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LECTURER: Dr Keerthi Ranasinghe (k.ranasinghe@uwtsd.ac.uk)

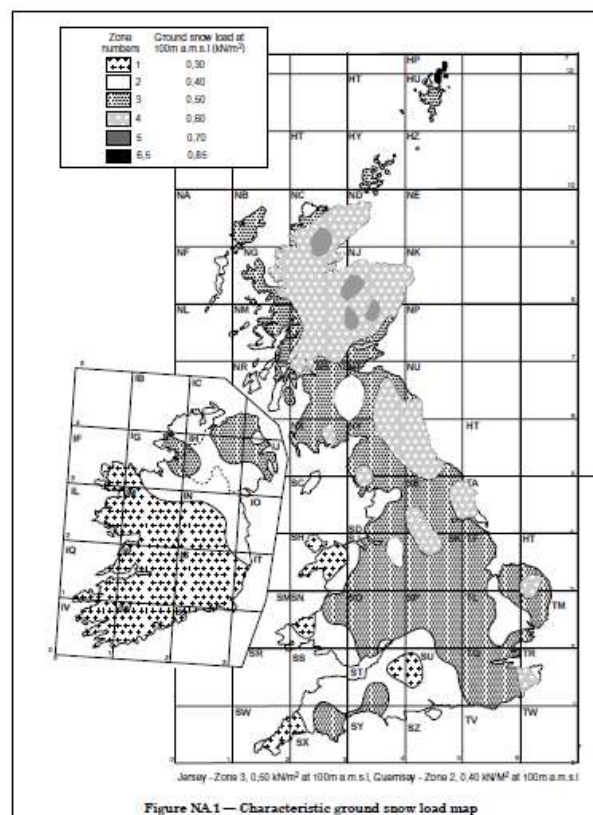
Date: 06.04.2018

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Types of Roof Loads:

1. Snow Loads:

Most countries with potential snow loads have a map indicating the severity of the load to expect. In the UK, the National Annex to BS EN 1991-1-4 has a map showing the snow intensity expected on the ground for different areas of the country as seen below.

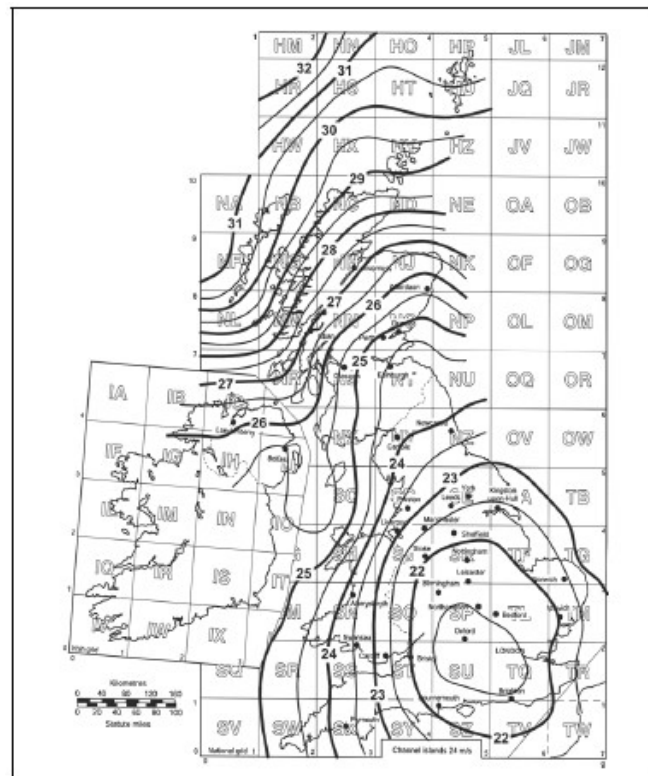


From the ground snow intensity, equations within the standard allows us to calculate the intensity of snow loads (in kN/m^2) that could be expected on the roof.

Assignment Note: We are ignoring the snow loads for this assignment. If you are interested to know more about snow loads please talk to me separately.

2. Wind Loads

As with snow loads, wind too has an associated map showing the wind speeds expected around the country. Many factors, such as the topography of the area, proximity to the sea or other tall buildings around affect the wind speeds. In the UK, the National Annex to BS EN 1991-1-4 has the following map.



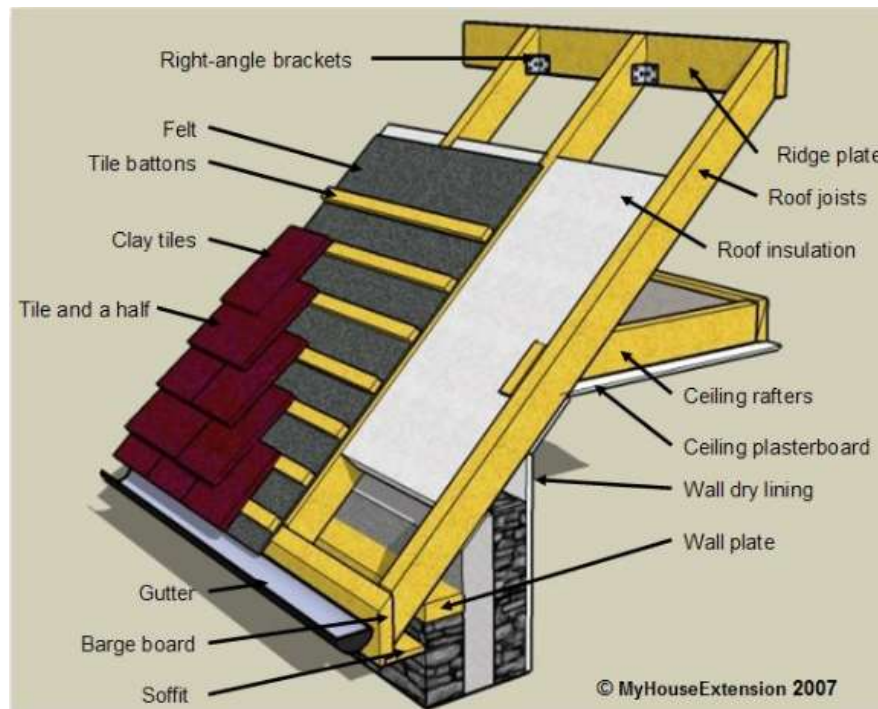
The wind speed alone is not suitable to be used as a load, so equations within the standard allow us to convert these wind speeds to wind pressures (in kN/m^2). Due to the wind outside, certain parts of a roof may experience negative wind pressures, which could result in the roof getting blown away if not properly designed.

Assignment Note: We are ignoring the wind loads for this assignment. If you are interested to know more about wind loads please talk to me separately.

3. Dead Loads

Dead loads are the self-weights associated with the different components of a roof. For example, the weights of roof tiles, battens and rafters etc. Different roof tiles will have different weights. Same type of roof tile coming from different manufacturers can have different self-weights. There are standard tables giving some generic values for these self-weights, but we normally tend to rely on manufacturer data. We look at architects' drawings

to understand how the roof is built-up, meaning put together with different elements and materials, and then add each load component to come up with a roof dead load (in kN/m^2).



Assignment Note: We are ignoring the dead loads for this assignment. If you are interested to know more about dead loads please talk to me separately.

4. Roof Imposed Loads

By roof imposed loads we mean the loads we apply on roofs from time to time, for example for maintenance work. If we install solar panels on the roof, this too could become an imposed load. The way some standards present the imposed loads could also cover simple situations of snow loads as well. In the UK, the National Annex to BS EN 1991-1-1 gives us the following table.

Table NA.7 — Imposed loads on roofs not accessible except for normal maintenance and repair

Roof slope, α degrees	q_k kN/m^2	Q_k kN
$\alpha < 30^\circ$	0,6	0,9
$30^\circ \leq \alpha < 60^\circ$	$0,6[(60 - \alpha)/30]$	
$\alpha \geq 60^\circ$	0	
NOTE 1 All roof slopes α are measured from the horizontal and all loads should be applied vertically.		
NOTE 2 In evaluating Table NA.7 for curved roofs the roofs should be divided into not less than five equal segments and the mean slope of each segment considered to be equivalent to the roof slope, α .		
NOTE 3 BS EN 1991-1-1:2002, Note 3 to Table 6.10 states that q_k may be assumed to act on an area A . It is recommended that the value of A should be the whole area of the roof.		

Assignment Note: This is the load that I would like you to use for your assignment.

I have already assigned a roof pitch (angle of slope of the roof, α) to each one of you in class. So based on that angle please work out your specific roof imposed load. I have used the angle assigned to me to show you below how the load is calculated.

$$\alpha = 57.5^\circ$$

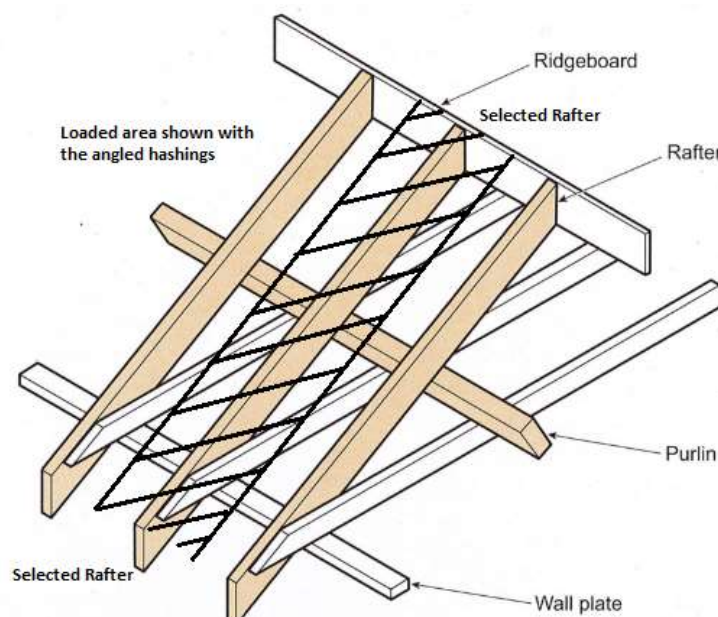
$$q_k = 0.6 \times \left[\frac{(60 - 57.5)}{30} \right] = 0.05 \text{ kN/m}^2$$

There is also a $Q_k = 0.9 \text{ kN}$ load in the above table. This is to denote something like a man standing on the roof. For calculation of maximum bending moment, we are required to assume that the man is standing right at the centre of a rafter. However, in calculating the maximum shear, you may quickly realise the maximum shear force is always the support reaction for the simple situations we are dealing with. So, to get the maximum reaction, we may have to move the man over on to a support. Please don't think of this as complicated. I will explain why later.

Read the NOTE 1 in the table above. It says these loads are to be applied vertically downwards. Which means the load I calculated above is to be applied to the rafters vertically downwards. However, before we do that, we need to understand what the above distributed loading really means.

Read it aloud, it says 0.05 kN per square metre of roof area. Which means that for each square metre of roof area there is a 0.05 kN load applied vertically downwards. Now let's look at how rafters are arranged in a roof. There are usually a number of rafters within one roof elevation, with a regular spacing between them. Spacing between rafters could be anything, but traditionally, to allow fitting of regular sized sheathing panels between them, we have spacing of 400 mm , 450 mm & 600 mm . More common is 600 mm , so we will use this in our assignment.

Now please look at the diagram below.



The diagram shows the area of roof that is loading one rafter. The areas of roofs adjacent to the marked areas are not loading the rafter shown, as these other areas can be considered as loading the adjacent rafters in a similar manner. Is this clear? After all, we are only designing (sizing) one rafter, and then use the same size everywhere. I selected the area by finding the mid-point between two rafters. The whole loading area on top of this selected rafter therefore is equal to;

$$\text{spacing between rafters} \times \text{the length of the rafter}$$

Through our many lessons in class about bending moments and shear forces, we are already familiar with Uniformly Distributed Loads (UDL's). Can you see a way of converting the load we found above as loading the rafter (0.05 kN/m^2) to a UDL? UDL's have units of kN per metre . In other words, for every metre of length of the rafter there is a load. See what I do below.

$$\text{spacing} = 600 \text{ mm} = 0.6 \text{ m}$$

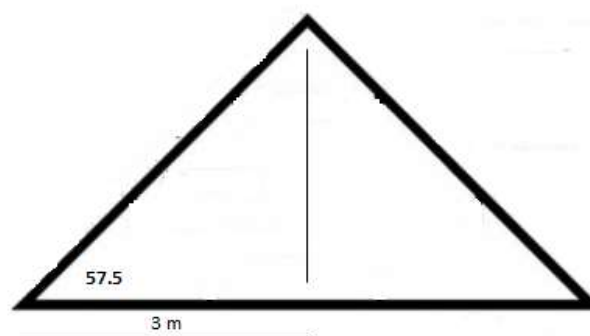
$$\text{our roof load, } q_k = 0.05 \text{ kN/m}^2$$

If I multiply the above two together;

$$\text{spacing} \times \text{our roof load, } q_k = 0.6 \text{ m} \times 0.05 \text{ kN/m}^2 = 0.03 \text{ kN/m}$$

What we found above is a UDL of $0.03 \text{ kN per metre}$ along the length of the rafter. What is the length of the rafter though?

All of you were assigned a half-width to the roof. I worked on the board with a 3m assigned to myself. If I then show the angle assigned and the roof half-width assigned on a right angled triangle (which is what half of a roof is), then it looks like below.

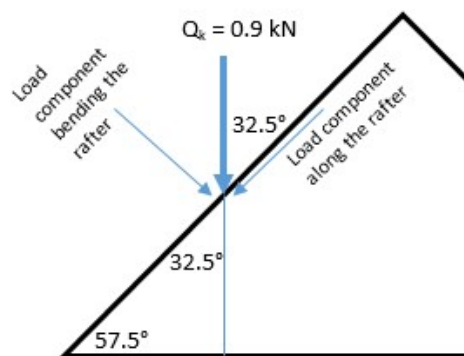


From the right-angled triangle formed;

$$\cos(57.5) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{3 \text{ m}}{\text{Length of the rafter}}$$

$$\text{Length of the rafter} = 5.584 \text{ m}$$

We now have all the information we need about the roof for the assignment, but there is only one problem. The loads are vertically downwards, whereas the rafter is at an angle to the horizontal. When loads are applied like that, part of the load tends to bend the rafters and the other part acts along the rafter with no bending but with compression. For our assignment we will find the part of the load that tends to bend the rafter and ignore the part of the load that goes along the rafter (although in real-life situations we will have to consider this too). Please look at the diagram below, where I have taken the $Q_k = 0.9 \text{ kN}$ load we found earlier to explain what we are about to do next.



From what we learned in class about the trigonometric functions (i.e. “sin”, “cos” etc.), we can see that;

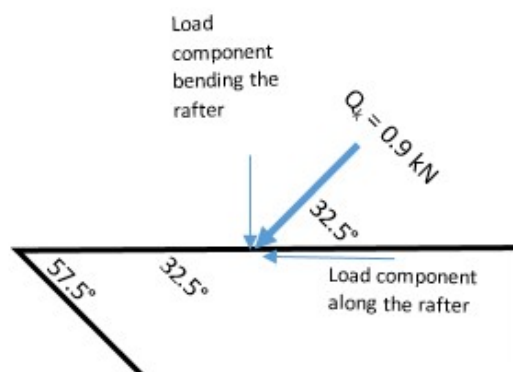
$$\text{Load component along the rafter} = 0.9 \text{ kN} \times \cos(32.5) = 0.759 \text{ kN}$$

$$\text{Load component bending the rafter} = 0.9 \text{ kN} \times \sin(32.5) = 0.484 \text{ kN}$$

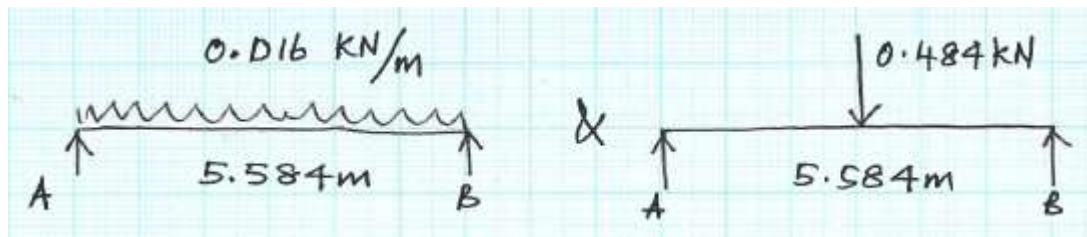
Likewise, if we look at the UDL we found earlier, the load component of the UDL that tends to bend the beam is;

$$= 0.03 \text{ kN/m} \times \sin(32.5) = 0.016 \text{ kN/m}$$

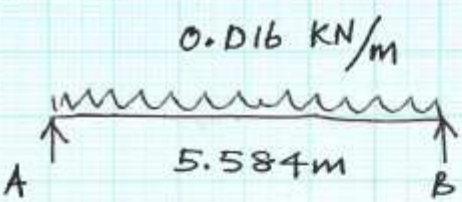
Now we have everything we need to complete this part of the assignment. Since the loads and the angles are now dealt with, we can even start drawing the rafter “horizontally”, as that’s how we are used to seeing “beams” in bending.



So for my numbers, what I have is as follows;



What we then normally do is find the maximum bending moment and shear forces from each of these and only use the higher values for our design. This is because we know from statistics the chances of both these loads being applied at the same time is very low. So if we design for the higher of the two, then when the other load appears, it's already looked after by the design.

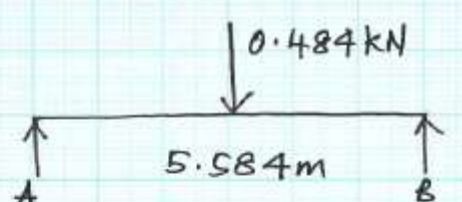


TOTAL DOWNWARD LOAD
 $= 0.016 \times 5.584$
 $= 0.089 \text{ kN}$

THEREFORE, DUE TO SYMMETRY
 $R_A = R_B = \frac{0.089 \text{ kN}}{2}$
 $= 0.0445 \text{ kN}$

THESE REACTIONS ARE ALSO
 THE MAXIMUM SHEAR FORCES.
 DRAW A SHEAR FORCE
 DIAGRAM AND SEE.

MAXIMUM BENDING MOMENT
 $= \frac{wL \times \text{SPAN}^2}{8}$
 $= \frac{0.016 \times 5.584^2}{8}$
 $= 0.062 \text{ kNm}$



DUE TO SYMMETRY
 $R_A = R_B = \frac{0.484 \text{ kN}}{2}$
 $= 0.242 \text{ kN}$

MAXIMUM BENDING MOMENT:
 IS UNDERNEATH THE LOAD
 $= R_A \times \text{DISTANCE TO THE LOAD}$
 $= R_B \times \text{DISTANCE TO THE LOAD}$
 $= 0.242 \times \frac{5.584}{2} \text{ kNm}$
 $= 0.676 \text{ kNm}$

So for my design, I will select the maximum bending moment of 0.676 kNm and the maximum shear force of 0.242 kN. However, remember our earlier discussion about moving the man over on to a support to get the maximum reaction. My man load is 0.484 kN. Therefore, instead of 0.242 kN here, we use 0.484 kN as the maximum shear force.

We will now look at the Ceilings too and do the timber designs together later.

Types of Ceiling Loads

1. Dead Loads:

Ceiling dead loads are found similar to what we discussed regarding roofs. Look at all the materials that make up a ceiling (joists, plasterboards etc.) and add their individual weights to find the self-weight of the ceiling.

Assignment Note: We are ignoring the dead loads for this assignment. If you are interested to know more about dead loads please talk to me separately.

2. Ceiling Imposed Loads

Similar to roof imposed loads, ceiling imposed loads are given in BS EN 1991-1-1 as follows. Please note that this time it's not the UK National Annex as the UK decided to adopt the European Standard values.

- (8) The following loads should be used for the design of frames and coverings of access hatches (other than glazing), the supports of ceilings and similar structures :
- a) without access: no imposed load;
 - b) with access: 0,25 kN/m² distributed over the whole area or the area supported, and the concentrated load of 0,9 kN so placed so as to produce maximum stresses in the affected member.

So the loads for our designs are;

- a. a distributed load of 0.25 kN/m², AND
- b. a point load of 0.9 kN placed in the centre of a ceiling joist.

I highlighted AND there because statistics do show that these two loads can be acting together. So, just like previously, we do our calculations separately, but then ADD the maximum shear and bending moment values to find the values we should design the timbers for.

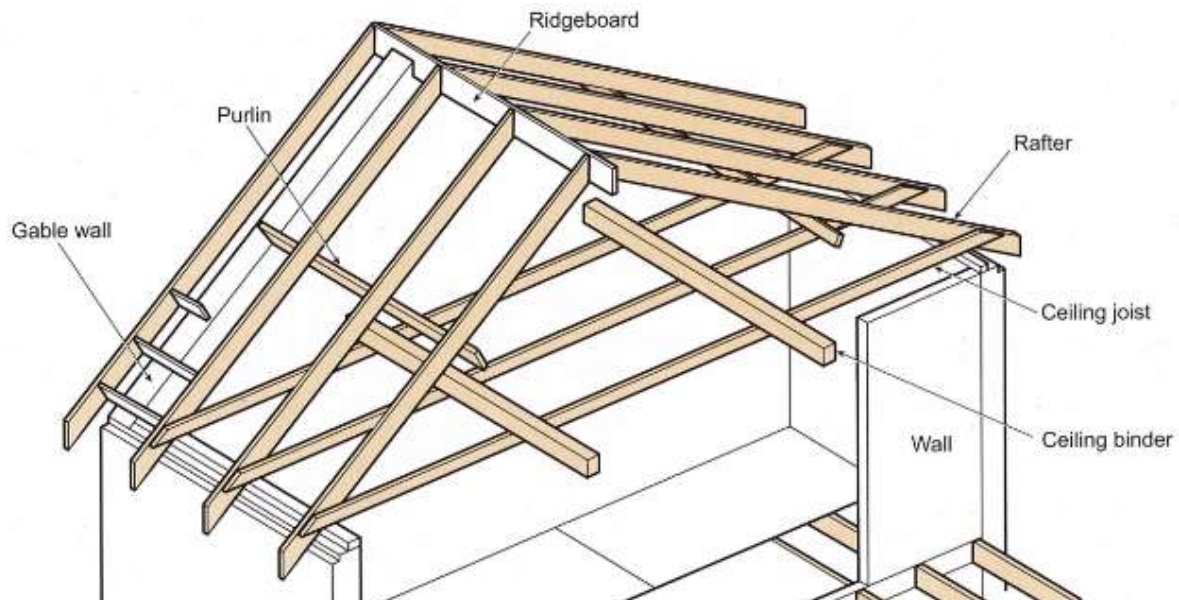
Like with rafters, ceiling joists are also arranged with a regular spacing between them. Again we will assume 600 mm. This is necessary because they have to match the rafter spacing, so that the rafters are held back from spreading out by the ceiling joists. The calculation of the UDL therefore, similar to previously, is;

$$\text{spacing} \times 0.25 \text{ kN/m}^2 = 0.6 \text{ m} \times 0.25 \text{ kN/m}^2 = 0.15 \text{ kN/m}$$

With these loads (0.15 kN/m & 0.9 kN) once again calculate the maximum bending moment and shear forces, which we will use to design our ceiling joists with.

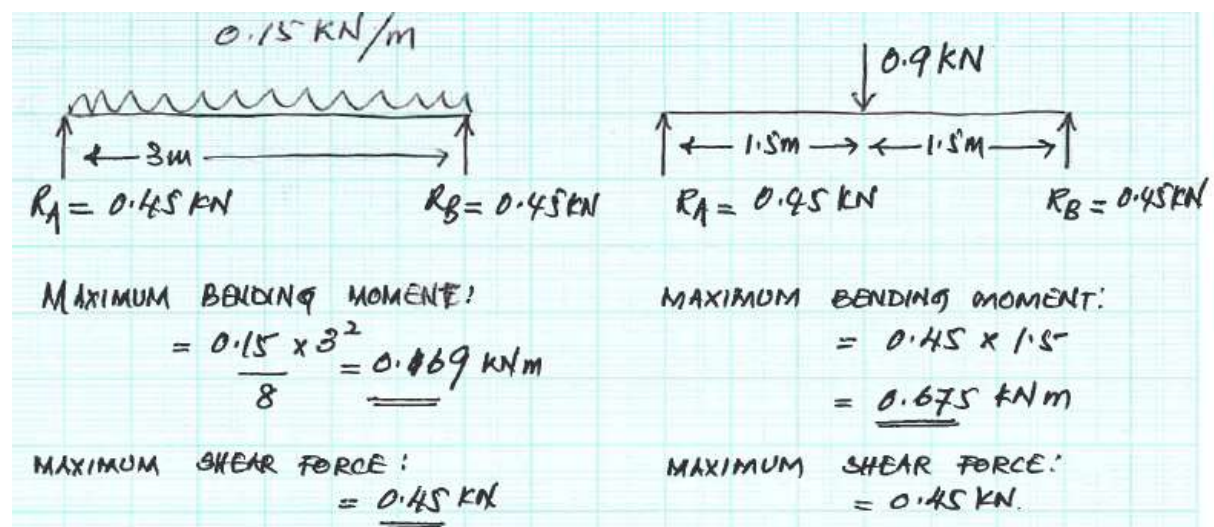
The building half-width that I assigned to myself was 3m. Which means, the ceiling joists will have to be 3m + 3m = 6m in length. With timber, most commonly available lengths are unfortunately a maximum of around 5.5m. Although nothing prevents us from ordering a 6m length from a saw-mill, what usually happens is that we try to work with the lengths readily

available to us from builders merchants. I am quite happy that I would be able to find timber lengths of 3m of whatever cross-section size that may come out of our design. So, I will make use of what is known as a “ceiling binder” to span my ceiling joists from the rafter end (wall) to the binder and then from the binder to the other wall. Please look at the diagram below where there are two binders shown. I can do with just one.



Interesting thing to note here is that the ceiling joist is “hung from” the binder, instead of being “supported on”. It doesn’t matter, as both these provide a “support”. Ceiling binders usually run from one gable wall to the other, supported on the brickwork/blockwork on pad stones.

So my decision is therefore to design a 3m long ceiling joist, which will then be supported by a ceiling binder. Following is what I have (please note I have done several steps of the calculation in one go);



So for the design, the maximum bending moment and shear forces are to be added. Also remember to move the man across on to a support.

Timber Design

Rafters:

My maximum bending moment and shear forces from above are;

- maximum bending moment = 0.676 kNm
- maximum shear force = ~~0.242 kN~~ 0.484 kN (moving the man across to a support)

Because we didn't apply load factors in the calculation above, we now apply the load factors as below. Load factor is a method whereby we increase the design load on a structure to account for any uncertainty in the calculation process. For example, what if we thought of one particular type of material for the roofing (say one manufacturer) but the contractors have used a slightly heavier material. This is just one example, but load factors also account for things like uncertainty of the statistical models used for example to come up with the snow loadings.

Load Factors:

The load factor to be used for imposed loads are given in BS EN 1990, and for our loads it is 1.5. We simply multiply the above moment and force by 1.5 to get;

- maximum bending moment = $0.676 \text{ kNm} \times 1.5 = 1.014 \text{ kNm}$
- maximum shear force = $0.484 \text{ kN} \times 1.5 = 0.726 \text{ kN}$

Section Sizes:

Timber design is unique in many senses. First the cross-section sizes available. Timber merchants have over the years developed a series of comfortable section sizes, mostly in imperial units. For example, what we commonly call a 2 x 2 refers to a 2 inches by 2 inches square cross section. In metric units this will be something like 50 mm x 50 mm square. Then we also have section sizes such as 38 mm, 44mm, 47mm, 50 mm, 63 mm, 75 mm, 100 mm, 125 mm, 150 mm, 175 mm, 200 mm, 300 mm etc. Anything above 300 mm may not be readily available. Also, when we have larger cross sections, as the timber dry out due to loss of moisture, they tend to split a lot. So for most building situations you may find the above range limited to between about 50 mm to 225 mm, or something near that. Larger sections are used in the heavy industries, such as for railway sleepers.

From the above we have to find a suitable rafter size for our use. Rafter sizes are also governed by the depth of the roof build-up the architect has in mind. This is mostly limited to around 100 mm to 200 mm. So, for my design I'm going to select a rafter depth of 150 mm. Rafter with is selected based on factors such as ease of nailing of battens on top. For example, if we don't have enough width, when we fix battens on top of the rafters, the nails driven may split the rafter. So we tend to have a higher width. For my design I decided to use a 50 mm width.

So, for my rafter design, all I have to do now is show that the rafter selected, 50 mm x 150 mm, is sufficient to carry the moment and forces I found above.

Stresses Applied:

Bending Stress:

We divide the maximum bending moment we found above by the “Section Modulus” of the cross-section selected to find the stress. The section modulus (W) of a rectangular cross-section used in the “stronger orientation” (when the same cross-section is used “flat-wise”, we say it is used in the “weaker orientation”) is always calculated as;

$$W = \frac{\text{width of the beam} \times \text{depth of the beam}^2}{6}$$

For 50 mm x 150 mm cross-section I selected above;

$$W = \frac{50 \text{ mm} \times (150 \text{ mm})^2}{6} = 187500 \text{ mm}^3$$

So the bending stress applied is;

$$\sigma_m = \frac{1.014 \text{ kNm}}{187500 \text{ mm}^3} = \frac{1014000 \text{ Nmm}}{187500 \text{ mm}^3} = 5.408 \text{ N/mm}^2$$

Shear Stress:

Shear stress is found simply by dividing the maximum shear force we found above by the area of the cross section we selected and then multiplying by 1.5 to find the maximum stress.

$$\tau_v = \frac{0.726 \text{ kN}}{50 \text{ mm} \times 150 \text{ mm}} \times 1.5 = \frac{726 \text{ N}}{50 \text{ mm} \times 150 \text{ mm}} \times 1.5 = 0.145 \text{ N/mm}^2$$

Strength Properties:

The second important thing is determining the strength of timber to be used. This is such a variable material that no one knows exactly what strength a single piece of timber may exhibit, unless that particular piece is tested. The test will give the strength for that piece, but it may not even indicate what strength to be expected from another piece of similar tree species, coming from the same tree even. This is such a vast subject, so at this stage we only need to know that there are tables of strength classes, together with the associated strength properties. The standard BS EN 338, which we are required to use in the UK, has the following table.

Table 1 — Strength classes - Characteristic values

		Softwood species												Hardwood species							
		C14	C16	C18	C20	C22	C24	C27	C30	C35	C40	C45	C50	D18	D24	D30	D35	D40	D50	D60	D70
Strength properties (in N/mm ²)																					
Bending	$f_{m,k}$	14	16	18	20	22	24	27	30	35	40	45	50	18	24	30	35	40	50	60	70
Tension parallel	$f_{t0,k}$	8	10	11	12	13	14	16	18	21	24	27	30	11	14	18	21	24	30	36	42
Tension perpendicular	$f_{t90,k}$	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,6	0,6	0,6	0,6	0,6	0,6	0,6	0,6
Compression parallel	$f_{c,0,k}$	16	17	18	19	20	21	22	23	25	26	27	29	18	21	23	25	26	29	32	34
Compression perpendicular	$f_{c,90,k}$	2,0	2,2	2,2	2,3	2,4	2,5	2,6	2,7	2,8	2,9	3,1	3,2	7,5	7,8	8,0	8,1	8,3	9,3	10,5	13,5
Shear	f_{vk}	3,0	3,2	3,4	3,6	3,8	4,0	4,0	4,0	4,0	4,0	4,0	4,0	3,4	4,0	4,0	4,0	4,0	4,0	4,5	5,0
Stiffness properties (in kN/mm ²)																					
Mean modulus of elasticity parallel	$E_{0,mean}$	7	8	9	9,5	10	11	11,5	12	13	14	15	16	9,5	10	11	12	13	14	17	20
5 % modulus of elasticity parallel	$E_{0,05}$	4,7	5,4	6,0	6,4	6,7	7,4	7,7	8,0	8,7	9,4	10,0	10,7	8	8,5	9,2	10,1	10,9	11,8	14,3	16,8
Mean modulus of elasticity perpendicular	$E_{90,mean}$	0,23	0,27	0,30	0,32	0,33	0,37	0,38	0,40	0,43	0,47	0,50	0,53	0,63	0,67	0,73	0,80	0,86	0,93	1,13	1,33
Mean shear modulus	G_{mean}	0,44	0,5	0,56	0,59	0,63	0,69	0,72	0,75	0,81	0,88	0,94	1,00	0,59	0,62	0,69	0,75	0,81	0,88	1,06	1,25
Density (in kg/m ³)																					
Density	ρ_k	290	310	320	330	340	350	370	380	400	420	440	460	475	485	530	540	550	620	700	900
Mean density	ρ_{mean}	350	370	380	390	410	420	450	460	480	500	520	550	570	580	640	650	660	750	840	1080
NOTE 1 Values given above for tension strength, compression strength, shear strength, 5 % modulus of elasticity, mean modulus of elasticity perpendicular to grain and mean shear modulus, have been calculated using the equations given in Annex A.																					
NOTE 2 The tabulated properties are compatible with timber at a moisture content consistent with a temperature of 20 °C and a relative humidity of 65 %.																					
NOTE 3 Timber conforming to classes C45 and C50 may not be readily available.																					
NOTE 4 Characteristic values for shear strength are given for timber without fissures, according to EN 408. The effect of fissures should be covered in design codes.																					

Most timbers sold in the UK are either C16 or C24. “C” in the table denotes “coniferous”, meaning softwoods and “D” denotes “deciduous”, meaning hardwoods. Timbers used for building works are mostly softwoods.

As we are only dealing with bending moments and shear forces, can you read out the strength values for bending and shear from the table above, if I say that we are going to use C24 strength class? There are so many other numbers, all equally important, but in carrying out our design we should not get distracted by all these numbers.

I selected the following values from the table. Can you agree?

Characteristic bending strength = 24 N/mm².

Characteristic shear strength = 4 N/mm².

Two things to note.

- NOTE 2 of the table says these properties are true for a temperature of 20°C and a relative humidity of 65%. These are the values to be expected inside a heated building in the UK. A roof, may or may not be heated. We assume it is unheated. So, the strength values above needs to be adjusted to suit. This is done by what is referred to as a “modification factor”. We will do that below.
- Second thing to note is that these values are given as “characteristic” properties. Meaning, they have not being modified at all to take any uncertainties into account. Like what we did with loads, we now use a “material factor” to modify (divide) the above values to account for these uncertainties. The material factor for timber, is always 1.3.

To deal with the first point we discussed, BS EN 1995-1-1, the British/European timber design standard gives the following table.

Table 3.1 – Values of k_{mod}

Material	Standard	Service class	Load-duration class				
			Permanent action	Long term action	Medium term action	Short term action	Instantaneous action
Solid timber	EN 14081-1	1	0,60	0,70	0,80	0,90	1,10
		2	0,60	0,70	0,80	0,90	1,10
		3	0,50	0,55	0,65	0,70	0,90
Glued laminated timber	EN 14080	1	0,60	0,70	0,80	0,90	1,10
		2	0,60	0,70	0,80	0,90	1,10
		3	0,50	0,55	0,65	0,70	0,90
LVL	EN 14374, EN 14279	1	0,60	0,70	0,80	0,90	1,10
		2	0,60	0,70	0,80	0,90	1,10
		3	0,50	0,55	0,65	0,70	0,90
Plywood	EN 636 Type EN 636-1 Type EN 636-2 Type EN 636-3	1	0,60	0,70	0,80	0,90	1,10
		2	0,60	0,70	0,80	0,90	1,10
		3	0,50	0,55	0,65	0,70	0,90
OSB	EN 300 OSB/2 OSB/3, OSB/4 OSB/3, OSB/4	1	0,30	0,45	0,65	0,85	1,10
		1	0,40	0,50	0,70	0,90	1,10
		2	0,30	0,40	0,55	0,70	0,90
Particle-board	EN 312 Type P4, Type P5 Type P5 Type P6, Type P7 Type P7	1	0,30	0,45	0,65	0,85	1,10
		2	0,20	0,30	0,45	0,60	0,80
		1	0,40	0,50	0,70	0,90	1,10
		2	0,30	0,40	0,55	0,70	0,90
Fibreboard, hard	EN 622-2 HB.LA, HB.HLA 1 or 2 HB.HLA1 or 2	1	0,30	0,45	0,65	0,85	1,10
		2	0,20	0,30	0,45	0,60	0,80
Fibreboard, medium	EN 622-3 MBH.LA1 or 2 MBH.HLS1 or 2 MBH.HLS1 or 2	1	0,20	0,40	0,60	0,80	1,10
		1	0,20	0,40	0,60	0,80	1,10
		2	–	–	–	0,45	0,80
Fibreboard, MDF	EN 622-5 MDF.LA, MDF.HLS MDF.HLS	1	0,20	0,40	0,60	0,80	1,10
		2	–	–	–	0,45	0,80

Once again, it's a huge table covering all timber related materials. Don't get distracted though. To get our required modification factor we need a way to "enter" the table. Remember what we discussed above? A heated indoor environment? This, according to the table above, is "Service Class 1". If it is indoor but not heated, like our roof, then it's Service Class 2. If it is external, then it's Service Class 3. So, from the discussion so far, we refer to the second line of numbers under "Solid Timber" from the above table. But which column?

Our imposed load is a short-term, maintenance type load. What load should be associated with what "duration" then? Why do we need to associate loads with durations? Timber, and most other materials too, lose strength over time. In timber it is more pronounced in comparison to materials such as steel. This is why we associate the loads with durations. The following table, taken from the National Annex to BS EN 1995-1-1 shows most common building loads and their durations.

Table NA.1 Load-duration classes

Load-duration class	Duration	Examples of loading
Permanent	More than 10 years	Self-weight
Long-term	6 months to 10 years	Storage loading (including in lofts), water tanks
Medium-term	1 week to 6 months	Imposed floor loading
Short-term	Less than 1 week	Snow, maintenance or man loading on roofs, residual structure after accidental event
Instantaneous		Wind, impact loading, explosion

See “man on roof” and “maintenance” above? They are both short-term loads.

With “Service Class 2” and “Short-term”, can you find the “modification factor, k_{mod} ” value then? I read it as 0.9.

In summary then, the two factors to deal with the two “points” noted above;

- 0.9
- 1.3

We use the above information to modify the characteristic strength properties we found as follows;

Bending Strength:

Characteristic bending strength, $f_{m,k} = 24 \text{ N/mm}^2$.

$$\text{Design Bending Strength, } f_{m,d} = \frac{f_{m,k}}{1.3} \times 0.9 = \frac{24}{1.3} \times 0.9 = 16.615 \text{ N/mm}^2$$

Shear Strength:

Characteristic shear strength, $f_{v,k} = 4 \text{ N/mm}^2$.

$$\text{Design Shear Strength, } f_{v,d} = \frac{f_{v,k}}{1.3} \times 0.9 = \frac{4}{1.3} \times 0.9 = 2.769 \text{ N/mm}^2$$

Can you see that once this conversion has taken place the strength properties are now called “design” properties.

Verification:

All we now got to do is see if the applied stresses are lower than the design strengths found. If they are, the section size selected is OK for the rafters.

Bending Verification:

$$\text{Applied bending stress} = 5.408 \text{ N/mm}^2$$

$$\text{Design bending strength} = 16.615 \text{ N/mm}^2$$

In bending the section is OK for the rafter as the applied bending stress is lower than the design bending strength.

Shear Verification:

$$\text{Applied shear stress} = 0.145 \text{ N/mm}^2$$

$$\text{Design shear strength} = 2.769 \text{ N/mm}^2$$

In shear the section is OK for the rafter as the applied shear stress is lower than the design shear strength.

As both these verifications are OK, the rafter size selected is OK. We may have even gone for a smaller section size and still satisfied the verifications above.

Ceiling Joists:

My maximum bending moment and shear forces from above are;

- c. maximum bending moment = $0.169 \text{ kNm} + 0.675 \text{ kNm} = 0.844 \text{ kNm}$
- d. maximum shear force = $0.45 \text{ kN} + 0.45 \text{ kN} = 0.9 \text{ kN}$ (moving the man across) = 1.35 kN.

Because we didn't apply load factors in the calculation above, we now apply the load factors as below.

Load Factors:

- c. maximum bending moment = $0.844 \text{ kNm} \times 1.5 = 1.266 \text{ kNm}$
- d. maximum shear force = $1.35 \text{ kN} \times 1.5 = 2.025 \text{ kN}$

Section Sizes:

For my design I decided to use a 50 mm width and a 150 mm depth, so I can order the same timber as the rafters.

Stresses Applied:

Bending Stress:

For 50 mm x 150 mm cross-section I selected above;

$$W = \frac{50 \text{ mm} \times (150 \text{ mm})^2}{6} = 187500 \text{ mm}^3$$

So the bending stress applied is;

$$\sigma_m = \frac{1.266 \text{ kNm}}{187500 \text{ mm}^3} = \frac{1266000 \text{ Nmm}}{187500 \text{ mm}^3} = 6.752 \text{ N/mm}^2$$

Shear Stress:

$$\tau_v = \frac{2.025 \text{ kN}}{50 \text{ mm} \times 150 \text{ mm}} \times 1.5 = \frac{2025 \text{ N}}{50 \text{ mm} \times 150 \text{ mm}} \times 1.5 = 0.405 \text{ N/mm}^2$$

Strength Properties:

Characteristic bending strength = 24 N/mm².

Characteristic shear strength = 4 N/mm².

One thing to note in addition to what we discussed earlier.

Table NA.1 Load-duration classes

Load-duration class	Duration	Examples of loading
Permanent	More than 10 years	Self-weight
Long-term	6 months to 10 years	Storage loading (including in lofts), water tanks
Medium-term	1 week to 6 months	Imposed floor loading
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Instantaneous		Wind, impact loading, explosion

As can be seen from the table above, which you have seen earlier too under rafters, storage loading is classified as a long-term load. Ceiling loads are, in the UK, mostly storage loads and therefore long term. So our k_{mod} value will be slightly different for the ceiling joists from the one that was selected for the rafters.

With “Service Class 2” and “Long-term”, I read the k_{mod} as 0.7.

We use the above information to modify the characteristic strength properties we found as follows;

Bending Strength:

Characteristic bending strength, $f_{m,k} = 24 \text{ N/mm}^2$.

$$\text{Design Bending Strength, } f_{m,d} = \frac{f_{m,k}}{1.3} \times 0.7 = \frac{24}{1.3} \times 0.9 = 12.923 \text{ N/mm}^2$$

Shear Strength:

Characteristic shear strength, $f_{v,k} = 4 \text{ N/mm}^2$.

$$\text{Design Shear Strength, } f_{v,d} = \frac{f_{v,k}}{1.3} \times 0.7 = \frac{4}{1.3} \times 0.9 = 2.154 \text{ N/mm}^2$$

Verification:

Bending Verification:

$$\text{Applied bending stress} = 6.752 \text{ N/mm}^2$$

$$\text{Design bending strength} = 12.923 \text{ N/mm}^2$$

In bending the section is OK for the ceiling joist as the applied bending stress is lower than the design bending strength.

Shear Verification:

$$\text{Applied shear stress} = 0.405 \text{ N/mm}^2$$

$$\text{Design shear strength} = 2.154 \text{ N/mm}^2$$

In shear the section is OK for the ceiling joist as the applied shear stress is lower than the design shear strength.

As both these verifications are OK, the ceiling size selected is OK. We may have even gone for a smaller section size and still satisfied the verifications above.