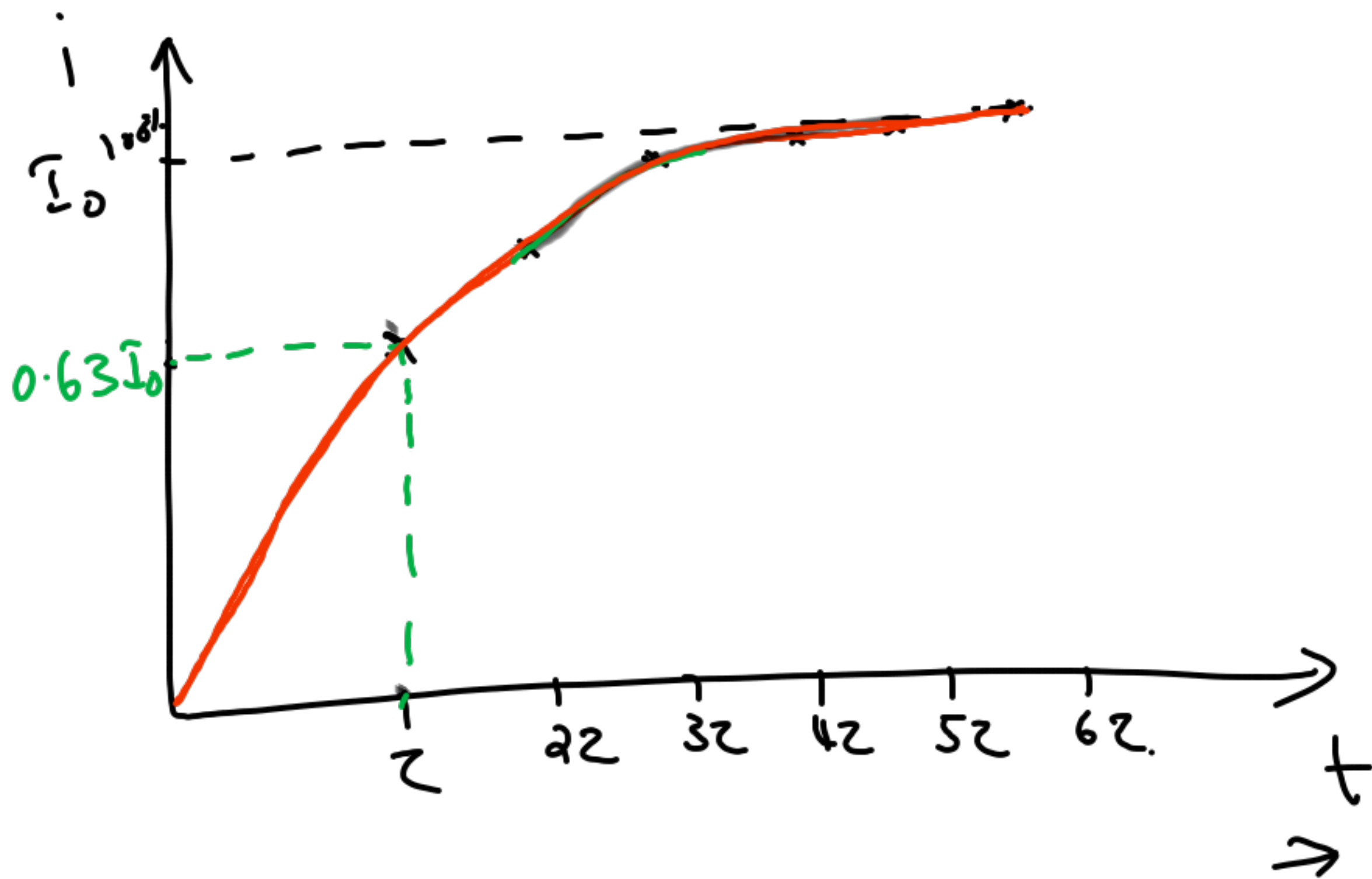
$$i = 0.99 I_0$$

When $t = 6\tau$ $- 6\tau/\tau$

$$i = I_0(1 - e^{-6})$$

$$i = I_0(1 - e^{-6})$$

$$i = 1 I_0$$



V_L V_R

To find V_R

$$V_R = i R$$

$$= I_0(1 - e^{-t/\tau}) \cdot R$$

$$= \frac{V_S}{R} (1 - e^{-t/\tau}) \cdot R$$

$$V_R = V_S(1 - e^{-t/\tau})$$

To find V_L

$$V_S = V_R + V_L \quad , \text{ from KVL}$$

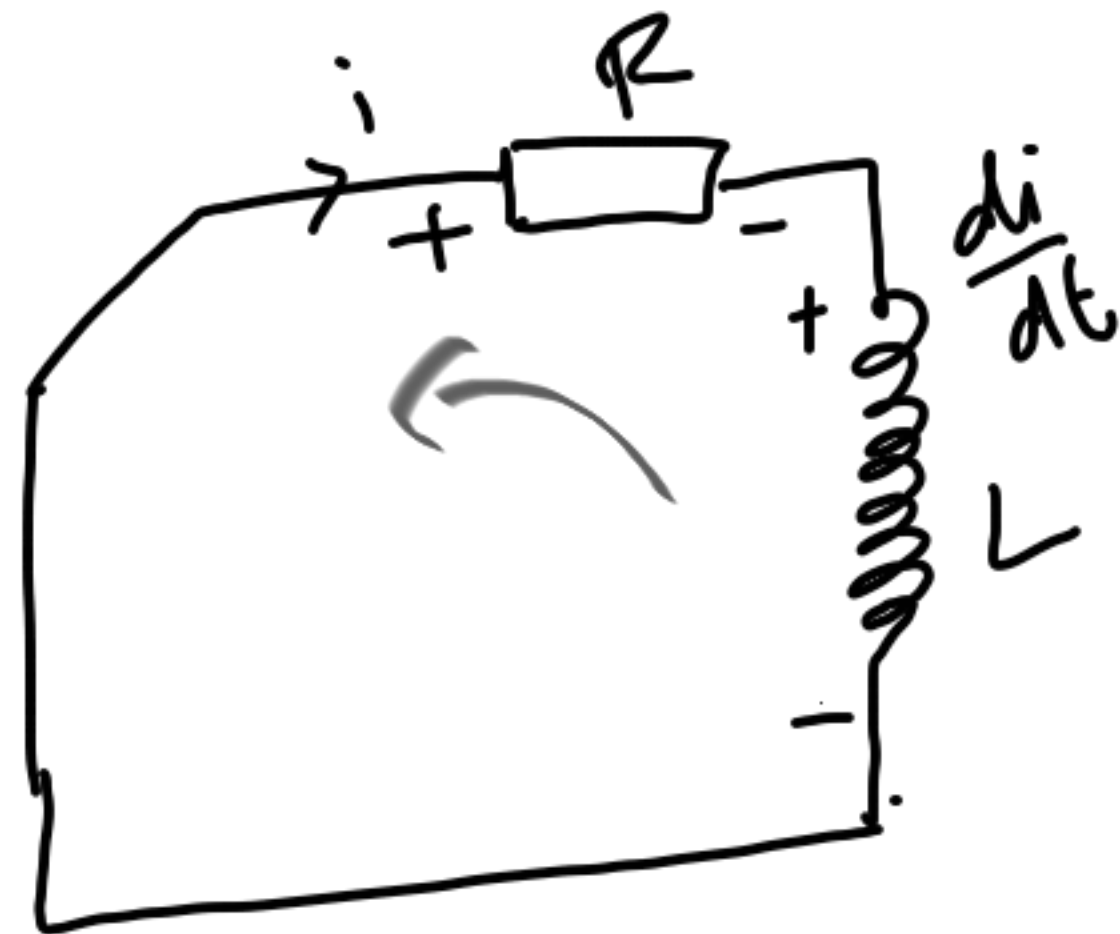
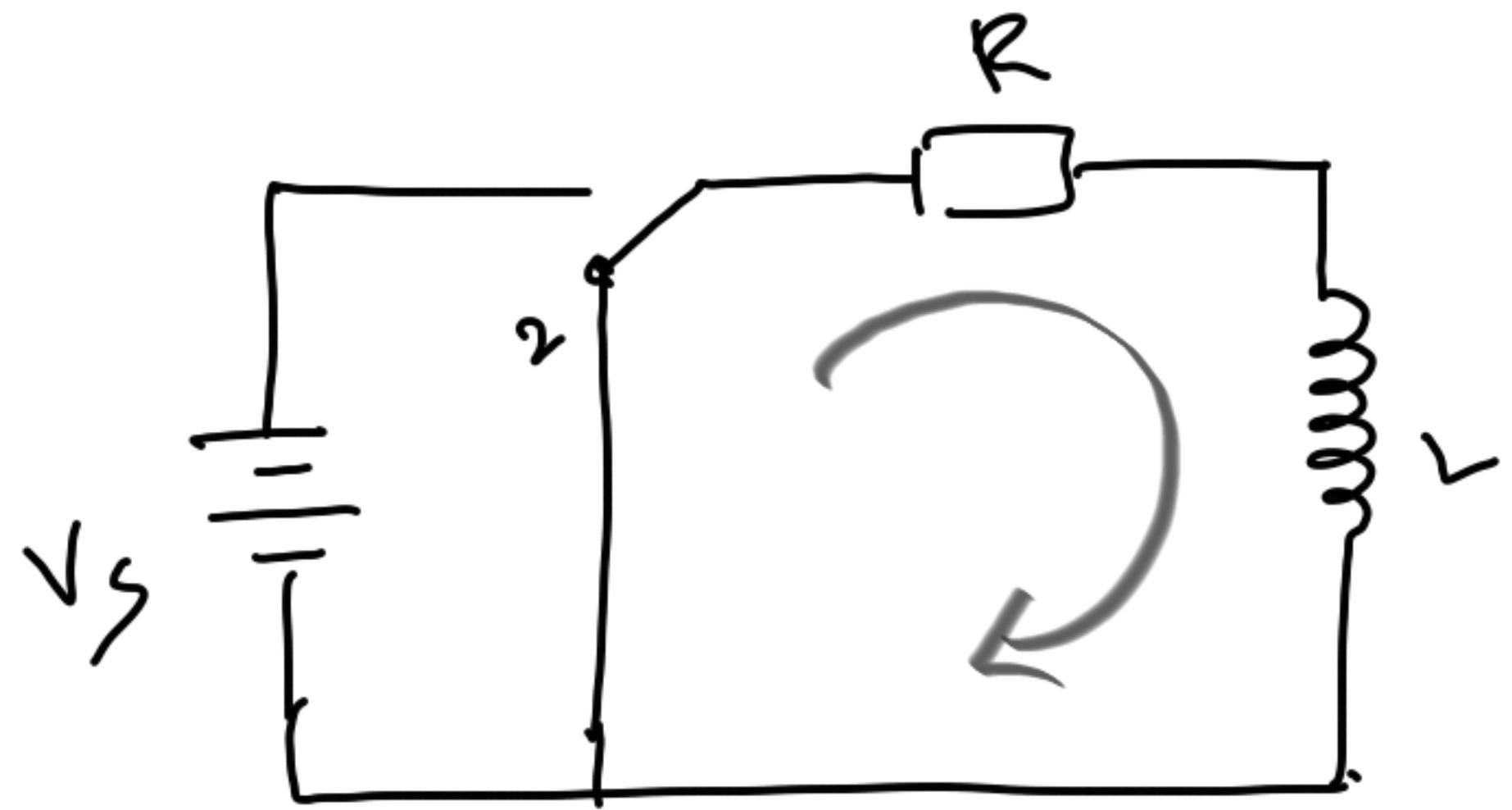
$$V_L = V_S - V_R \quad \checkmark$$

$$V_L = V_S - V_S(1 - e^{-t/\tau})$$

$$V_L = \cancel{V_S} - \cancel{V_S} + V_S e^{-t/\tau}$$

$$V_L = V_S e^{-t/\tau}$$

RL circuit discharging.



Apply KVL in above circuit.

$$-iR - L \frac{di}{dt} = 0$$

$$-iR = L \frac{di}{dt}$$

$$-\frac{R}{L} dt = \frac{di}{i}$$

Integrating both the side.

$$-\frac{R}{L} \int dt = \int \frac{di}{i}$$

t ranges from, 0 to t
 i ranges from, I_0 to i

$$-\frac{R}{L} \int_0^t dt = \int_{I_0}^i \frac{di}{i}$$

$$-\frac{R}{L} \left[t \right]_0^t = \left[\ln(i) \right]_{I_0}^i$$

$$-\frac{Rt}{L} = \ln[i - I_0]$$

$$= \ln\left(\frac{i}{I_0}\right)$$

$$-\frac{Rt}{L} = \ln\left(\frac{i}{I_0}\right)$$

taking exponent.

$$e^{-Rt/L} = \frac{i}{I_0} \Rightarrow i = I_0 e^{-Rt/L}$$

We know that $\tau = \frac{L}{R}$.

$$i = I_0 e^{-t/\tau}$$

When $t = \tau$.

$$i = I_0 e^{-\tau/\tau}$$

$$i = I_0 e^{-1}$$

$$i = 0.37 I_0$$

When $t = 2\tau$

$$i = I_0 e^{-2\tau/\tau}$$

$$i = I_0 e^{-2}$$

$$i = 0.14 I_0$$

When $t = 3\tau$

$$i = I_0 e^{-3\tau/\tau}$$

$$i = 0.05 I_0$$

When $t = 4\tau$

$$i = I_0 e^{-4\tau/\tau}$$

$$i = 0.02 I_0$$

When $t = 5\tau$

$$i = 0.01 I_0$$

When $t = 6\tau$

$$i = 0$$

