

ACDA 4008 Paper 2 Solutions

2025/26

JANUARY 2026 PAPER 2 SOLUTIONS

Q1 [30 marks]

i) $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

[2]

ii) $y = x^2 + 4x^{1/2}$

$$\Rightarrow \frac{dy}{dx} = 2x + 4 \cdot \frac{1}{2} \cdot x^{-1/2}$$

$$= 2x + 2x^{-1/2}$$

$$= 2x + \frac{2}{\sqrt{x}}$$

[3]

iii) $y = 5 + x^{-7} + x^{-2}$

$$\frac{dy}{dx} = 0 - 7x^{-8} - 2x^{-3}$$

$$= \frac{7}{x^8} - \frac{2}{x^3}$$

[3]

$$iv) y = \cos(10x)$$

$$\frac{dy}{dx} = \underline{-10 \sin(10x)}$$

[2]

$$v) y = x^2 \sin(x) \quad (\text{Product Rule})$$

$$\frac{dy}{dx} = 2x \cdot \sin(x) + x^2 \cos(x)$$

$$= x \underline{[2 \sin(x) + x \cos(x)]} \quad [5]$$

$$vi) y = (2x+7)^{10} \quad (\text{Chain Rule})$$

$$\frac{dy}{dx} = 10(2x+7)^9 \cdot 2$$

$$= \underline{20(2x+7)^9}$$

[3]

$$(vii) y = \tan(x) + 2x$$

$$\underline{\frac{dy}{dx} = \sec^2(x) + 2}$$

[2]

$$\underline{1b)} \quad y = \sin(2x)$$

$$\frac{dy}{dx} = 2\cos(2x)$$

$$\frac{d^2y}{dx^2} = -4\sin(2x)$$
$$= -4y$$

$$\Rightarrow \underline{\frac{d^2y}{dx^2} + 4y = 0}$$

[4]

$$(c) \quad y = \frac{(x+1)}{(2+x^2)} \quad (\text{Quotient Rule})$$

$$\frac{dy}{dx} = \frac{(2+x^2) \cdot 1 - (x+1)(2x)}{(2+x^2)^2}$$

$$= \frac{2+x^2 - 2x^2 - 2x}{(2+x^2)^2}$$

$$= \frac{2 - 2x - x^2}{(2+x^2)^2}$$

$$\text{when } x=0 \quad \frac{dy}{dx} = \frac{2}{(2)^2} = \frac{1}{2}$$

[6]

Q2 [30 marks]

$$\begin{aligned} \text{a i)} \quad \int (10x + x^3) dx &= \frac{10x^2}{2} + \frac{x^4}{4} + C \\ &= \underline{\underline{5x^2 + \frac{x^4}{4} + C}} \quad [3] \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \int (2x - 4e^x) dx &= \frac{2x^2}{2} - 4e^x + C \\ &= \underline{\underline{x^2 - 4e^x + C}} \quad [3] \end{aligned}$$

$$\text{(iii)} \quad \int \left(\frac{1}{x} + 2 \right) dx = \ln x + 2x + C \quad [3]$$

$$\begin{aligned} \text{(iv)} \quad \int x(x+4) dx &= \int (x^2 + 4x) dx \\ &= \frac{x^3}{3} + 4 \frac{x^2}{2} + C \\ &= \underline{\underline{\frac{x^3}{3} + 2x^2 + C}} \quad [3] \end{aligned}$$

$$\begin{aligned}
 \text{v) } \int_0^3 (4x^3 + 2x) dx &= \left[\frac{4x^4}{4} + \frac{2x^2}{2} \right]_0^3 \\
 &= (3^4 + 3^2) - (0^4 + 0^2) \\
 &= 81 + 9 - 0 \\
 &= \underline{90} \quad [3]
 \end{aligned}$$

$$(b) \quad I = \int_0^1 \frac{x \, dx}{\sqrt{x+5}}$$

$$\text{let } u = x+5 \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$$

$$\text{when } x=0, \quad u=5$$

$$x=1, \quad u=6$$

$$I = \int_{u=5}^6 \frac{(u-5) du}{\sqrt{u}}$$

$$I = \int_5^6 u^{-1/2} (u-5) du$$

$$I = \int_5^6 (u^{1/2} - 5u^{-1/2}) du$$

$$I = \left[\frac{u^{3/2}}{3/2} - 5 \frac{u^{1/2}}{1/2} \right]_5^6$$

$$I = \left[\frac{2}{3} \sqrt{u^3} - 10 \sqrt{u} \right]_5^6$$

$$I = \left\{ \frac{2}{3} \sqrt{6^3} - 10 \sqrt{6} \right\} - \left\{ \frac{2}{3} \sqrt{5^3} - 10 \sqrt{5} \right\}$$

$$I = \{ 9.79796 - 24.49491 \} - \{ 7.45356 - 22.36068 \}$$

$$= -14.69695 - \{ -14.90712 \}$$

$$= +14.90712 - 14.69695$$

$$= 0.21017$$

$$= \underline{0.21 \text{ (2dps)}}$$

(5)

$$\begin{aligned}
 c) \quad i) \int 2x \cos(x) dx &= \sin(x) \cdot 2x - \int \sin(x) \cdot 2 dx \\
 &= 2x \sin(x) - (-2 \cos(x)) + C \\
 &= 2x \sin(x) + 2 \cos(x) + C \\
 &= 2 [x \sin(x) + \cos(x)] + C.
 \end{aligned}$$

[5]

$$\begin{aligned}
 ii) \int_1^e 3 \ln x dx &= [3x \ln x]_1^e - \int_1^e 3x \cdot \frac{1}{x} dx \\
 &= \left[\cancel{3e \ln e} - \cancel{3(1) \ln(1)} \right]_1^e - [3x]_1^e \\
 &= 3e - (3e - 3(1)) \\
 &= \cancel{3e} - \cancel{3e} + 3 \\
 &= \underline{+3}
 \end{aligned}$$

[5]

Q3 [30 Marks]

$$\begin{aligned} \text{i) } (1-i)(1+i) &= 1 - \cancel{i} + \cancel{i} - i^2 \\ &= 1 - (-1) \\ &= 2 \\ &= \underline{2+0i} \end{aligned}$$

[2]

$$\begin{aligned} \text{(ii) } \frac{3}{(1-i)} &= \frac{3}{(1-i)} \frac{(1+i)}{(1+i)} \\ &= \frac{3(1+i)}{1 - \cancel{i} + \cancel{i} - i^2} \\ &= \frac{3(1+i)}{1 - (-1)} \\ &= \frac{3(1+i)}{2} \\ &= \underline{\frac{3}{2} + \frac{3}{2}i} \end{aligned}$$

[2]

$$\begin{aligned} \text{(iii) } \frac{1}{i^5} &= \frac{1}{i^2} \cdot \frac{1}{i^2} \cdot \frac{1}{i} \\ &= \frac{1}{(-1)} \cdot \frac{1}{(-1)} \cdot \frac{1}{i} \end{aligned}$$

$$\begin{aligned}
 \frac{1}{j^5} &= \frac{1}{j} \cdot \frac{j}{j} \\
 &= \frac{j}{j^2} \\
 &= \frac{j}{-1} \\
 &= \underline{0 - j}
 \end{aligned}$$

[2]

3b

$$z_1 = \frac{(1-j)(1-j)}{(1+j)(1-j)}$$

$$z_1 = \frac{1 - j - j + j^2}{1 + \cancel{j} - \cancel{j} - j^2}$$

$$z_1 = \frac{2j}{2}$$

$$z_1 = j$$

$$\underline{z_1 = 0 + j}$$

[3]

$$z_2 = \frac{(3+j)(3+j)}{(3-j)(3+j)}$$

$$z_2 = \frac{9 + 3j + 3j + j^2}{9 - \cancel{3j} + \cancel{3j} - j^2}$$

$$z_2 = \frac{8 + 6j}{10}$$

$$z_2 = \frac{2(4 + 3j)}{10}$$

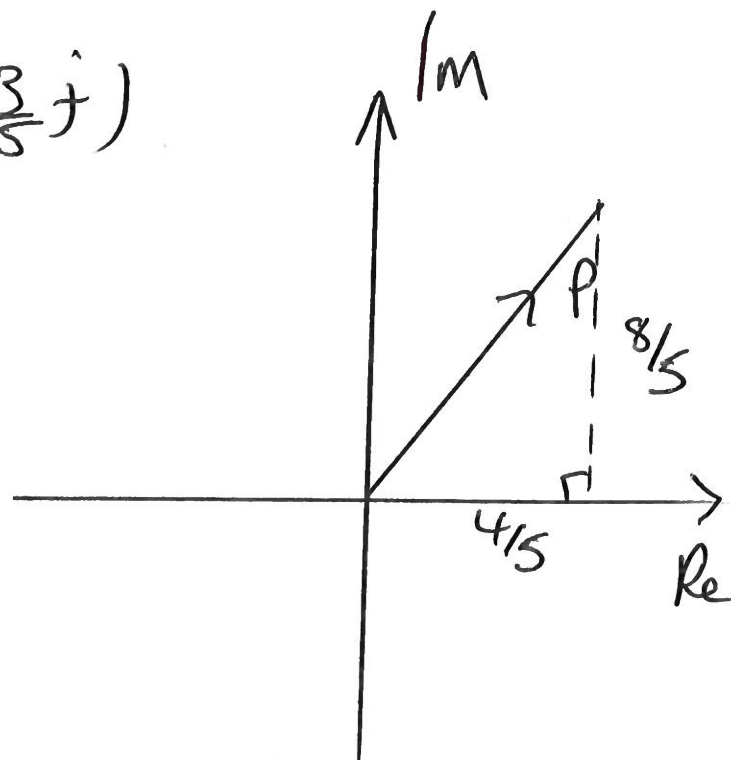
$$\underline{z_2 = \frac{4}{5} + \frac{3}{5}j}$$

[3]

c) $p = z_1 + z_2$

$$p = (0+j) + \left(\frac{4}{5} + \frac{3}{5}j\right)$$

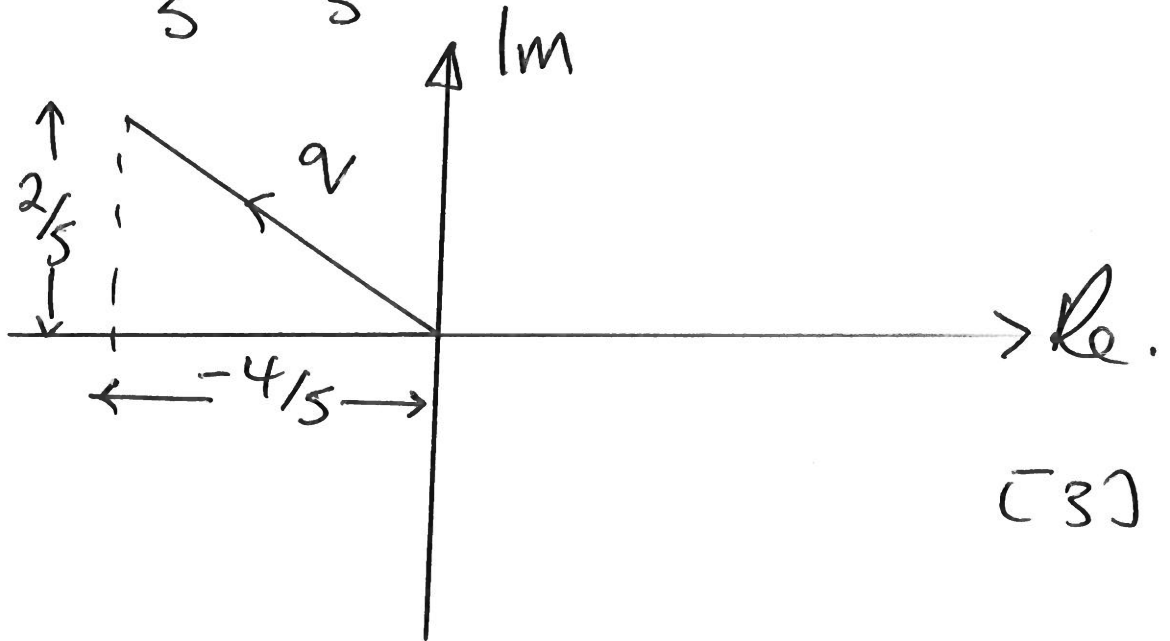
$$p = \frac{4}{5} + \frac{8}{5}j$$



$$(ii) q = z_1 - z_2$$

$$q = (0 + j) - \left(\frac{4}{5} + \frac{3}{5}j\right)$$

$$q = -\frac{4}{5} + \frac{2}{5}j$$



$$d) z = 1 + i\sqrt{3}$$

$$|z| = \sqrt{(1)^2 + (\sqrt{3})^2}$$

$$|z| = \sqrt{1+3}$$

$$|z| = r = 2$$



$$\begin{aligned} \text{Arg}(z) &= \tan \theta = \frac{\sqrt{3}}{1} \\ \Rightarrow \theta &= \tan^{-1}(\sqrt{3}) \\ \theta &= 60^\circ \end{aligned}$$

$$\therefore z = r(\cos \theta + i \sin \theta)$$

$$z = 2(\cos 60^\circ + i \sin 60^\circ)$$

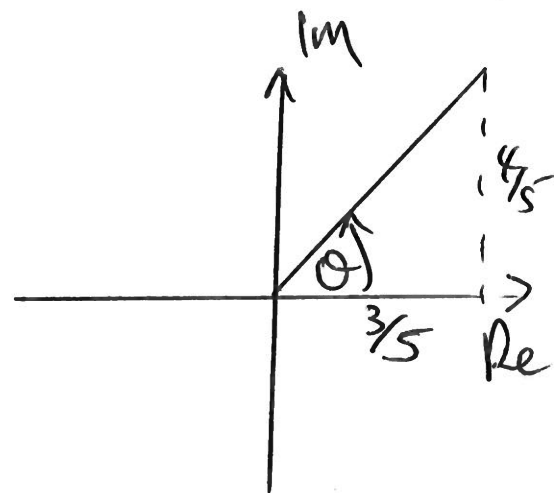
[6]

$$e) \quad z = \frac{(2+i)(2+i)}{(2-i)(2+i)}$$

$$z = \frac{4 + 2i + 2i + i^2}{4 - \cancel{2i} + \cancel{2i} - i^2}$$

$$z = \frac{3 + 4i}{5}$$

$$z = \frac{3}{5} + \frac{4}{5}i$$



$$\begin{aligned} |z| = |r| &= \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} \\ &= \sqrt{\frac{25}{25}} \\ &= 1 \end{aligned}$$

$$\text{Arg}(z) = \tan \theta = \frac{4/5}{3/5} = 4/3$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\theta = 53.13^\circ$$

$$\begin{aligned} z &= r [\cos \theta + i \sin \theta] \\ &= 1 [\cos(53.13^\circ) + i \sin(53.13^\circ)] \end{aligned}$$

[6]

Q4 [10 marks]

a i) $7^{2x} = 10$

$$\log_{10} 7^{2x} = \log_{10} 10$$

$$2x \log_{10} 7 = 1$$

$$x = \frac{1}{2 \log_{10} 7}$$

$$\underline{x = 0.5916 \text{ (4 d.p.s)}} \quad [4]$$

(ii) $2 \ln x = \ln(5x-6)$

$$\Rightarrow \ln x^2 = \ln(5x-6)$$

$$\Rightarrow x^2 = 5x-6$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$\Rightarrow \underline{x=2 \text{ and } x=3} \quad [2]$$

$$\begin{aligned} 4b \text{ i) } \log(x^5 y z) &= \log x^5 + \log y + \log z \\ &= \underline{5 \log x + \log y + \log z} \quad [2] \end{aligned}$$

$$\begin{aligned} \text{ii) } \ln \left[\frac{x^4 y}{z} \right] &= \ln[x^4 y] - \ln(z) \\ &= \ln(x^4) + \ln(y) - \ln(z) \\ &= \underline{4 \ln x + \ln y - \ln(z)} \quad [2] \end{aligned}$$