

## SCALAR AND VECTOR QUANTITIES

### I. DEFINITION

There are 2 types of (physical) quantities in mechanics:

- scalar quantities and,
- vector quantities.

1. **A scalar quantity** is one that is defined by a single number with appropriate units, e.g. length, mass, energy, etc. Once the units are stated, the quantity is denoted entirely by its size or magnitude.
2. **A vector quantity** is defined when we know not only its magnitude (with units) but also the direction in which it operates, e.g. force, velocity, acceleration. A vector quantity necessarily involves direction as well as magnitude.
3. Examples:
  - A speed of "10 km/h" is a scalar quantity, but
  - A velocity of "10 km/h due north" is a vector quantity.

### II. VECTOR REPRESENTATION

- ❖ A vector quantity is represented graphically by a line, drawn so that:
  - The length of the line denotes the magnitude of the quantity.
  - The direction of the line denotes the direction in which the vector quantity acts.The sense of the direction is indicated by an arrow.

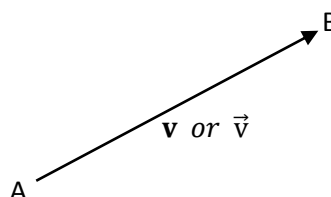
Example: An horizontal force of 35 N acting to the right would be indicated by a line:   
and if the chosen vector scale were 1 cm  $\equiv$  10 N, the line would be 3.5 cm long.

- ❖ Usually vectors are represented by a single letter in bold:

- for example: **v**

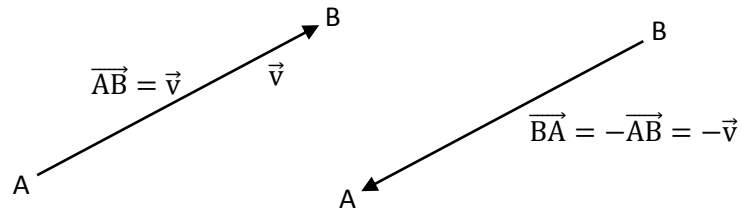
or like so:

- $\overrightarrow{AB}$  or  $\vec{v}$



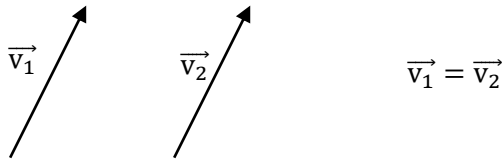
- ❖ The magnitude of the vector quantity is written  $|\overrightarrow{AB}|$  or  $|\mathbf{v}|$ , or simply AB or the single letter in italic  $v$  and not in bold anymore.

Note that  $\overrightarrow{BA}$  would represent a vector quantity of same magnitude but opposite sense.



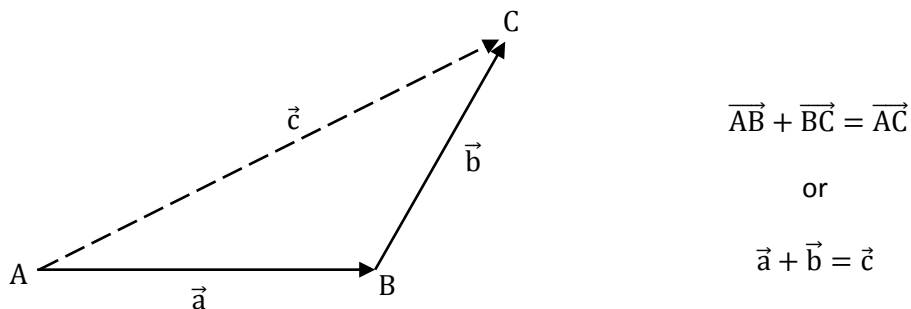
### III. TWO EQUAL VECTORS

If 2 vectors  $\vec{v}_1$  and  $\vec{v}_2$  are said to be equal, then they have the same magnitude and the same direction.



### IV. ADDITION OF VECTORS

- ❖ The sum of 2 vectors  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$  is defined as the resultant vector  $\overrightarrow{AC}$



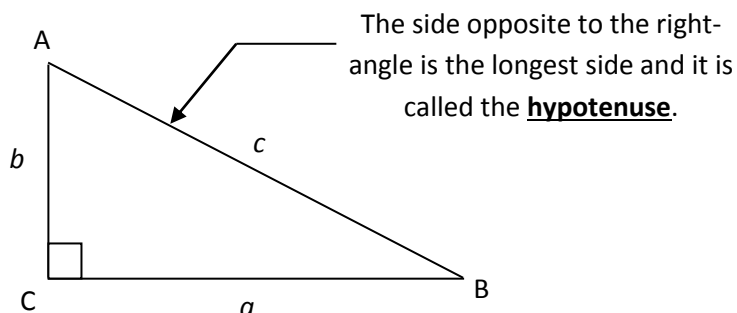
To find the sum of 2 vectors  $\vec{a}$  and  $\vec{b}$ , we draw them as a chain, starting the 2<sup>nd</sup> where the 1<sup>st</sup> ends: the sum  $\vec{c}$  is given by the resultant vector joining the start of the 1<sup>st</sup> to the end of the 2<sup>nd</sup>.

## ❖ RIGHT-ANGLED TRIANGLES

### ➤ Pythagoras' theorem:

All right-angled triangles have a property in common that is expressed in Pythagoras' theorem:

**The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.**

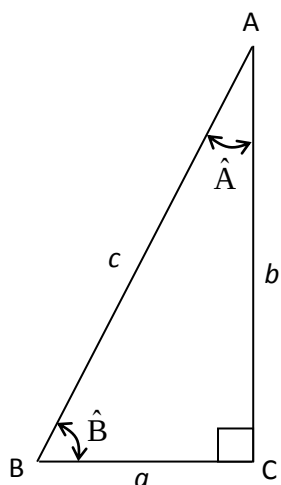


So in the above figure we have:

$$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{a^2 + b^2}$$

### ➤ Trigonometric ratios:

Given the right-angled triangle ABC as shown in the following figure, we define the trigonometric ratios as:



|   |   |   |   |   |   |
|---|---|---|---|---|---|
| S | O | C | A | T | O |
|   | H |   | H |   | A |

$$\sin \hat{A} = \frac{\text{side opposite to angle } \hat{A}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \hat{A} = \frac{\text{side adjacent to angle } \hat{A}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \hat{A} = \frac{\text{side opposite to angle } \hat{A}}{\text{side adjacent to angle } \hat{A}} = \frac{a}{b}$$

$$\sin \hat{B} = \frac{\text{side opposite to angle } \hat{B}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos \hat{B} = \frac{\text{side adjacent to angle } \hat{B}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan \hat{B} = \frac{\text{side opposite to angle } \hat{B}}{\text{side adjacent to angle } \hat{B}} = \frac{b}{a}$$

Tangent can be expressed in terms of sine and cosine as follows:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

❖ **NON-RIGHT TRIANGLES:**

We deal here with triangles which have no right angle.

Every triangle consists of 6 elements: 3 angles & 3 sides.

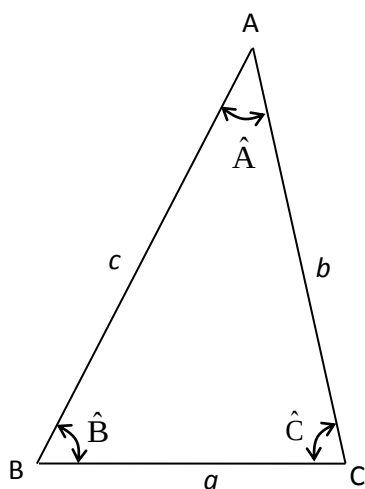
If we are given any 3 of these 6 elements we can find the other 3 by using either the sine rule or the cosine rule.

➤ **The sine rule:**

The sine rule is used when given:

1. Two angles and any side.

**THE AMBIGUOUS CASE** → 2. Two sides and an angle opposite to one of the given sides.



The sine rule states:

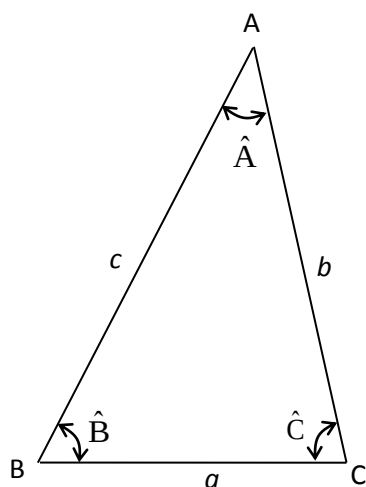
$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}}$$

➤ **The cosine rule:**

The cosine rule is used when given:

1. Two sides and the angle between them.

2. All three sides.



The cosine rule states:

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \hat{A}$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \hat{B}$$

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \hat{C}$$