

Vehicle Technology

Steady State Cornering

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Ackerman Steering

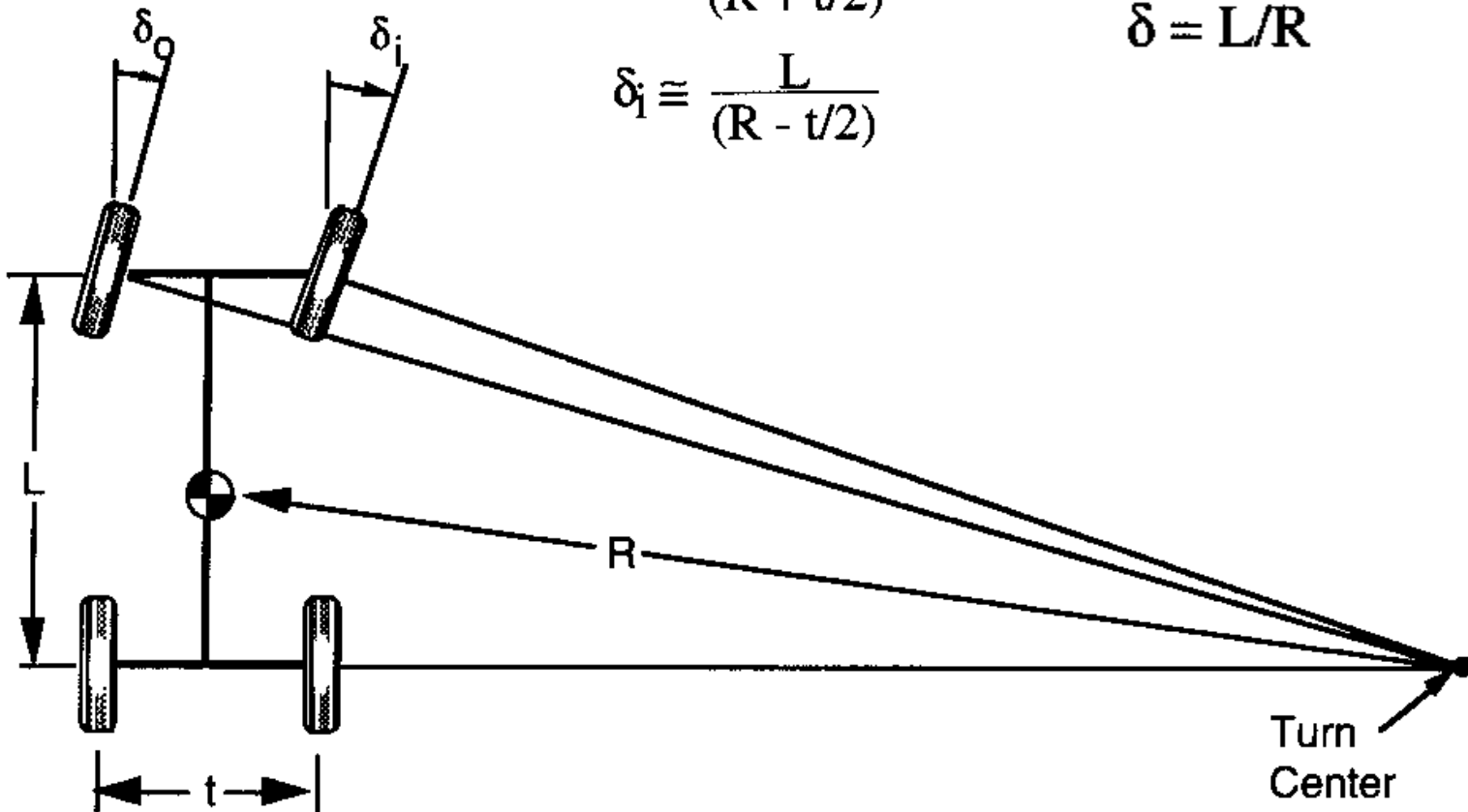
- At low speeds rear wheels do not generate slip angles
- Steering angle of inner and outer wheel must differ
- Outer wheel is further away from the centre of turn by the length of the vehicle track – t .
- So inner and outer wheel angles must be applied

Ackerman Steering

$$\delta_o \equiv \frac{L}{(R + t/2)}$$

$$\delta_i \equiv \frac{L}{(R - t/2)}$$

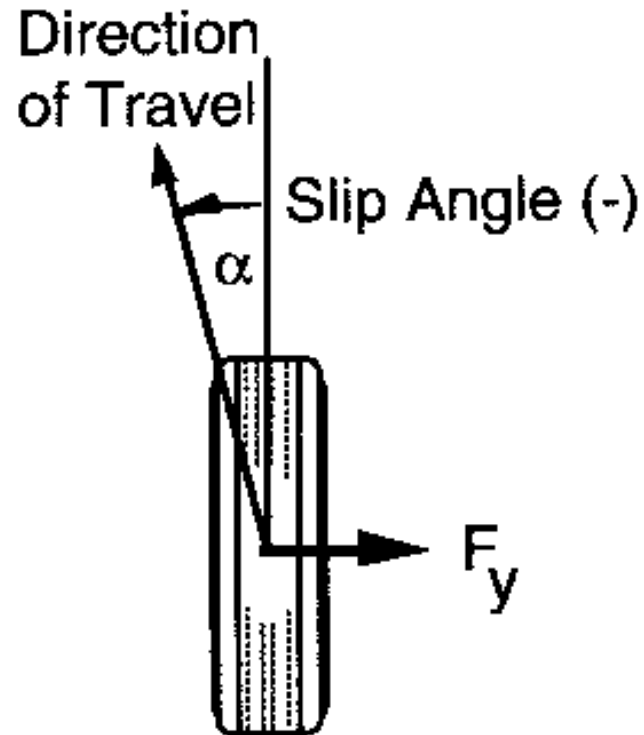
$$\delta = L/R$$



Gillespie, 1992

High Speed Cornering

While cornering the tyre will have a slip angle as it generates cornering forces

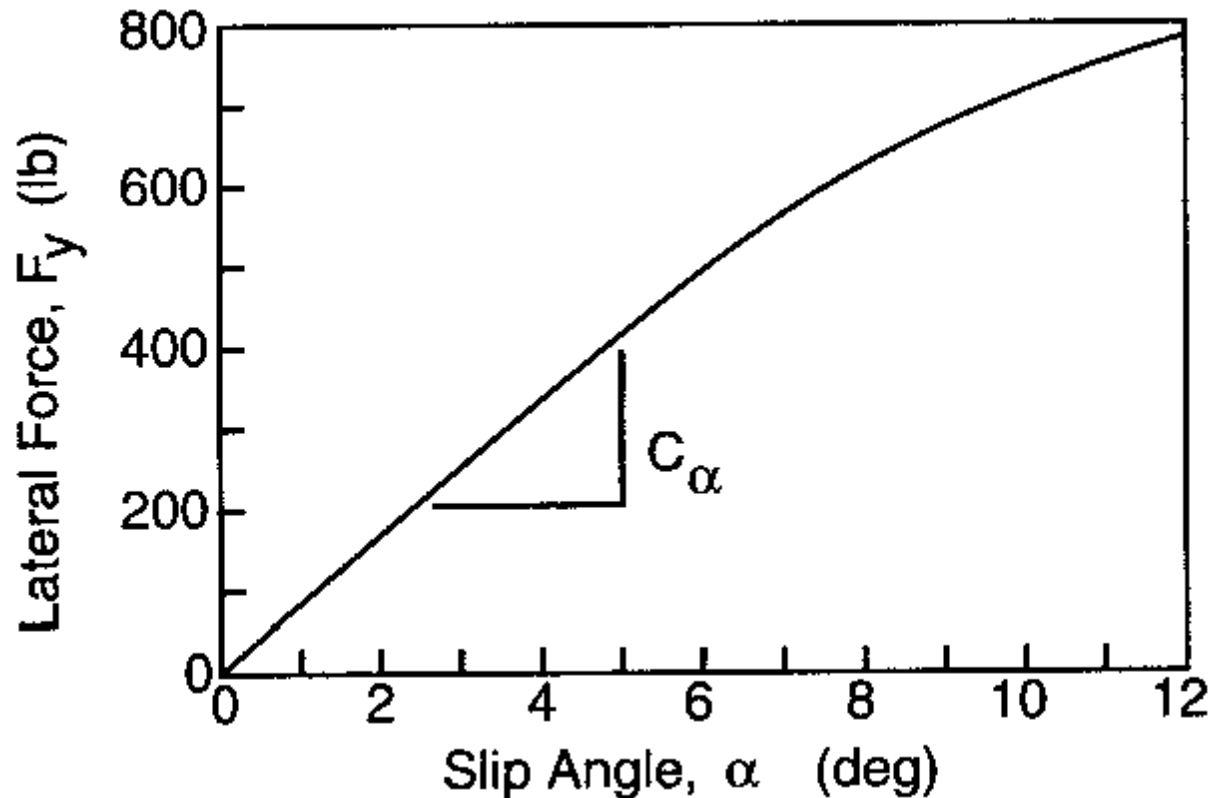


Gillespie, 1992

Gillespie, 1992

High Speed Cornering

- The relationship between the cornering force created and the slip angle generated is the cornering stiffness, C_α

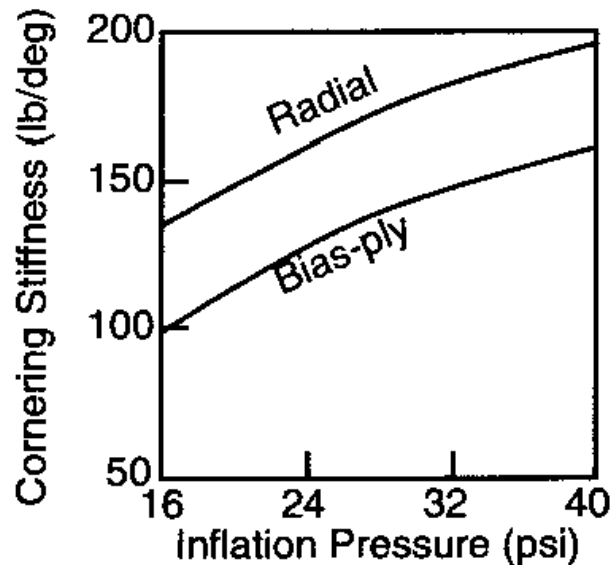


High Speed Cornering

- So we can say that cornering force will be related to slip angle and cornering stiffness by:

$$F_y = C_\alpha \alpha$$

- Cornering Stiffness is dependant on many variables such as tyre construction, size, inflation pressure and load, etc. The latter two for a same tyre will be the most relevant.

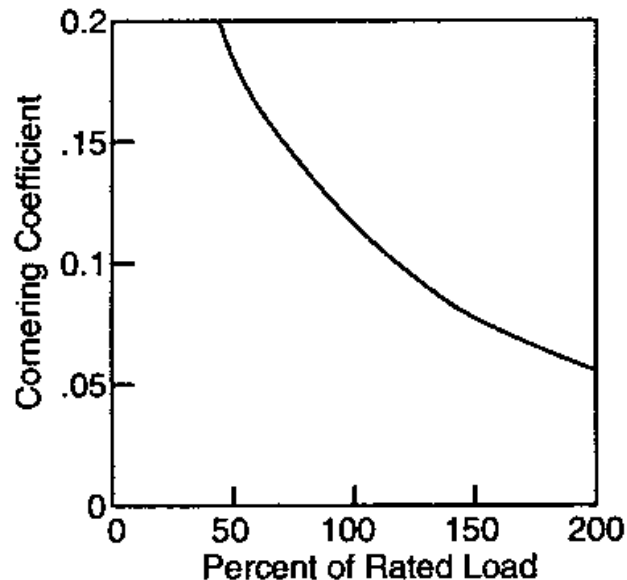


High Speed Cornering

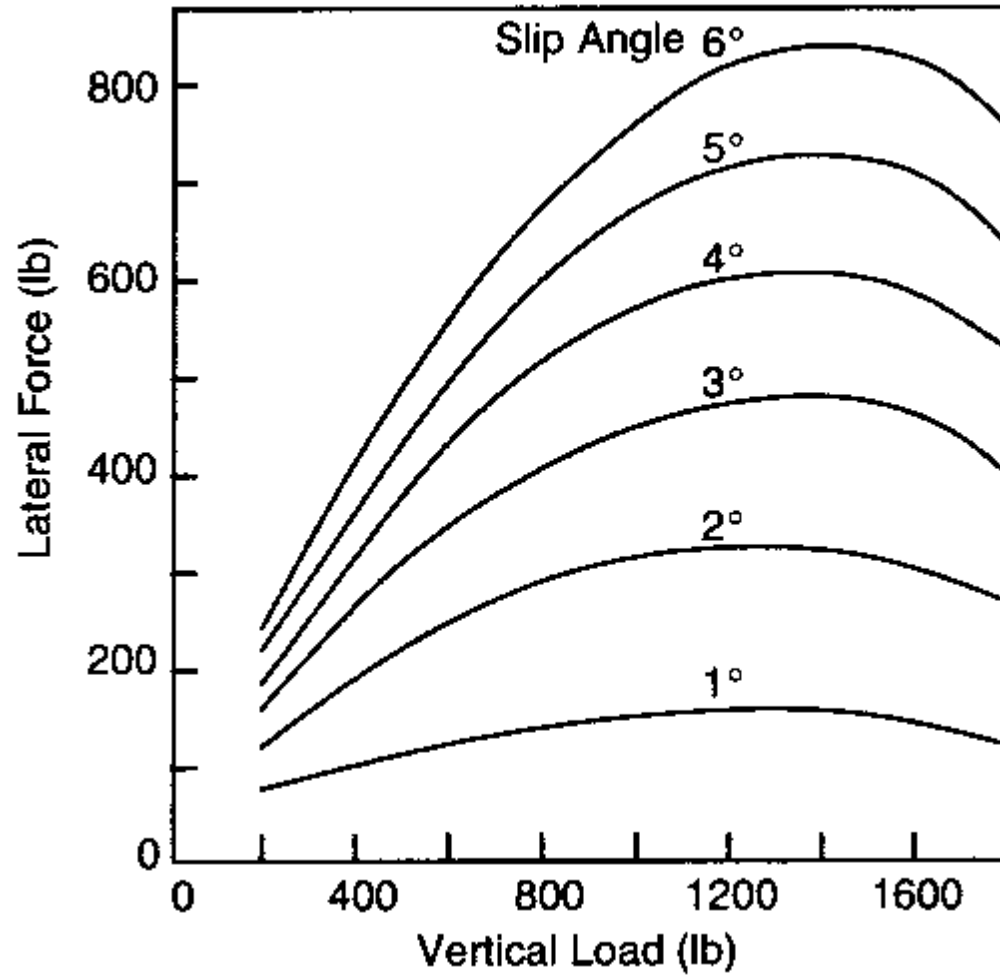
- Another way of describing tyre cornering characteristics is to, for a set tyre, introduce the relationship between load on the tyre and the Cornering Stiffness.
- This is called tyre cornering coefficient:

$$CC_{\alpha} = C_{\alpha}/F_z$$

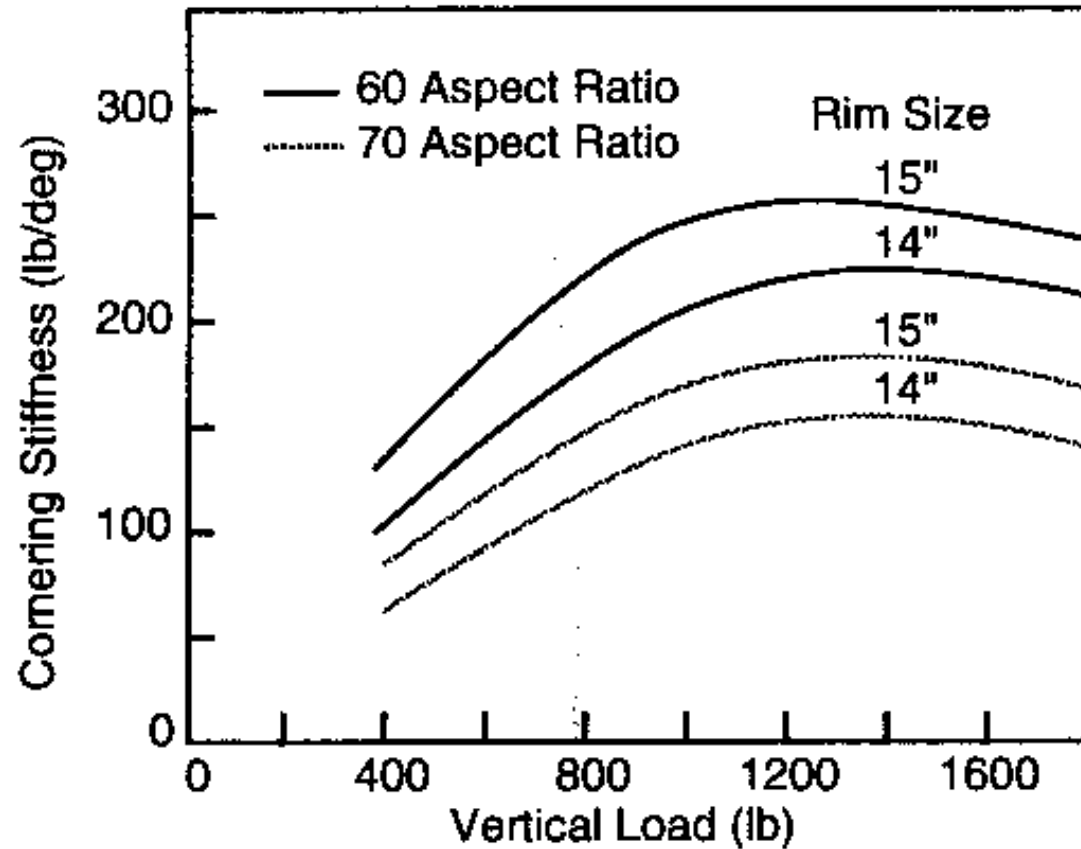
- Units will be Ny/Nz/deg



High Speed Cornering



High Speed Cornering



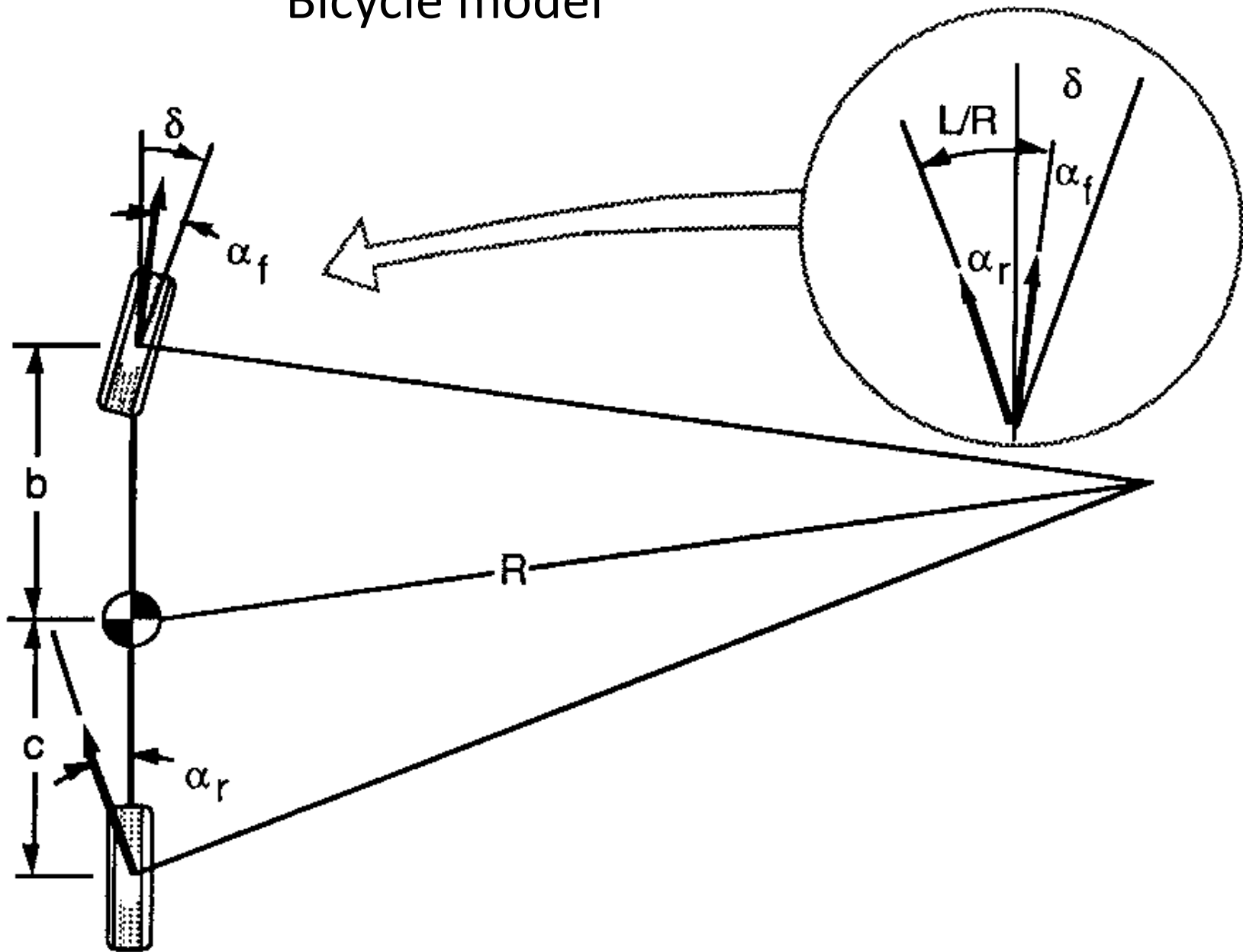
Cornering Equations

- For steady state cornering conditions – radius and speed constant – we can assume a condition of equilibrium between the lateral forces being generated by the tyres and the centripetal acceleration.
- Therefore we can state:

$$\Sigma F_y = F_{yf} + F_{yr} = M V^2/R$$

- For ease of analysis of vehicle characteristics while cornering under steady state conditions we shall assume the two wheels on axle as one since at high speeds radius of turn much larger than vehicle track.

Bicycle model



Cornering Equations

- From the bicycle model diagram we can also state that for equilibrium about the centre of gravity:

$$F_{yf} b - F_{yr} c = 0$$

- Or

$$F_{yf} = F_{yr} c/b$$

- Substituting that into our original equilibrium condition:

$$M V^2/R = F_{yr} (c/b + 1) = F_{yr} (b + c)/b = F_{yr} L/b$$

$$F_{yr} = M b/L (V^2/R)$$

Cornering Equations

- But the component Mb/L is nothing more than the weight of the vehicle acting on the rear axle and can be substituted by W_r/g , where g is the acceleration due to gravity.
- Knowing the lateral accelerations being generated the slip angles can now be found.

Where:

$$\alpha_f = W_f V^2 / (C_{\alpha f} g R)$$

and

$$\alpha_r = W_r V^2 / (C_{\alpha r} g R)$$

Cornering Equations

- Looking at the vehicle characteristics in the turn from the bicycle model we can state that the steering angle is given by.

$$\delta = 57.3 \frac{L}{R} + \alpha_f - \alpha_r$$

- And substituting for the values of slip angle from above.

$$\delta = 57.3 \frac{L}{R} + \frac{W_f V^2}{C_{\alpha f} g R} - \frac{W_r V^2}{C_{\alpha r} g R}$$

Cornering Equations

■ Or

$$\delta = 57.3 \frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right) \frac{V^2}{g R}$$

δ = Steer angle at the front wheels (deg)

L = Wheelbase (ft)

R = Radius of turn (ft)

V = Forward speed (ft/sec)

g = Gravitational acceleration constant :

W_f = Load on the front axle (lb)

W_r = Load on the rear axle (lb)

$C_{\alpha f}$ = Cornering stiffness of the front tires

$C_{\alpha r}$ = Cornering stiffness of the rear tires (

Understeer Gradient

We can further simplify the equation by substituting

$$\delta = 57.3 L/R + K a_y$$

K = Understeer gradient (deg/g)

a_y = Lateral acceleration (g)

Understeer Gradient

Neutral Steer

$$W_f / C_{\alpha f} = W_r / C_{\alpha r} \rightarrow K = 0 \rightarrow \alpha_f = \alpha_r$$

Understeer

$$W_f / C_{\alpha f} > W_r / C_{\alpha r} \rightarrow K > 0 \rightarrow \alpha_f > \alpha_r$$

Oversteer

$$W_f / C_{\alpha f} < W_r / C_{\alpha r} \rightarrow K < 0 \rightarrow \alpha_f < \alpha_r$$

Critical and Characteristic Speeds

Characteristic speed is the vehicle's forward speed at which an understeer vehicle requires a steering input that is twice that of the Ackerman angle. This can be calculated as:

$$V_{\text{char}} = \sqrt{57.3 L g/K}$$

Critical speed is that at which an oversteer car will become directionally unstable and is given by:

$$V_{\text{crit}} = \sqrt{-57.3 L g/K}$$

Revision Example

A vehicle of mass 100kg, wheelbase 1.75 metres, track 1.2 metres and a weight distribution of 60/40 front/rear is fitted with tyres developing cornering stiffness of 450 and 500 N/deg, front and rear respectively. The car is taking a corner of 80 metre radius at 70 mph.

- a) Calculate the vehicles Ackerman steering angle and explain its use.
- b) Calculate the lateral acceleration the car is witnessing.
- c) Calculate the under steer coefficient.
- d) Based on c) explain what the vehicle's cornering characteristics are and what that means to an inexperienced driver.
- e) Calculate and explain their impact
 - a. Characteristic Speed
 - b. Critical Speed

$$\delta = 57.3 \frac{L}{R} + \left(\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right) \frac{V^2}{g R}$$

$$\delta = 57.3 L/R + K a_y$$

$$V_{\text{char}} = \sqrt{57.3 L g/K}$$

$$V_{\text{crit}} = \sqrt{-57.3 L g/K}$$