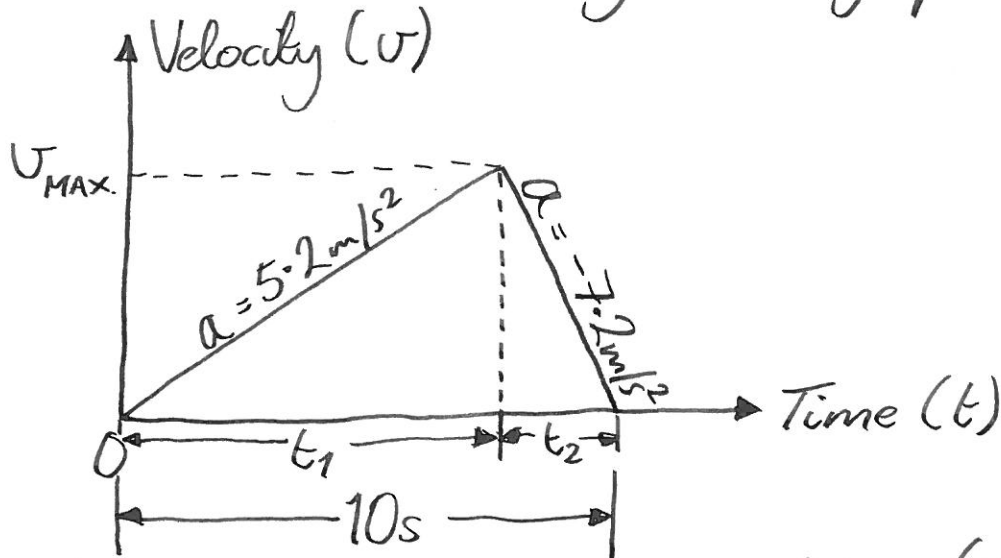


DYNAMICS REVISION L5.

(R1)

Q1. A vehicle is capable of a maximum acceleration of 5.2 m/s^2 and a maximum retardation of 7.2 m/s^2 . Calculate the maximum distance it can travel in 10s, starting from rest, and finishing at rest, assuming there is no upper limit to its speed.

Sketch a velocity-time graph showing info.



NOTE: Three unknown quantities (v_{MAX} , t_1 , t_2) hence we require three equations to solve.

From, $v = u + at$;

$$v_{\text{MAX}} = 0 + 5.2 t_1 \quad \text{--- (1)}$$

$$\text{and, } 0 = v_{\text{MAX}} + (-7.2) t_2 \quad \text{--- (2)}$$

$$\text{also, } t_1 + t_2 = 10. \quad \text{--- (3)}$$

$$\text{From (1), } t_1 = \frac{v_{\text{MAX}}}{5.2}, \quad \text{from (2), } t_2 = \frac{v_{\text{MAX}}}{7.2}$$

$$\text{Substituting into (3), } \frac{v_{\text{MAX}}}{5.2} + \frac{v_{\text{MAX}}}{7.2} = 10.$$

$$v_{\text{MAX}} (0.1923 + 0.1389) = 10$$

$$\therefore v_{\text{MAX}} = 30.19 \text{ m/s.}$$

$$\text{Hence, } t_1 = \frac{30.19}{5.2} = 5.806s$$
$$t_2 = \frac{30.19}{7.2} = 4.193s$$
$$\left\{ \begin{array}{l} \text{where, } t_1 + t_2 \\ \text{is } 9.999s \end{array} \right\}$$

Now possible to calculate the distances covered,

$$\text{Using, } S = \frac{1}{2}(u+v)t$$

$$S_1 = \frac{1}{2}(0 + 30.19) \cdot 5.806 = 87.642m$$

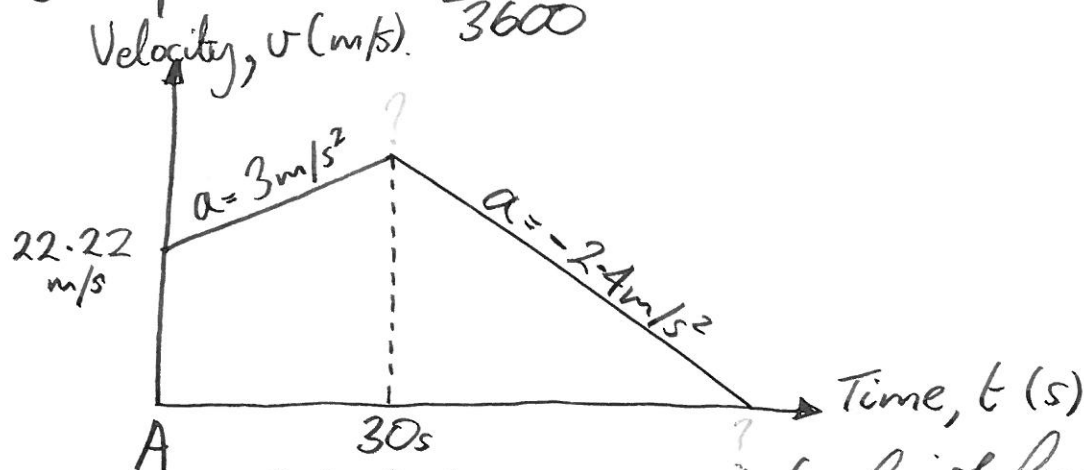
$$\text{and, } S_2 = \frac{1}{2}(30.19 + 0) \cdot 4.193 = 63.293m$$

Therefore, maximum distance travelled
is; $S = S_1 + S_2 = \underline{150.935m}$

Q2. A train passes station A with velocity 80 kph and acceleration 3 m/s^2 . After 30s it passes a signal which instructs the driver to slow down to a halt. The train then decelerates at 2.4 m/s^2 until it stops. Find the total time and total distance from station A when it comes to rest. (R2)

Sketch a velocity-time graph.

$$80 \text{ kph} = 80 \times \frac{1000}{3600} = 22.22 \text{ m/s}$$



To find total time, need to first find velocity at $t = 30 \text{ s}$;

$$v = u + a \cdot t$$

$$\therefore v = 22.22 + (3 \times 30) = 112.22 \text{ m/s}$$

Now possible to find time when train comes to rest, i.e. $v = 0$;

$$t = \frac{v - u}{a} = \frac{0 - 112.22}{-2.4}$$

$$\therefore t = 46.76 \text{ s}$$

$$\text{Hence, total time} = 30 + 46.76 = \underline{76.76 \text{ s}}$$

Distance covered in first 30s;

$$s = \frac{1}{2}(u + v)t = \frac{1}{2}(22.22 + 112.22) \cdot 30$$

$$\text{so, } s_1 = 2016.6 \text{ m}$$

$$\text{and, } s_2 = \frac{1}{2}(112.22 + 0)46.76 = 2623.7 \text{ m}$$

} TOTAL DISTANCE,

$$s_T = \underline{4640.3 \text{ m}}$$

Determine the average velocity of the journey from station A to rest.

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}} = \frac{4640.3 \text{ m}}{76.76 \text{ s}}$$

$$\therefore \underline{V_{\text{AVG.}} = 60.45 \text{ m/s (217.62 kph)}}.$$

If the train had a mass of 72 tonnes, find the kinetic energy possessed by the train at the point when the signal was given to slow down to a halt.

$$\text{K.E.} = \frac{1}{2} MV^2, \text{ i.e. } \text{K.E.} = \frac{1}{2} (72 \times 1000) (112.22)^2$$

$$\therefore \text{K.E.} = 453359822.4 \text{ J}$$

$$\text{or, } \underline{453.36 \text{ MJ.}}$$

Q3.

(R3)

For a reciprocating piston moving with SHM in a straight line, and given conditions;

(i) velocity of 15 m/s at 85 mm from mid-position

(ii) " " 4.5 m/s " 105 mm " " " ,

determine the amplitude and frequency of motion.

Find amplitude from, $V = \omega \sqrt{(r^2 - x^2)}$

$$\text{So, (i)} \quad 15 = \omega \sqrt{(r^2 - 0.007225)} \quad \text{--- (1)}$$

$$\text{(ii)} \quad 4.5 = \omega \sqrt{(r^2 - 0.011025)} \quad \text{--- (2)}$$

Divide (1) by (2) to eliminate ω , and square both sides;

$$11.111 = \frac{r^2 - 0.007225}{r^2 - 0.011025}$$

$$11.111r^2 - 0.011025 = r^2 - 0.007225$$

$$\therefore \text{amplitude, } r = 0.10678 \text{ m}$$

$$(r = 106.78 \text{ mm})$$

To find frequency, find angular velocity ω ;

$$\text{Using Equation (1) above, } 15 = \omega \sqrt{0.011402 - 0.007225}$$

$$\therefore \omega = 232.09 \text{ rad/s}$$

$$\text{and, } f = \frac{\omega}{2\pi} = 36.94 \text{ Hz}$$

Find the maximum acceleration experienced by the reciprocating piston.

$$\text{Using, } a = \omega^2 x, \text{ where } x = r$$

$$\therefore a_{\text{MAX}} = (232.09)^2 (0.10678) = 5751.8 \text{ m/s}^2$$

$$(c. 586g)$$

- Q4. A solid disc flywheel of diameter 1.2m and mass 92kg, is accelerated from rest to a speed of 1450rpm in a period of 45s. (24)
- (i) Neglecting any losses, calculate the necessary accelerating torque.

$$\text{Torque, } T = I \alpha \quad \text{where, } I = \frac{1}{2} m r^2$$

Using, $\omega_2 = \omega_1 + \alpha t$ we can find α .

$$\omega_1 = 0 \text{ rad/s (rest); } \omega_2 = \frac{1450 \times 2\pi}{60} = 151.84 \text{ rad/s.}$$

$$\therefore \alpha = \frac{\omega_2 - \omega_1}{t} = 3.374 \text{ rad/s}^2$$

$$\text{Hence, } T = \frac{1}{2} (92) (0.6)^2 \times 3.374$$

$$T = 55.87 \text{ Nm (assuming no losses).}$$

- (ii) If the bearings supporting the flywheel partially collapsed, causing a constant frictional torque of 2.2Nm acting on the flywheel, what would be the speed of the flywheel 45s after it had reached the speed of 1450rpm?

$$\text{'New' available torque, } T = 55.87 - 2.2 = 53.67 \text{ Nm}$$

$$\therefore \alpha = \frac{T}{I} = \frac{53.67}{\frac{1}{2} (92) (0.6)^2} = 3.241 \text{ rad/s}^2$$

$$\text{So, 'new' speed, } \omega_2 = 151.84 + (3.241 \times 45) = 297.685 \text{ rad/s}$$

$$\therefore \text{ speed is } \underline{2842.7 \text{ rpm.}}$$

(iii) If the original flywheel was to be replaced by one of a composite design, having the same mass and moment of inertia, find the required radius of gyration.

$$I = m.k^2 \Rightarrow k = \sqrt{\frac{I}{m}}$$

$$\therefore k = \sqrt{\frac{mr^2}{2m}} = \sqrt{\frac{r^2}{2}} = \sqrt{\frac{0.6^2}{2}} \left\{ k = \frac{r}{\sqrt{2}} \right\}$$

$$\text{so, } \underline{k = 0.424m.}$$

(iv) Find the loss in kinetic energy of the original flywheel, when considering the partial failure of the supporting bearings, when the flywheel has been rotating for a total time of $1\frac{1}{2}$ minutes.

Find speed of flywheel at 90s if no bearing failure; $\omega_2 = \omega_1 + \alpha.t = 0 + (3.374 \times 90)$

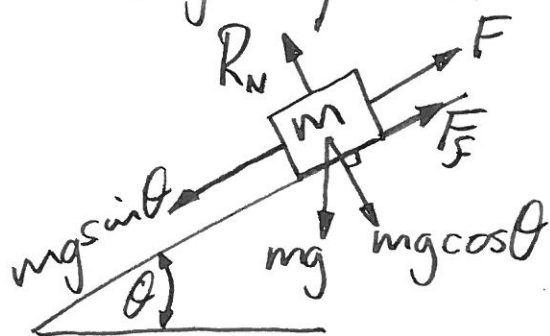
$$\therefore \omega_2 = 303.66 \text{ rad/s}$$

and, from part (ii), $\omega_2' = 297.685 \text{ rad/s}$

$$\begin{aligned} \therefore \text{Loss in K.E.} &= \frac{1}{2} I (\omega_2 - \omega_2')^2 \\ &= 0.5 \times (0.5 \times 92 \times 0.6^2) (5.975)^2 \\ \text{" " " } &= \underline{295.6 \text{ J}} \end{aligned}$$

- Q5. A body of mass 3kg rests on a rough inclined plane at 60° to the horizontal. If the coefficient of static friction between the body and the plane is 0.35 , determine:
- the force which must be applied parallel to the plane to just prevent motion down the plane;
 - the force which must be applied horizontally to just prevent motion down the plane.

(a)



If we were to consider forces perpendicular to the inclined plane, then they would cancel, i.e.

$$mg \cos \theta = R_N = 0.$$

Body is in static equilibrium.

Resolve forces parallel to the plane;

$$F + F_f = mg \sin \theta \quad \text{where, } F_f \text{ is the frictional force.}$$

$$\therefore F + \mu R_N = mg \sin \theta \quad \text{where, } R_N = mg \cos \theta$$

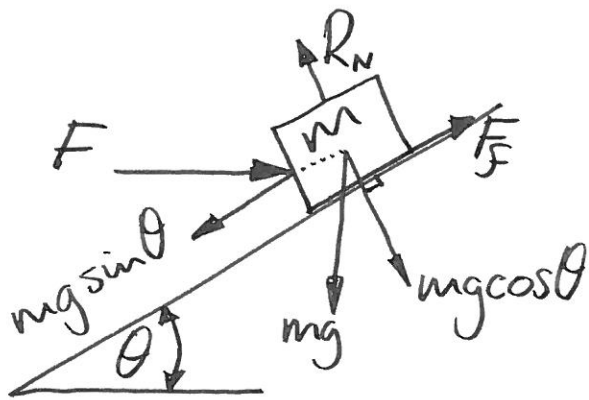
$$\text{So, } F = mg \sin \theta - \mu mg \cos \theta$$

$$F = mg (\sin \theta - \mu \cos \theta)$$

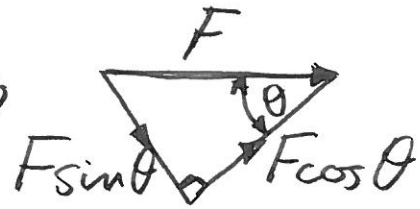
$$\therefore F = (3 \times 9.81) (\sin 60^\circ - 0.35 \cos 60^\circ)$$

so, $F = 20.34\text{N}$ force required parallel to plane to prevent sliding downwards.

(b).



where,



For components of forces perpendicular to the inclined plane;

$$F \sin \theta + mg \cos \theta = R_N$$

For components parallel to the plane;

$$F \cos \theta + F_f = mg \sin \theta$$

$$\text{so, } F \cos \theta + \mu R_N = mg \sin \theta$$

$$\therefore F \cos \theta + \mu (F \sin \theta + mg \cos \theta) = mg \sin \theta$$

$$F \cos \theta + \mu F \sin \theta + \mu mg \cos \theta = mg \sin \theta$$

$$F (\cos \theta + \mu \sin \theta) = mg \sin \theta - \mu mg \cos \theta$$

$$\therefore F = \frac{mg (\sin \theta - \mu \cos \theta)}{(\cos \theta + \mu \sin \theta)}$$

$$= \frac{(3 \times 9.81) (\sin 60^\circ - 0.35 \cos 60^\circ)}{(\cos 60^\circ + 0.35 \sin 60^\circ)}$$

$$\therefore \underline{F = 25.32 \text{ N}} \quad \text{horizontal force required to prevent sliding downwards.}$$