



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD
MAY 2017

CYFNOD ARHOLIADAU
MAI 2017

MODULE CODE: M6X00420
ADVANCED STRESS ANALYSIS & FEA

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

Adnoddau neu ddefnyddiau sydd eu hangen – Dim

A formula sheet is provided for reference.

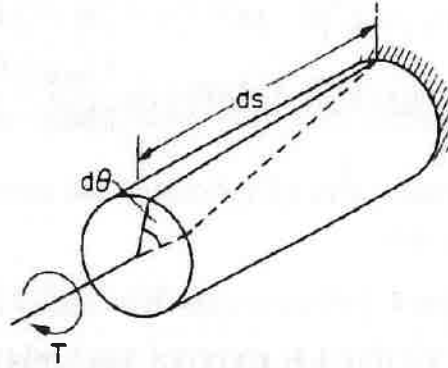
University approved calculators are permitted.

Assume 'g' = 9.81 m/s².

ONE

- a) (i) Consider the bar below subjected to torsion. Prove using the simple theory of elastic torsion ($\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$) that the change in angle, $d\theta$, due to the torque is given by $d\theta = \frac{Tds}{GJ}$, where T is the Torque, ds is the length of the bar, G is the shear modulus and J is the polar second moment of area.

[2 Marks]



- (ii) Hence prove that the strain energy, U , for a bar in Torsion is given by; $U = \frac{T^2 L}{2GJ}$

[4 Marks]

- (iii) What is meant by the term Resilience? Hence prove from the equation for a bar in tension ($U = \frac{P^2 L}{2AE}$, where P is the load applied, A is the cross sectional area and E is

the Youngs Modulus), that resilience may be defined as Resilience = $\frac{\sigma^2}{2E}$

[4 Marks]

b) (i)

A steel rod has a square cross section 20mm x 20mm & a length of 4m. Calculate the strain energy when a stress of 300MPa is produced by stretching it. ($E=200\text{GPa}$)

(360J)

[3 Marks]

- (ii) The rod then has a metal collar fitted at the lower end, and a mass, m , is dropped onto the collar. Show, by considering the energy transferred from the falling weight to the rod, that the height, h , is related to the stress, σ , by the following relationship;

$$h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$$

Where V is the volume of the bar, and L is the length. Assume that the extension of the bar is significant in the energy transfer.

[4 Marks]

- (iii) If the stress must not exceed 75MPa, from what height could 200kg fall onto the collar, without exceeding this?

(9.97mm)

[3 Marks]

TWO

A steel rectangular strut of hollow cross-section is to be used in the construction of an elevated test bed for a turbine. The external dimensions of the strut are 60 mm by 40 mm, and the wall thickness is a constant value of 4 mm for both dimensions. The strut is to be 950 mm in length and is assumed to be pinned at both ends to adjacent structural members.

The value of Young's Modulus, E , for the steel is 200 GN/m^2 , the compressive yield stress, σ_c , is 325 MN/m^2 , and the constant ' a ' of $1/7500$ for a pinned end condition.

- (a) By means of a suitable sketch of the cross-section of the strut, indicating given dimensions and appropriate axes through the centroid, determine about which axis the strut is likely to buckle.

(x-axis)

(4 marks)

- (b) According to the Euler theory of buckling, at what load will the strut buckle?

(389.33 kN)

(6 marks)

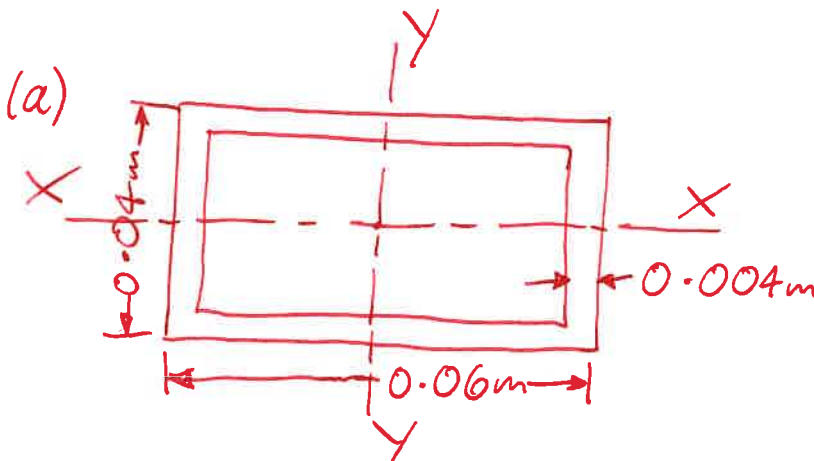
- (c) According to the Rankine-Gordon theory of buckling, at what load will the strut buckle?

(159.73 kN)

(6 marks)

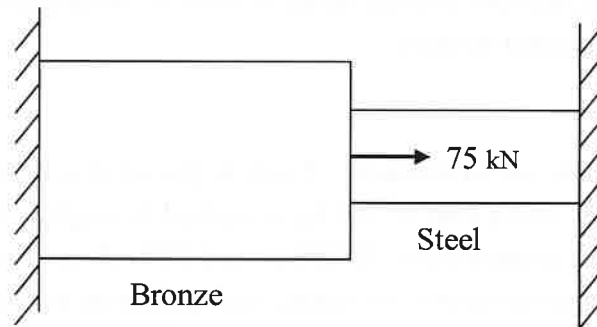
- (d) Based upon the results obtained in parts (b) and (c) above, and the use of an appropriate diagram, explain the difference between experimental results and buckling loads determined according to the Euler theory.

(4 marks)



THREE

Two dissimilar metals, bronze and low carbon steel, are to be friction welded together and placed between two rigid end supports as shown below. A force of 75 kN is applied as indicated in the diagram.



- (a) Draw a simple annotated sketch to show how the system above can be represented as two linear springs when applying the Finite Element Method (FEM). Indicate on your diagram the action of forces and displacements by using recognised notation.

(4 marks)

- (b) The bronze material has values of Young's Modulus of 105 GN/m^2 , cross-sectional area of 400 mm^2 and length 200 mm , and the steel body has values of Young's Modulus of 210 GN/m^2 , cross-sectional area of 200 mm^2 and length 125 mm . By means of Finite Element Analysis (FEA), analytically determine:

- (i) The magnitude and direction of the displacement at the interface between the two materials.

(12 marks)

- (ii) The magnitude and direction of the forces exerted on the end supports.

(3 marks)

- (iii) What can be deduced from the values of force found in part (ii) above?

(1 mark)

FOUR

The founding theory of contact mechanics involves analysis of *Hertzian* contact stresses, developed in the late 1800s. Fundamentally it is concerned with the deformation of solids that are in contact with each other at one or more points.

- (a) State the general assumptions that are made in order to analytically solve a problem involving *Hertzian* contact stresses.

(3 marks)

- (b) A spherical aluminium ball of diameter 15 mm is placed on a flat horizontal surface made from mild steel, and a load of 100 kg is applied through its vertical axis. If Young's Modulus for aluminium is 72 GN/m^2 and 210 GN/m^2 for mild steel, determine the maximum pressure (i.e. stress) and deflection at the point where the two bodies are in contact.

$$(p_0 = 2273.6 \text{ MN/m}^2; \delta = 0.0275 \text{ mm})$$

(10 marks)

- (c) If the flat surface in part (b) above was replaced by a mild steel spherical ball of diameter 30 mm, what would be the deflection for the same applied load of 100 kg?

$$(\delta = 0.0315 \text{ mm})$$

(4 marks)

- (d) Comment on the differences found in the values of deflection in parts (b) and (c) with particular reference to the theory of contact mechanics.

(3 marks)