

Q2) (a) $\int \frac{2x}{(4+3x^2)} dx$

$$\frac{\ln(3x^2+4)}{3} + C$$

$$u = 3x^2 + 4$$

$$\frac{du}{dx} = 6x$$

$$\frac{2}{6} \int \frac{1}{u} du$$

$$\int \frac{\ln u}{3}$$

$$= \frac{\ln(3x^2+4)}{3} + C$$

(ii) $\int_0^4 e^{1/2 x} dx$

$$\int e^{x/2} = 2e^{x/2}$$

$$\left[2e^{x/2} - 2e^{0/2} \right]$$

$$= 12.778 = \underline{2e^2 - 2}$$

(iii) $\int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$

$$5x+13 = A(x+4) + B(2x+1)$$

when $x = -4$

$$-7 = -7B$$

$$B = +1$$

Q 1) a) (vi) $y = \frac{1 - 10x}{(2x - 1)^2}$

~~$y = \frac{1 - 10x}{(2x - 1)^2}$~~

$= \frac{2(10x + 3)}{(2x - 1)^3}$

$$\begin{aligned}
 \text{(ii)} \quad & \frac{3}{(1+i)(2+i)} = \frac{3}{i^2 + 3i + 2} \\
 & = \frac{3}{3i + 1} \\
 & = \underline{\underline{\frac{3}{10} - \frac{9}{10}i}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i} \\
 & 3 - 7i
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & 2i(i-1) + (\sqrt{3}-i) + (1+i)(1+i) \\
 & = -2 + \sqrt{3} - i
 \end{aligned}$$

$$b) \quad z = (4+7i)(3-2i)$$

$$r = 13\sqrt{5} \quad \theta = \tan^{-1} 0.5$$

$$z = 26 + 13i$$

$$r = \sqrt{26^2 + 13^2} = 13\sqrt{5}$$

$$\theta = \tan^{-1} \left(\frac{\text{opp}}{\text{adj}} \right) = \tan^{-1} 0.5$$

$$c) \quad 1+3i \div 1+i = \underline{\underline{2+i}}$$

Q2) b) $f(x) = \frac{4}{x(1+4\ln x)^2} dx$

$$u = 4\ln x + 1 \quad \frac{du}{dx} = \frac{4}{x}$$

$$\frac{dx}{x} = \frac{1}{4} du$$

$$\int \frac{1}{u^2} du$$

$$= -\frac{1}{u}$$

$$= -\frac{1}{4(\ln(x)+1)} + C$$

c) $\int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$

$$u = e^x - 1 \quad \frac{du}{dx} = e^x$$

$$dx = e^{-x} du$$

$$e^{2x} = (u+1)^2$$

$$\int \frac{u+1}{\sqrt{u}} du$$

$$= \int \sqrt{u} du + \int \frac{1}{\sqrt{u}} du$$

$$\left[2\sqrt{e^x - 1} (e^x + 2) \right]_{\ln 2}^{\ln 5}$$

$$= 20$$

$$Q2) \text{ c) (ii) } \int_1^e \ln x \, dx$$

$$\left[x(\ln x - 1) \right]_1^e$$

$$(e(\ln e - 1)) - (1(\ln 1 - 1))$$

$$= e(1 - 1) - (0 - 1)$$

$$= \underline{\underline{+1}}$$

$$Q3) \text{ a) (i) } \left(\frac{1+i}{1-i} \right)^3$$

$$\frac{(1+i)(1+i)(1+i)}{(1-i)(1-i)(1-i)}$$

$$= \frac{-j^3 - 3i^2 - 3i - 1}{i^3 - 3i^2 - 3i - 1}$$

$$= \frac{i + 3 - 3i - 1}{-i + 3 - 3i - 1}$$

$$= \frac{+2i + 2}{+2i + 2}$$

$$\underline{\underline{1 + i}}$$

$$= \underline{\underline{0 - 1i}}$$

when $x = 0$

$$13 = 4A + 1$$

$$12 = 4A$$

$$A = 3$$

$$\int_0^4 \frac{3}{2x+1} + \frac{1}{x+4}$$

$$3 \int \frac{1}{2x+1} dx + \int \frac{1}{x+4} dx$$

$$= \frac{\ln(2x+1)}{2} + \ln(x+4)$$

$$\int = \frac{3 \ln(12x+1)}{2} + \ln(1x+4) + C$$

$$\left[\frac{3 \ln(12x+1)}{2} + \ln(1x+4) \right]_0^4$$

$$= \left(\frac{3 \ln(12(4)+1)}{2} + \ln(1(4)+4) \right) -$$

$$\left(\frac{3 \ln(12(0)+1)}{2} + \ln(1(0)+4) \right)$$

$$= \left(\frac{3}{2} \ln 9 + \ln 8 \right) - \left(\frac{3}{2} \ln 1 + \ln 4 \right)$$

$$= \frac{3}{2} \ln(9) + \ln(8) - \ln(4)$$

$$(v) \quad y = (x + \sin 2x)$$

on separate page
page $\rightarrow$ (vi)

(b)
(vii)

$$\frac{dy}{dx} = 1 + 2\cos 2x$$

$$y = \frac{1}{2}x + \cos 2x \quad \text{find } \frac{d^5 y}{dx^5}$$

$$\frac{dy}{dx} = -x^{-2} - 2\sin 2x$$

$$\frac{d^2 y}{dx^2} = 2x^{-3} - 4\cos 2x$$

$$\frac{d^3 y}{dx^3} = -6x^{-4} + 8\sin 2x$$

$$\frac{d^4 y}{dx^4} = 24x^{-5} + 16\cos 2x$$

$$\frac{d^5 y}{dx^5} = -120x^{-6} - 32\sin 2x$$

c)

$$f(x) = \ln(\operatorname{cosec} x - \cot x) \quad f'(x) = \operatorname{cosec} x$$

$$u = \operatorname{cosec} x - \cot x$$

$$f(x) = \frac{1}{u}$$

$$\frac{du}{dx} = -\cot x \operatorname{cosec} x + \operatorname{cosec}^2 x$$

$$= -\frac{\cos x}{\sin^2 x} + \frac{1}{\sin^2 x}$$

$$= \frac{1 - \cos x}{\sin^2 x}$$

$$= \sin x$$

$$f'(x) = \frac{1}{\sin x} = \underline{\underline{\operatorname{cosec} x}}$$

Q1) a) (i) $y = \frac{x^2}{2x+1}$

$$u = 2x+1$$

$$\frac{dy}{dx} = \frac{2x(x+1)}{(2x+1)^2}$$

ii) $y = \sqrt{x^2+1}$

$$u = x^2+1$$

$$y = u^{1/2}$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$$

$$\frac{dy}{dx} = x(x^2+1)^{-1/2}$$

(iii) $y = 3x^{3/2} + \frac{3}{x^2} - 6x$

$$\frac{dy}{dx} = \frac{9}{2} x^{1/2} - 6x^{-3} - 6$$

(iv) $y = \sec^3 4x$

$$u = 4x$$

$$y = \sec^3 u$$

$$\frac{dy}{du} = 3 \sec^3 u \tan u$$

$$\frac{du}{dx} = 4$$

$$3 \sec^3 4x \tan 4x$$