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2nd Maths Exam
11/01/21

Mathematics: ACDA4008 (EES)

Q1 a i $y = \frac{x^2}{2x+1} = x^2(2x+1)^{-1}$

~~$f(x) = x^2$
 $f'(x) = 2x$~~

~~$g(x) = (2x+1)^{-1}$
 $g'(x) = -2(2x+1)^{-2}$~~

~~$f(x)g'(x) + g(x)f'(x) =$~~

~~$\frac{x^2(-2(2x+1)^{-2})}{(2x+1)^2} + \frac{(2x+1)^{-1}2x}{2x+1}$~~

$f(x) = x^2$

$f'(x) = 2x$

$g(x) = (2x+1)$

$g'(x) = 2$

$\frac{2x(2x+1) - 2(x^2)}{(2x+1)^2} =$

$\frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \frac{2x^2 + 2x}{(2x+1)^2}$

ii $y = \sqrt{x^2+1} = (x^2+1)^{1/2}$

~~$u = x^2+1$ $\frac{du}{dx} = 2x$~~

~~$\frac{dy}{dx} = u^{1/2}$~~

~~$\frac{dy}{dx} = 2x \cdot \frac{1}{2} u^{-1/2}$~~

$$\frac{dy}{du} = \frac{1}{2} u^{-1/2}$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$\begin{aligned} \frac{dy}{du} \times \frac{du}{dx} &= \frac{1}{2} u^{-1/2} \times 2x \\ &= \frac{1}{2} (x^2 + 1)^{-1/2} \times 2x \\ &= \left(\frac{1}{2\sqrt{x^2 + 1}} \right) 2x \\ &= \frac{2x}{2\sqrt{x^2 + 1}} = \frac{x}{\sqrt{x^2 + 1}} \end{aligned}$$

$$\begin{aligned} \text{iii} \quad y &= 3x^{3/2} + \frac{3}{x^2} - 6x \\ &= 3x^{3/2} + 3x^{-2} - 6x \\ &= \frac{3}{2}x^{1/2} - 6x^{-3} - 6 \end{aligned}$$

$$\text{iv} \quad y = \sec^2 4x$$

$$\frac{dy}{dx} = 2(\sec(4x))^2$$

$$\frac{dy}{du} = 2(u)^2$$

$$u = \sec 4x$$

$$\frac{du}{dx} = \sec 4x \tan 4x \cdot 4$$

$$\frac{dy}{du} \times \frac{du}{dx} = 2(u)^2 \times (\sec 4x \tan 4x \cdot 4)$$

$$\begin{aligned} &= 2(\sec(4x))^2 \times (\sec 4x \tan 4x \cdot 4) \\ &= 2(\sec 4x)^3 (\tan 4x \cdot 4) \\ &= 8 \sec^3 4x \tan 4x \end{aligned}$$

$$v \quad y = x + \sin 2x$$

$$\frac{dy}{dx} = 1 + 2 \cos 2x$$

$$vi \quad y = \frac{1-10x}{(2x-1)^2}$$

$$f(x) = 1-10x$$

$$f'(x) = -10$$

$$g(x) = (2x-1)^2$$

$$g'(x) = 4(2x-1)$$

$$\frac{-10(2x-1)^2 - (8x-4)(1-10x)}{(2x-1)^4}$$

$$\frac{(-20x+10)^2 - (8x-80x^2-4+40x)}{(2x-1)^4}$$

$$\frac{(-20x+10)^2 - (-80x^2+48x-4)}{400x^2+10}$$

$$\frac{-10(2x-1)^2 - 4(2x-1)(1-10x)}{(2x-1)^4}$$

$$\frac{-10(2x-1)(2x-1) - 4(2x-1)(1-10x)}{(2x-1)^4}$$

$$\frac{-10(4x^2-4x+1) - 4(-20x^2+12x-1)}{(2x-1)^4}$$

$$\frac{-40x^2+40x-10+80x^2-48x+4}{(2x-1)^4}$$

$$\frac{40x^2-8x-6}{(2x-1)^4}$$

$$\frac{2(20x^2 - 4x - 3)}{(2x - 1)^4}$$

$$\frac{2(2x - 1)(10x + 3)}{(2x - 1)^4}$$

$$\frac{2(10x + 3)}{(2x - 1)^3}$$

b $y = \frac{1}{x} + \cos 2x = x^{-1} + \cos 2x$

$$\frac{dy}{dx} = -x^{-2} + (-2 \sin 2x)$$

$$\frac{d^2y}{dx^2} = x^{-3} + (4 \sin 2x)$$

$$\frac{d^3y}{dx^3} = -3x^{-4} + (8 \cos 2x)$$

$$\frac{d^4y}{dx^4} = 12x^{-5} + (-16 \sin 2x)$$

$$\frac{d^5y}{dx^5} = -60x^{-6} + (-32 \cos 2x)$$

c $f(x) = \ln(\operatorname{cosec} x - \cot x)$

$$f(x) = \ln(u)$$

$$f'(x) = u^{-1}$$

$$u = \operatorname{cosec} x - \cot x$$

$$\frac{du}{dx} = (-\operatorname{cosec} x \cot x) - (-\operatorname{cosec}^2 x)$$

$$= (-\operatorname{cosec} x \cot x) + \operatorname{cosec}^2 x$$

$$\frac{1}{\operatorname{cosec}(x) - \cot(x)} \cdot (-\operatorname{cosec}(x) \cot(x) + \operatorname{cosec}^2(x))$$

$$\frac{-\csc x \cot x + \csc^2 x}{\csc x - \cot x}$$

$$\frac{\csc x (\csc x - \cot x)}{\csc x - \cot x}$$

Q 2

a i $\int \frac{2x}{(4+3x^2)} dx$

$$2 \int \frac{x}{4+3x^2} dx$$

$$u = 4+3x^2$$

$$\frac{du}{dx} = 6x$$

$$du = 6x dx$$

$$2 \int \frac{x}{u} dx$$

$$2 \times \frac{1}{6} \int \frac{1}{u} du$$

$$2 \times \frac{1}{6} [\ln u]$$

$$\frac{2}{6} \ln|u|$$

$$\frac{1}{3} \ln|4+3x^2| + C$$

ii $\int_0^4 e^{\frac{1}{2}x} dx$

$$u = \frac{1}{2}x$$

$$\frac{du}{dx} = \frac{1}{2}$$

$$2 \int_0^4 e^u du$$

$$du = \frac{1}{2} dx$$

$$2 du = dx$$

x	u
4	2
0	0

~~12.778~~

$$2 \int_0^{238...} e^u du = [2e^u]_0^{238...}$$

$$2e^2 - 2e^0 = 12.778$$

$$\text{iii} \quad \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$$

$$\frac{5x+13}{(2x+1)(x+4)} = \frac{A}{2x+1} + \frac{B}{x+4}$$

$$5x+13 = A(x+4) + B(2x+1)$$

$$\text{let } x = -4$$

$$(5x-4)+13 = A(-4+4) + B(2x-4)+1$$

$$-20+13 = A(0) + B(-8+1)$$

$$-7 = -7B$$

$$B=1$$

$$\text{let } x = -\frac{1}{2}$$

$$(5x - \frac{1}{2}) + 13 = A(-\frac{1}{2}+4) + B(2x - \frac{1}{2} + 1)$$

$$-5\frac{1}{2} + 13 = A(\frac{7}{2}) + B(0)$$

$$2\frac{1}{2} = \frac{7}{2}A$$

$$A = 3$$

$$\int_0^4 \frac{3}{2x+1} + \frac{1}{x+4} dx$$

$$\left[3 \ln(2x+1) + \ln(x+4) \right]_0^4$$

$$\left[3 \ln(2 \times 4 + 1) + \ln(4+4) \right] - \left[3 \ln(2 \times 0 + 1) + \ln(0+4) \right] = 3.988984047$$

$$\begin{aligned} \text{b} \quad f(x) &= \frac{4}{x(1+4\ln x)^2} \\ &= \frac{4}{x(1+4u)^2} \end{aligned}$$

$$u = \ln x$$

$$\frac{du}{dx} = x^{-1}$$

$$du = x^{-1} dx$$

$$4 \int \frac{1}{(1+4u)^2} du$$

$$v = 1+4u$$

$$\frac{dv}{du} = 4$$

$$\frac{1}{4} dv = du$$

$$4 \times \frac{1}{4} \int \frac{1}{\sqrt{v}} dv$$

$$4 \times \frac{1}{4} \left[-\frac{1}{\sqrt{v}} \right]$$

$$-\frac{1}{1+4u}$$

$$-\frac{1}{1+4 \ln x}$$

$$c \quad i \quad \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x - 1}} dx$$

$$u = 3e^{2x} \quad \frac{du}{dx} = 6e^{2x} \quad \frac{dv}{dx} = (e^x - 1)^{-1/2} \quad v =$$

$$\int (e^x - 1)^{-1/2} dx \quad u = e^x$$

$$Q3 a \quad i \quad \left(\frac{1+i}{1-i} \right)^3 = \left(\frac{(1+i)}{(1-i)} \times \frac{(1+i)}{(1+i)} \right)^3 = \left(\frac{1+2i+i^2}{1^2+i^2} \right)^3 = \left(\frac{2i}{2} \right)^3 = i^2 i = -i$$

$$\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2} = \frac{2i}{2} = i$$

$$ii \quad \frac{3}{(1+i)(2+i)} = \frac{3}{2+i+2i+i^2} = \frac{3}{2+3i-1} = \frac{3}{1+3i}$$

$$\frac{3}{1+3i} \times \frac{1-3i}{1-3i} = \frac{3-9i}{1-3i+3i-9i^2} = \frac{3+9i}{10}$$

$$\frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$$

$$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i} \right) - (2+2i+4i+4i^2) + \left(\frac{3-i}{1+i} \times \frac{1-i}{1-i} \right)$$

$$\frac{1+2i+i^2}{1+(-1)^2} - (2+6i+4(-1)) + \frac{3-3i-i+i^2}{1+(-1)^2}$$

$$\frac{2i}{2} - (-2+6i) + \frac{2-4i}{2}$$

$$i + 2 - 6i + 1 - 2i$$

$$3 - 7i$$