

FLUID FLOW

Some basic concepts:

Viscosity

A viscous fluid offers a resistance to the motion through it of a solid body, or equivalently, its own motion past a solid body. The layer of fluid in contact with solid is stationary with respect to that body.

Laminar flow

— all particles along a given streamline have same velocity. In a viscous fluid different layers of streamlines can have different velocities.

Turbulent flow

Disorderly flow.

In a viscous fluid, laminar flow becomes turbulent when the velocity exceeds a certain value for a given pipe of diameter D .

Reynold's number

$$R_e = \frac{\rho v D}{\mu}$$

ρ density
 v velocity
 μ coeff of viscosity

Bernoulli's Equation

For an incompressible, inviscid fluid undergoing steady flow, the pressure plus K.E. per unit volume plus the P.E. per unit volume is constant at all points on a streamline

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{constant}$$

- where
- P pressure in fluid
 - ρ density of fluid
 - v velocity of fluid
 - g acceleration due to gravity
 - h height of fluid

Viscosity arises from the frictional force between adjacent fluid layers that are in relative motion. Consider two layers of area A :



Newton suggested the frictional force F between the layers was proportional to area A and the velocity gradient $\frac{dv}{dy}$ perpendicular to flow direction.

$$F \propto A \frac{dv}{dy}$$

$$F = \eta A \frac{dv}{dy}$$

η Coefficient of viscosity for a Newtonian fluid.

Equation of Continuity for a fluid in 3D. (14)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

ρ density
 $\vec{u}$ fluid velocity.

Remember the ∇ operator is

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

so the continuity equation written in full is

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_x)}{\partial x} + \frac{\partial (\rho u_y)}{\partial y} + \frac{\partial (\rho u_z)}{\partial z} = 0$$

where (u_x, u_y, u_z) are components of $\vec{u}$

The Euler Equations of gas dynamics

The first Euler equation is concerned with conservation of mass. The other Euler equations relate to momentum and energy. They all have a similar form:

In 1D:

$$\frac{\partial}{\partial t} \begin{pmatrix} \rho \\ \rho u \\ E \end{pmatrix} + \frac{\partial}{\partial x} \begin{pmatrix} \rho u \\ \rho u^2 + p \\ H + \frac{u^2}{2} \end{pmatrix} = 0$$

where H is enthalpy

In steady state $\frac{\partial}{\partial t} = 0$

— and this will apply if we move in a frame of reference with a shock/discontinuity that has a constant speed.

then we get across a discontinuity:

p_2		p_1
u_2		u_1
h_2		h_1
ρ_2		ρ_1

$$\begin{aligned} \rho_1 u_1 &= \rho_2 u_2 \\ p_1 + \rho_1 u_1^2 &= p_2 + \rho_2 u_2^2 \\ H_1 + \frac{u_1^2}{2} &= H_2 + \frac{u_2^2}{2} \end{aligned}$$

HUGONIOT EQUATIONS