



12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	1302736
2	Title of Examination Paper	Control Engineering
3	Module Code	SGEE 5009
4	Date and Time of Examination	18/5/23 09:30
5	Examination Venue	C5G
6	Seat Number	31
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
1	9
2	22
3	5
Total	36

Mark checked and
agreed
L Williams

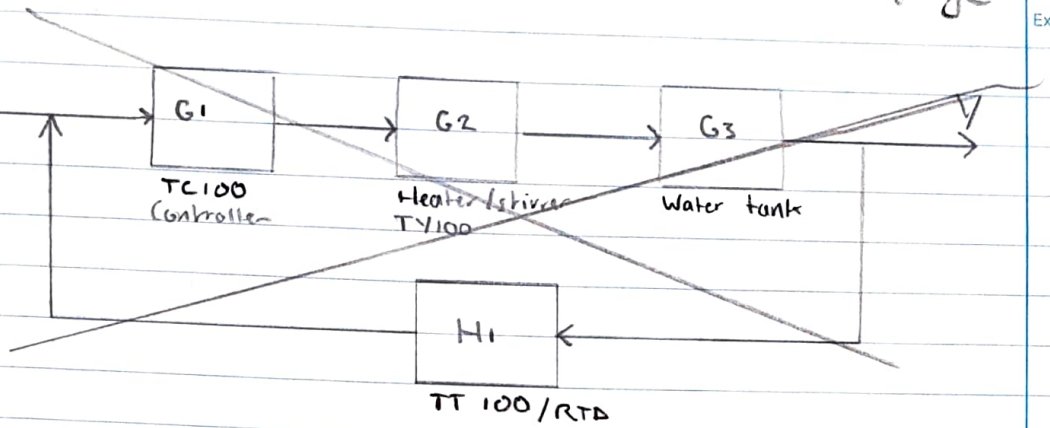
Block diagram below has been redrawn on the next page

Question No.

Examiner Only

Q1

a)



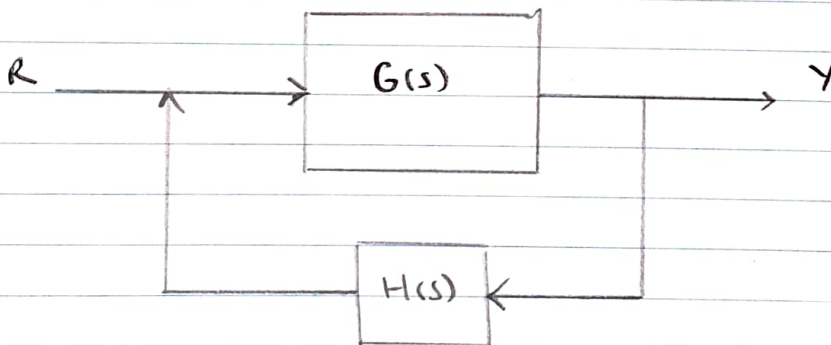
$$G_1 = k_p$$

$$G_2 = \frac{k_1}{\tau_1 s + 1} = \frac{0.5}{60s + 1}$$

$$H_1 = \frac{k_2}{\tau_2 s + 1} = \frac{1}{8s + 1}$$

$$\begin{aligned} b) \quad G(s) &= G_1 G_2 = k_p \times \frac{0.5}{60s + 1} \\ &= \frac{k_p 0.5}{60s + 1} \end{aligned}$$

$$T(s) = \frac{G(s)}{1 + H(s) G(s)}$$



$$G(s) = \frac{k_p 0.5}{60s + 1}$$

$$H(s) = \frac{1}{8s + 1}$$

$$T(s) = \frac{G(s)}{1 + H(s) G(s)}$$

$$T(s) = \frac{kp \cdot 0.5}{(60s + 1)}$$

$$1 + \frac{1}{(8s + 1)} \times \frac{kp \cdot 0.5}{(60s + 1)}$$

× top + bottom by $(60s + 1)(8s + 1)$

$$T(s) = \frac{kp \cdot 0.5 (8s + 1)}{480s^2 + 8s + 60s + 0.5}$$

$$= \frac{kp \cdot 0.5 (8s + 1)}{480s^2 + 68s + 0.5}$$

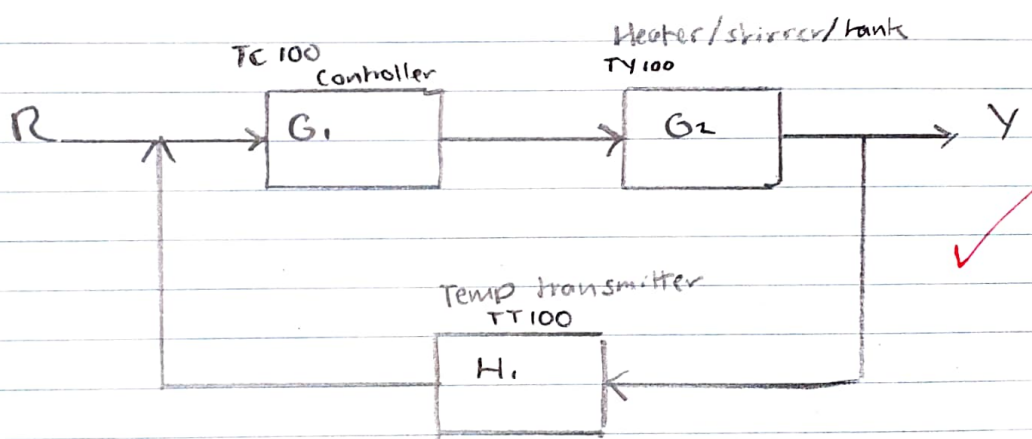
Compare above equation with characteristic equation below.

$$T(s) = \frac{k \cdot \omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

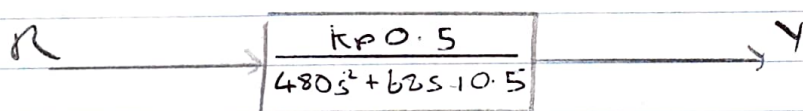
$$\omega_n^2 = 0.5$$

$$\therefore kp = 1$$

Q1a)



Q1b)



Q2.

$$a) \quad G(s) = \frac{25}{s(s+1)}$$

$$= \frac{25}{s^2 + s}$$

$$H(s) = 1$$

$$T(s) = \frac{G(s)}{1 + H(s)G(s)}$$

$$= \frac{25}{s(s+1)} \\ 1 + 1 \times \frac{25}{s(s+1)}$$

x top + bottom by (s+1)

$$T(s) = \frac{25}{s^2 + s + 25} \quad \checkmark$$

b) Compare the transfer function equation above with the characteristic equation below.

$$T(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \checkmark$$

$$i) \quad \omega_n = \sqrt{\omega_n^2} = \sqrt{25} = 5 \text{ rad/s} \quad \checkmark$$

$$ii) \quad \zeta = \frac{1}{2\omega_n} = \frac{1}{2 \times 5} = 0.1 \quad \checkmark$$

$$iii) \quad \omega = \omega_n \sqrt{1 - \zeta^2} = 5 \sqrt{1 - 0.1^2} = 4.975 \text{ rad/s} \quad \checkmark$$

$$iv) \quad t_r = \frac{\pi - \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}}{\omega} = 0.336 \text{ sec} \quad \checkmark$$

$$v) \quad t_p = \frac{\pi}{\omega} = \frac{\pi}{4.975} = 0.631 \text{ sec} \quad \checkmark$$

$$vi) \quad i.O.S = 100 \times e^{-\frac{\zeta\pi}{\sqrt{1 - \zeta^2}}} = 100 \times e^{-\frac{0.1\pi}{\sqrt{1 - 0.1^2}}} \\ = 72.92\% \quad \checkmark$$

$$vii) t_s = \frac{4}{\zeta \omega} = \frac{4}{0.1 \times 4.975} = 8.04 \text{ sec} \quad \checkmark$$

$$viii) n = \frac{1.5}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} \\ = \frac{1.5}{\pi} \sqrt{\frac{1}{0.1^2} - 1} = 4.751 \quad \times$$

c) Poles are found on the denominator of the transfer function: $s^2 + s + 25$

Using quadratic equation

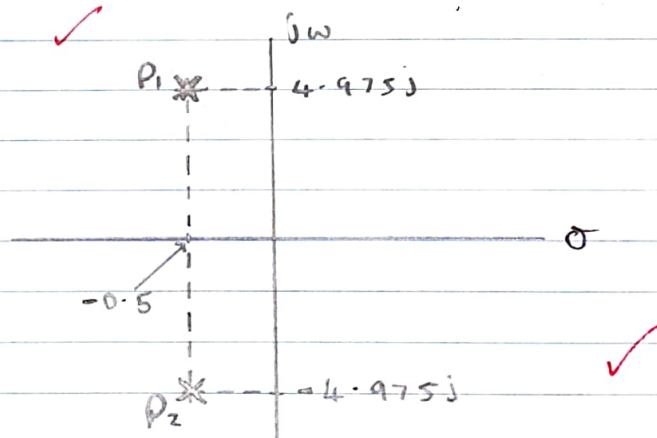
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-1 \pm \sqrt{1^2 - 4(1)(25)}}{2(1)}$$

$$\frac{-1 \pm \sqrt{-99}}{2}$$

$$\Rightarrow \frac{-1 \pm 9.95j}{2}$$

$$-0.5 \pm 4.975j$$



Poles are located on the left side of the argand diagram, therefore the system is stable.

Q3

a)

- i) All coefficients are present and none are negative $\therefore$ the system is stable. ✓
- ii) All coefficients are present but one is negative $\therefore$ system is unstable. ✓
- iii) One coefficient is missing but all are positive $\therefore$ system is at best critically stable. ✓

(3)

$$b) G_o(s) = \frac{3}{(s+2)} \times \frac{1}{s(s+4)}$$

$$= \frac{3}{s(s+2)(s+4)}$$

$$= \frac{3}{s(s^2 + 4s + 2s + 8)}$$

$$= \frac{3}{s^3 + 6s^2 + 8s} \quad \checkmark$$

(2)

$$T(s) = \frac{G_o(s)}{1 + H(s)G_o(s)}$$

$$= \frac{3}{(s^3 + 6s^2 + 8s)}$$

$$1 + k \times \frac{3}{(s^3 + 6s^2 + 8s)}$$

$$= \frac{3}{s^3 + 6s^2 + 8s}$$

Q1c)

$$G_1 = K_p K_i$$

$$G_2 = \frac{0.5}{60s + 1}$$

$$K_p + \frac{K_i}{s}$$

$$G(s) = \frac{K_p K_i 0.5}{60s + 1}$$

$$T(s) = \frac{G(s)}{1 + H(s) G(s)}$$

$$= \frac{K_p K_i 0.5}{(60s + 1)}$$

$$1 + \frac{1}{(8s+1)} \times \frac{K_p K_i 0.5}{(60s + 1)}$$

$\times$ top + bottom by $(60s+1)(8s+1)$

$$T(s) = \frac{K_p K_i 0.5}{480s^2 + 68s + 0.5}$$

Compare above with characteristic equation below

$$T(s) = \frac{K_p K_i \omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

$$\omega_n^2 = 0.5$$

$$K_p = 1$$

$$\therefore K_i = 1$$

