

1 a)

Piston assembly	0.24 kg
Con Rod	0.33 kg
BB shells	0.02 kg
Crankshaft	5 kg
R	45 mm
L	140 mm

$$M_{Rec} = 0.24 + \frac{0.33}{3}$$

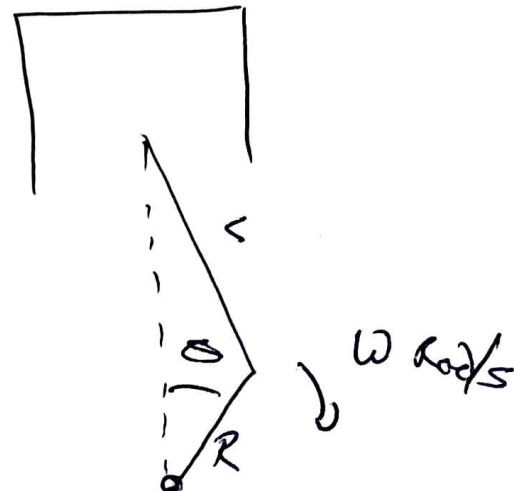
$$= 0.35 \text{ kg}$$

$$M_{Rot} = 2\left(\frac{0.33}{3}\right) + 0.02$$

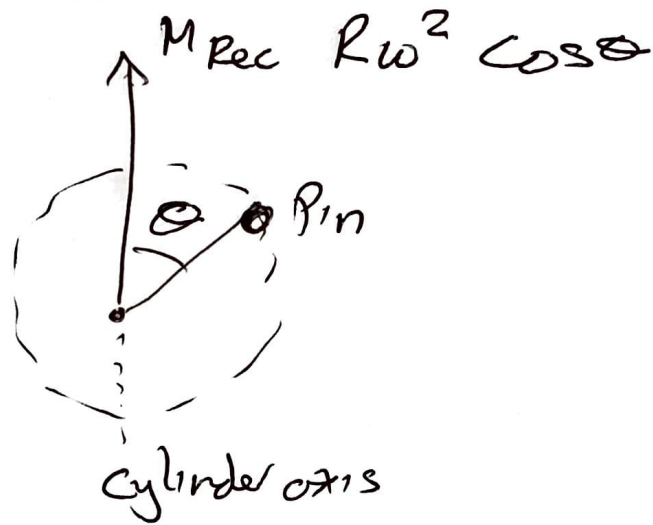
$$= 0.24 \text{ kg}$$

~~Rotating forces~~

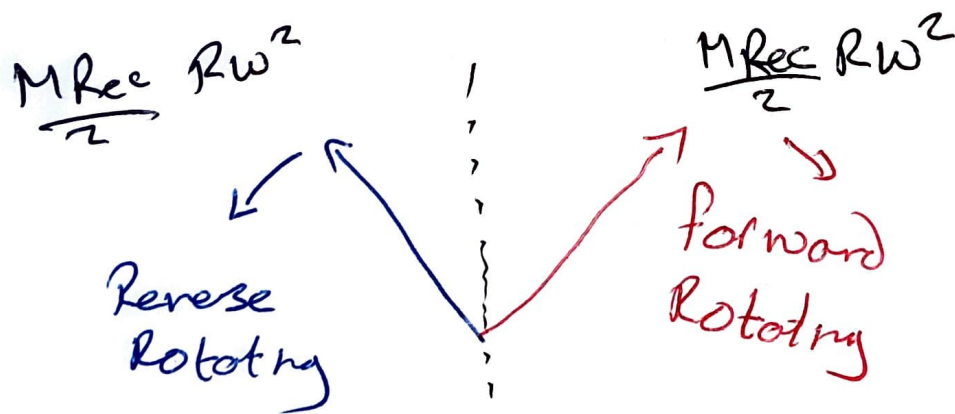
Reciprocating
forces



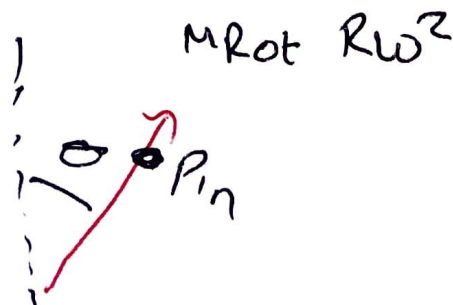
The first order vertical Forces can be expressed as:



this is mathematically identical to representing this force as two ~~can~~ constant amplitude counter rotating forces



The rotating mass creates a centrifugal force acting in the direction of the crank pin



The total first order force are represented as below



The mass moment of the resulting first order forward vector

$$= \left(M_{Rot} + \frac{M_{Rec}}{2} \right) R$$

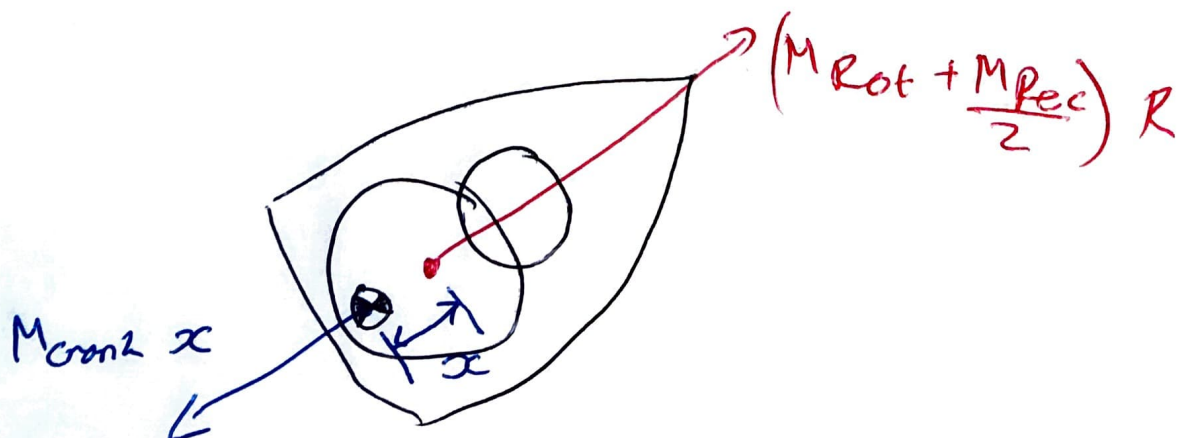
$$= \left(0.24 + \frac{0.35}{2} \right) \times 45$$

$$= 18.675 \text{ kg.m} \quad \text{Per cylinder}$$

$$= 37.35 \text{ kg.m} \quad \text{For both cylinders}$$

2 b) To balance the two moments must be equal

$$M_{crank} x = 2 \left(M_{Rot} + \frac{M_{Rec}}{2} \right) R$$



$$5x = 18.675 \times 2$$

$$x = \frac{3.735}{2} \text{ mm}$$

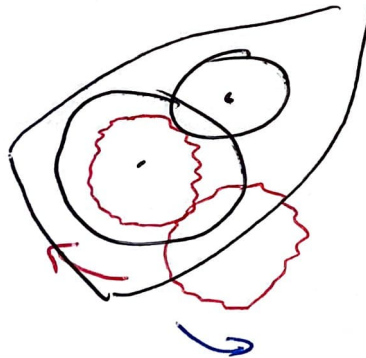
$$7.47 \text{ mm}$$

2c) M_m of reverse vector $= 2 \frac{M_{\text{pec}} R}{2}$

$$= \frac{0.35}{2} \times 45 \times 2$$

$$= \frac{7.875}{15.75} \text{ kg.mm}$$

2d)



I would choose gears of equal diameter because the shaft would run at equal speed but in opposite direction

2e)

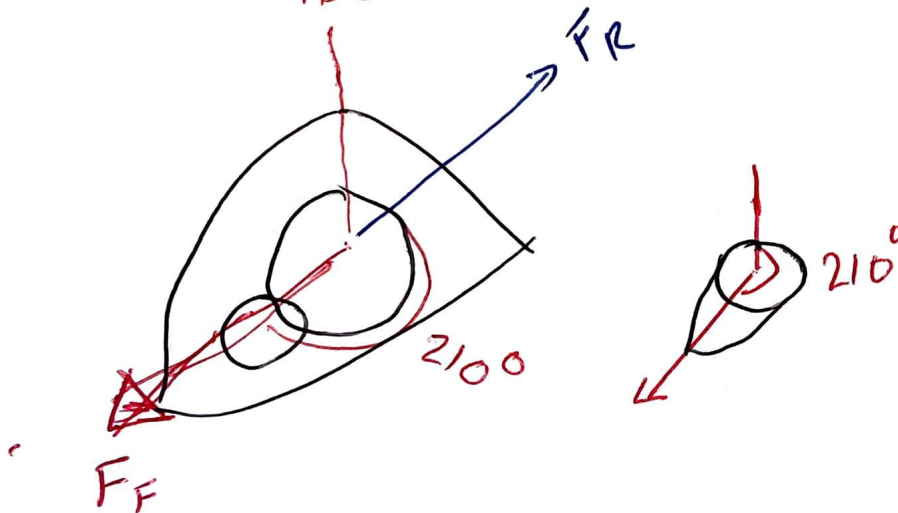
$$M_{bal} y = \frac{M_{Rec}}{2} R$$

$$2.5 y = \frac{15.75}{7.875}$$

$$y = \frac{6.3}{3.15} \text{ mm}$$

TDC

2f)



2g)

As the engine is mounted horizontally to minimise vertical forces the resultant forces need to be in the cylinder axis

$$(M_{Rot} + \frac{M_{Rec}}{2}) R \omega^2 - M_{Crank} x \omega^2 = \frac{M_{Rec}}{2} R \omega^2$$

$$M_{\text{crank}} \propto \omega^2 = \left(M_{\text{rot}} + \frac{M_{\text{rec}}}{2} \right) R \omega^2 - \frac{M_{\text{rec}}}{2} R \omega^2$$

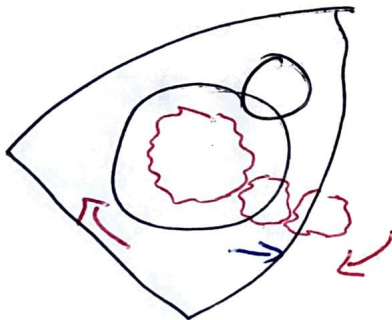
$$M_{\text{crank}} \propto \cancel{\omega^2} = M_{\text{rot}} R \cancel{\omega^2}$$

$$M_{\text{crank}} \propto = 2 M_{\text{rot}} R$$

$$M_{\text{crank}} \propto = 0.24 \times 45 \times 2$$

$$M_{\text{M}} = \cancel{10.8 \text{ kgm}} \\ 21.6 \text{ kgm}$$

h)



Use secondary gears
of half diameter
to achieve double speed
of rotation

Mass moment of second order

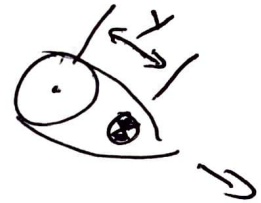
forward = back ward

$$= 2 \times \frac{1}{2} \frac{R}{L} M_{\text{rec}} R$$

$$= \frac{R}{L} M_{\text{rec}} R$$

$$\frac{45}{140} \times 0.35 \times 45 = 5.0625 \text{ kgmm}$$

$$\begin{aligned} \text{Balancing force} &= M_{\text{bal}} y (2\omega)^2 \\ &= 4 M_{\text{bal}} \cdot y \omega^2 \end{aligned}$$



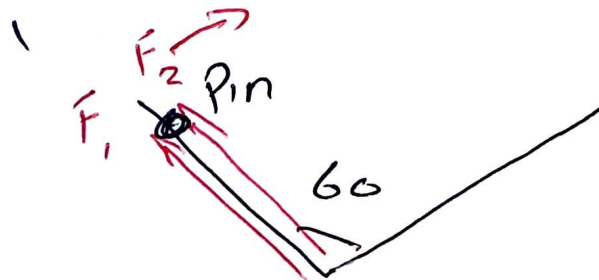
$$M_m = 4 M_{\text{bal}} \cdot y$$

$$\frac{M_m}{4} = 5.0625 \text{ kgmm}$$

$$\begin{aligned} 2a) \quad R &= 40 \text{ mm} \\ L &= 140 \text{ mm} \end{aligned}$$

$$M_{\text{Rot}} = 0.24 \text{ kg}$$

$$M_{\text{Rec}} = 0.36 \text{ kg}$$



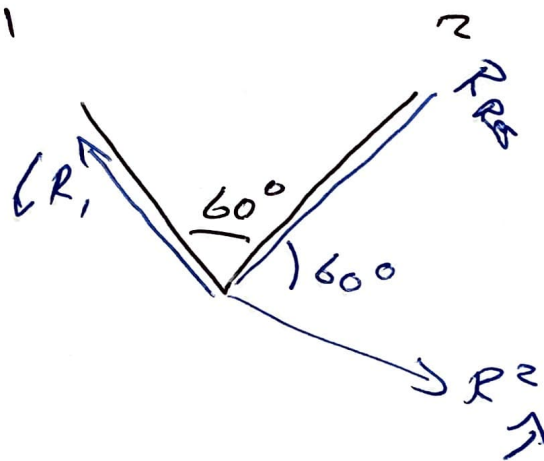
$$F_1 = F_2 = \left(M_{Rot} + \frac{M_{Rec}}{2} \right) R \omega^2$$

Resultant Mass moment = $2 \left(M_{Rot} + \frac{M_{Rec}}{2} \right) R$

$$= 2 \times \left(0.24 + \frac{0.36}{2} \right) \times 40$$

$$= 33.6 \text{ kg mm}$$

2 b)



$$R_{res} = \frac{M_{Rec}}{2} R \omega^2 \cos 60 + \frac{M_{Rec}}{2} R \omega^2 \cos 60$$

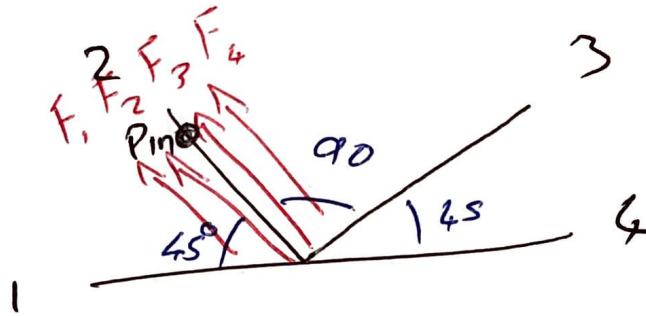
$$M_{res} = \frac{M_{Rec}}{2} R \cos 60 + \frac{M_{Rec}}{2} R \cos 60$$

$$= \frac{M_{Rec}}{2} R$$

$$\cos 60 = 0.5$$

$$= \frac{0.36}{2} \times 40 = 5.2 \text{ kg mm}$$

2 c)



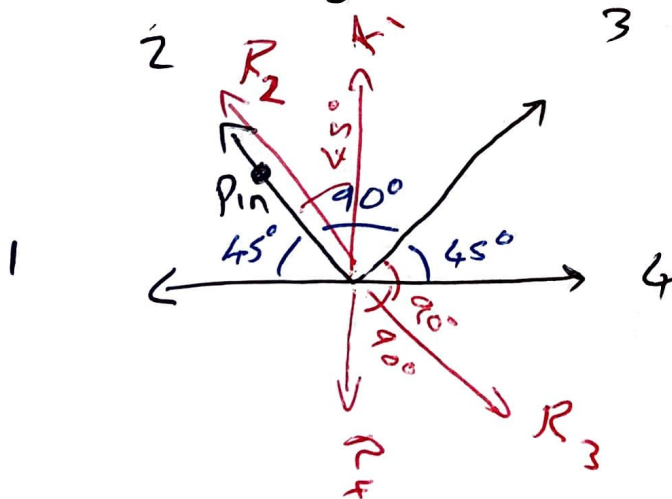
$$F_1 = F_2 = F_3 = F_4 = \left(M_{Rot} + \frac{M_{Rec}}{2} \right) R \omega^2$$

$$M_M = 4 \times \left(M_{Rot} + \frac{M_{Rec}}{2} \right) R$$

$$M_M = 4 \times (0.24 + 0.13) \times 40$$

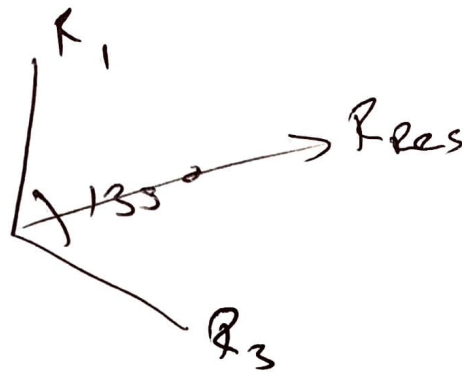
$$= 59.2 \text{ kg.m}$$

2d)



Resolve

 R_1 and R_3 R_2 and R_4



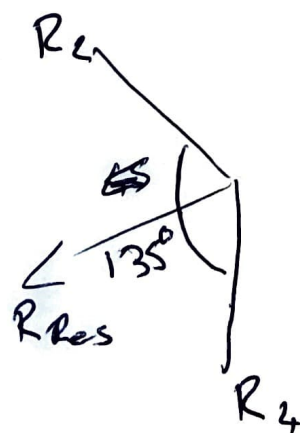
$$R_1 = R_3 = \frac{M_{Rec}}{r} 2w^2 \cos 67.5$$

$$R_{Res} = R_1 + R_3$$

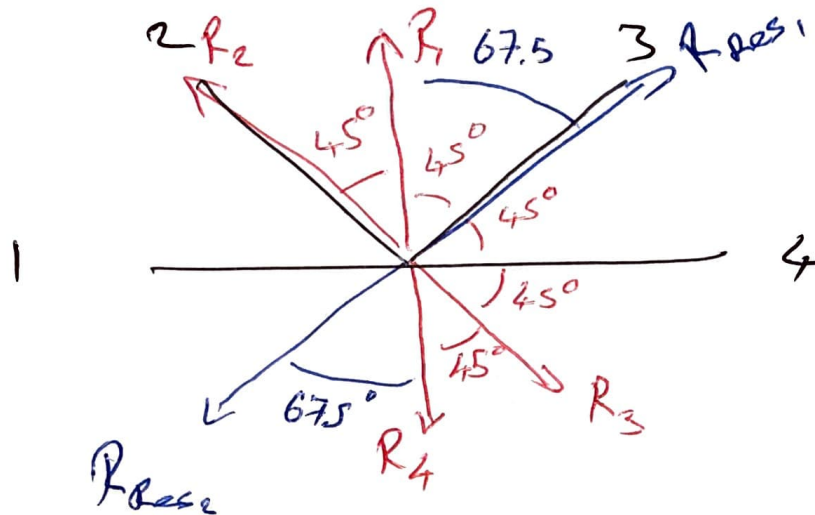
$$M_{Res} = \frac{M_{Rec}}{r} R \cos 67.5 + \frac{M_{Rec}}{r} R \cos 67.5$$

$$= (5.2 \cos 67.5) 2$$

$$= 3.98 \text{ kgmm}$$



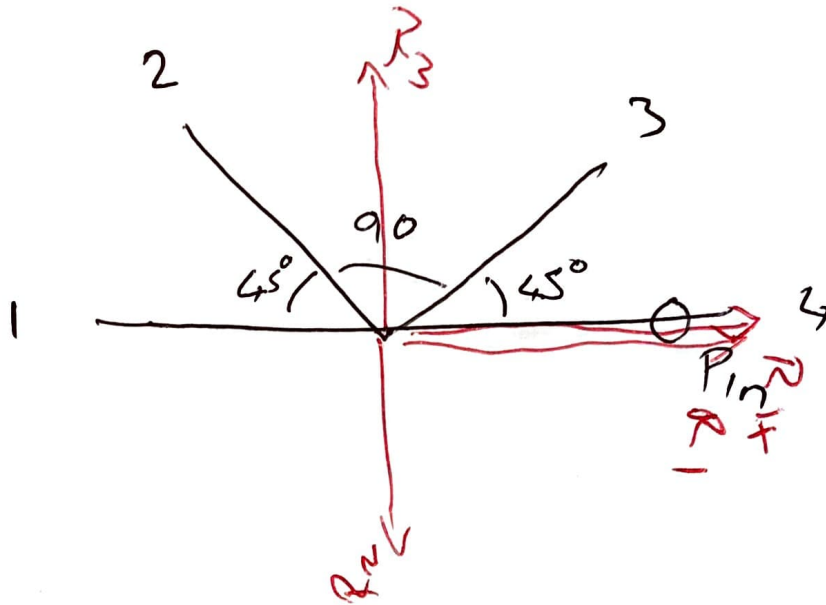
$$M_{Res} \text{ for } R_2, R_4 = M_{Res} \text{ for } R_3, R_1$$



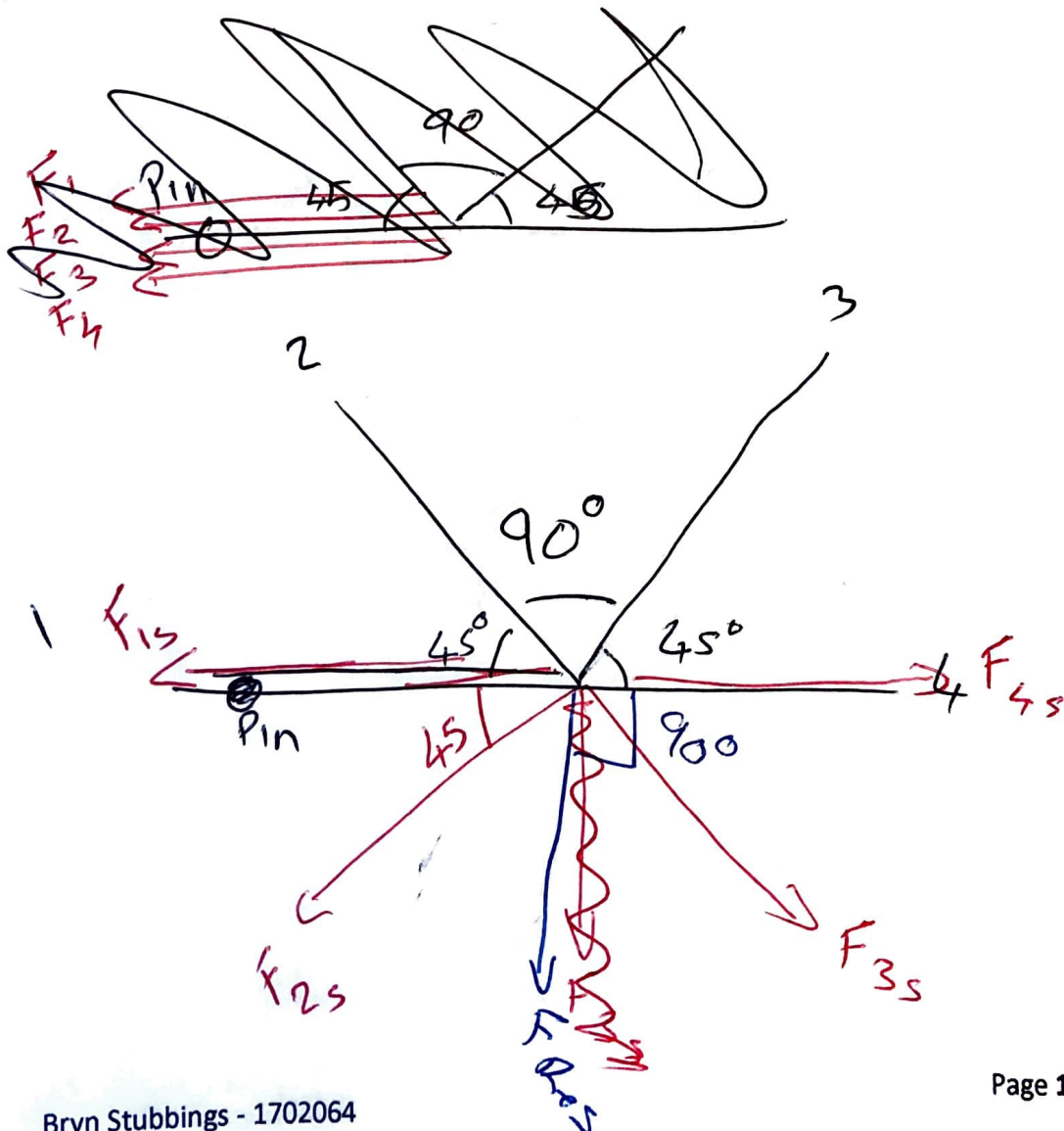
as they are equal and opposite
the resultant Mass moment = 0

2e) for the forward rotating forces
I would use the crank shaft to
balance ~~the~~ the forces
the reverse rotating forces do not
need balancing

2 f)



2 g)



$$F_{1s} = F_{2s} = F_{3s} = F_{4s} = \frac{1}{2} \frac{P}{L} M_{\text{rec}} R W^2$$

$$F_{\text{Res}} = F_{1s} \cos 90 + F_{2s} \cos 45 + F_{3s} \cos 45 + F_{4s} \cos 90$$

$$\cos 90 = 0$$

$$\therefore F_{\text{Res}} = F_{2s} \cos 45 + F_{3s} \cos 45$$

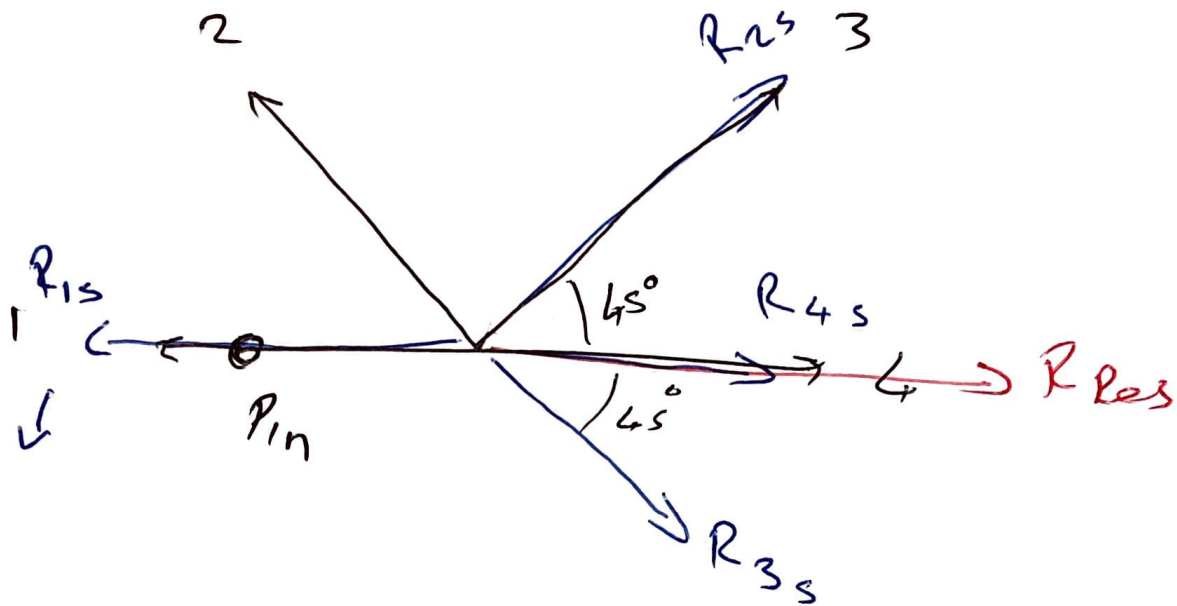
$$M_{\text{Res}} = \frac{1}{2} \times \frac{40}{140} \times 0.36 \times 40$$

$$= \cancel{0.05} \quad 2.06$$

$$M_{\text{Res}} = 2.06 \cos 45 + 2.06 \cos 45$$

$$= \cancel{0.673} \quad 2.91$$

2h)



as R_{1s} and R_{4s} are equal and opposite
the resultant mass moment is zero

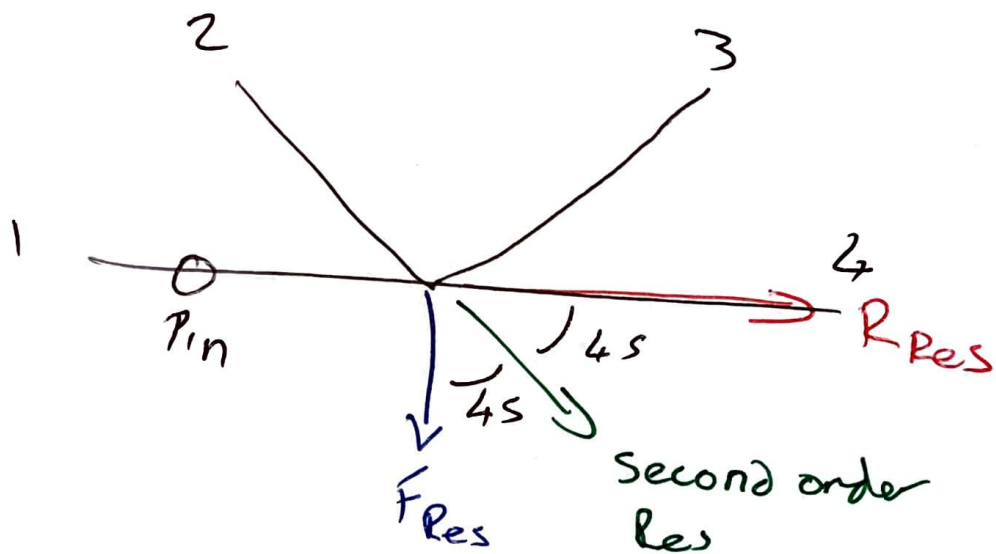
resultant mass moment of R_{2s} and R_{3s}

$$= \frac{1}{2} \frac{R}{L} M_{Rec} R \cos 45 + \frac{1}{2} \frac{R}{L} M_{Rec} R \cos 45$$

$$= \frac{1}{2} \times \frac{40}{140} \times 0.36 \cos 45 + \frac{1}{2} \times \frac{40}{140} \times 0.36 \cos 45$$

$$M_{Res} = \frac{2.91}{2.91} \text{ kgmm}$$

21)



$$M_{M_{Res}} = 2.91 \cos 45 + 2.91 \cos 45$$

$$\text{Second order Res} = 4.115 \text{ kg mm}$$