

Question Numbers Answered	1st Examiner	2nd Examiner (optional)	3rd Examiner (optional)
3	36		
2	33½		
TOTAL	69½		
RESULT			

$$3a) i) \int_0^2 (4x^3 - 6x^2 + 8x - 1) dx$$

$$\Rightarrow \left[\frac{4x^4}{4} - \frac{6x^3}{3} + \frac{8x^2}{2} - x \right]_0^2$$

$$\Rightarrow [x^4 - 2x^3 + 4x^2 - x]_0^2$$

$$\Rightarrow [(2)^4 - 2(2)^3 + 4(2)^2 - 2] - [0]$$

$$\Rightarrow [16 - 16 + 16 - 2] - [0] = \underline{14}$$

$$ii) \int_0^1 (e^{2x} - e^x + 3) dx$$

$$\Rightarrow \left[\frac{1}{2} e^{2x} - e^x + 3x \right]_0^1$$

$$\Rightarrow \left[\frac{1}{2} e^{2(1)} - e^{(1)} + 3(1) \right] - \left[\frac{1}{2} e^{2(0)} - e^{(0)} + 3(0) \right]$$

$$\Rightarrow [3.6945 - 2.7183 + 3] - \left[\frac{1}{2} - 1 + 0 \right]$$

$$\Rightarrow [3.9762] - \left[-\frac{1}{2} \right]$$

$$\Rightarrow \underline{4.4762}$$

$$3a.iii) \int (4 \sin 2x - 3 \cos 4x) dx$$

$$= 4 \int \sin 2x dx - 3 \int \cos 4x dx$$

$$\Rightarrow 4 \left(-\frac{1}{2} \cos 2x \right) + C - 3 \left(\frac{1}{4} \sin 4x \right) + C$$

$$\Rightarrow -2 \cos 2x - \frac{3}{4} \sin 4x + C$$

4

3b) Integration by Parts

$$\int x \sin 4x dx$$

$$\text{Let } u = x$$

$$\text{Let } \frac{dv}{dx} = \sin 4x$$

$$\frac{du}{dx} = 1$$

$$\therefore v = \int \sin 4x dx = -\frac{1}{4} \cos 4x$$

4

Parts Formula

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$= x \left(-\frac{1}{4} \cos 4x \right) - \int \left(-\frac{1}{4} \cos 4x \right) (1) dx$$

$$= x \left(-\frac{1}{4} \cos 4x \right) + \left(\frac{1}{4} \sin 4x \right) + C$$

2

c) Use the substitution $u = 5x^2 - 4$ to show

$$\int_1^2 \frac{10x}{5x^2 - 4} dx = \ln 16$$

$$u = 5x^2 - 4 \quad \frac{du}{dx} = 10x \quad \therefore dx = \frac{du}{10x}$$

$$\int_1^2 \frac{\cancel{10x}}{u} \frac{du}{\cancel{10x}}$$

$$\int_1^2 \frac{du}{u} = \int_1^2 \frac{1}{u} = [\ln(u)]_1^2$$

$$[\ln(5(2)^2 - 4)] - [\ln(5(1)^2 - 4)]$$

$$= \ln(16) - 0$$

8

↓) Use partial fractions to find

$$\int_5^6 \frac{5x-5}{(x+1)(x-4)} dx$$

$$\frac{5x-5}{(x+1)(x-4)} = \frac{A}{(x+1)} + \frac{B}{(x-4)}$$

$$A(x-4) + B(x+1) = 5x-5$$

$$\text{Let } x = -1$$

$$A(-1-4) + B(-1+1) = 5(-1)-5$$

$$A(-5) + B(0) = -10 \quad \therefore A = 2 \quad \checkmark$$

$$\text{Let } x = 4$$

$$A(4-4) + B(4+1) = 5(4)-5$$

$$A(0) + B(5) = 15 \quad \therefore B = 3 \quad \checkmark$$

$$\int_5^6 \frac{5x-5}{(x+1)(x-4)} dx = \int_5^6 \frac{2}{(x+1)} + \int_5^6 \frac{3}{(x-4)}$$

$$= \left[2 \ln(x+1) + 3 \ln(x-4) \right]_5^6$$

$$= [2 \ln(6+1) + 3 \ln(6-4)] - [2 \ln(5+1) + 3 \ln(5-4)]$$

$$= 3.8918 + 2.0794 - 3.5835 + 0$$

$$= \underline{\underline{2.39}}$$

8

$$e) \int \frac{x^2+1}{x^3+3x} dx$$

$$\text{let } u = x^3+3x$$

$$\int \frac{x^2+1}{u} dx$$

$$\frac{du}{dx} = 3x^2 + 3$$

$$\int \frac{x^2+1}{u} \frac{du}{3x^2}$$

$$\therefore dx = \frac{du}{3x^2}$$

X

$$= \int \frac{1}{u} \frac{du}{3x^2}$$

36

$$2a) \begin{pmatrix} 3 & 2x+4 & 2 \\ y^2 & 0 & 1 \\ \frac{2z-1}{5} & 5 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -7 & 2 \\ 49 & 0 & 1 \\ \frac{2}{3} & 5 & 3 \end{pmatrix}$$

$$2x+4 = -7$$

$$2x = -11$$

$$x = \frac{-11}{2} = -5.5 \quad \times$$

$$y = \sqrt{49} = \underline{\underline{+7}} \quad 1$$

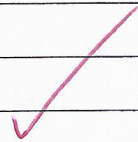
$$\frac{2z-1}{5} = \frac{2}{3}$$

$$3(2z-1) = 5 \times 2$$

$$6z-3 = 10$$

$$6z-3-10=0$$

$$6z-13=0 \quad \therefore \underline{\underline{z = \frac{13}{6}}}$$



2

b) Use determinants

$$\begin{vmatrix} x & 3 & -2 \\ 5 & x & -4 \\ 3 & -2 & 5 \end{vmatrix} = 0$$

$$= x(5x - (-4 \times -2)) - 3(5 \times 5 - (-4 \times 3)) + -2(5 \times -2 - x \times 3) = 0$$

$$= 5x^2 - 8x - 75 - 36 + 20 + 6x = 0$$

$$= 5x^2 - 2x - 91 = 0$$

Roots of $(5x^2 - 2x - 91)$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a=5 \quad b=-2 \quad c=-91$$

$$\frac{+2 \pm \sqrt{2^2 - 4 \times 5 \times -91}}{2 \times 5}$$

$$= \frac{2 \pm 42.708}{10}$$

$$\therefore x = 4.4708 \text{ or } x = -4.0708$$

$$\therefore \underline{x = 4.5} \text{ or } \underline{x = -4.1}$$

Question No.

$$2c) \quad P = \begin{pmatrix} -1 \\ -3 \\ -5 \end{pmatrix} \quad Q = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} \quad R = \begin{pmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 5 & -1 & 8 \end{pmatrix}$$

$3 \times 1 \qquad \qquad 2 \times 2 \qquad \qquad 3 \times 3$

i) $PQ = 3 \times 1 \quad 2 \times 2$ Inner numbers do not match = impossible

ii) $P^T = 3 \times 1 \quad 3 \times 1$ inner numbers do not match impossible

iii) $Q^2 = (2 \times 2)(2 \times 2)$

$$\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} 3 \times 3 + (-2) \times 5 & 3 \times (-2) + (-2) \times (-4) \\ 5 \times 3 + (-4) \times 5 & 5 \times (-2) + (-4) \times (-4) \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}$$

iv) $R^2 = (3 \times 3)(3 \times 3)$

$$\begin{pmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 5 & -1 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 5 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \times 2 + (-1) \times 0 + 6 \times 5 & 2 \times (-1) + (-1) \times 3 + 6 \times (-1) & 2 \times 6 + (-1) \times 0 + 6 \times 3 \\ 0 \times 2 + 3 \times 0 + 0 \times 5 & 0 \times (-1) + 3 \times 3 + 0 \times (-1) & 0 \times 6 + 3 \times 0 + 0 \times 3 \\ 5 \times 2 + (-1) \times 0 + 3 \times 5 & 5 \times (-1) + (-1) \times 3 + 3 \times (-1) & 5 \times 6 + (-1) \times 0 + 3 \times 3 \end{pmatrix}$$

$$= \begin{pmatrix} 34 & -11 & 30 \\ 0 & 9 & 0 \\ 25 & -9 & 39 \end{pmatrix}$$

v) PR $(3 \times 1)(3 \times 3)$ Impossible inner products do not match ✓ 1

vi) RP $(3 \times 3)(3 \times 1)$

$$\begin{pmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 5 & -1 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ -3 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \times -1 + -1 \times -3 + 6 \times -5 \\ 0 \times -1 + 3 \times -3 + 0 \times -5 \\ 5 \times -1 + -1 \times -3 + -3 \times -5 \end{pmatrix} = \begin{pmatrix} -29 \\ -9 \\ 13 \end{pmatrix} \checkmark$$

vii) P^T

$$P = \begin{pmatrix} -1 \\ -3 \\ -5 \end{pmatrix} \quad P^T = (-1 \quad -3 \quad -5) \quad \checkmark$$

viii) Q^T

$$Q = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} \quad Q^T = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix} \quad \checkmark$$

ix) $P^T R^T = (1 \times 3)(3 \times 3)$ $R^T = \begin{pmatrix} 2 & 0 & 5 \\ -1 & 3 & -1 \\ 6 & 0 & -3 \end{pmatrix}$ ✓ 1

$$\begin{pmatrix} -1 & -3 & -5 \end{pmatrix} \begin{pmatrix} 2 & 0 & 5 \\ -1 & 3 & -1 \\ 6 & 0 & -3 \end{pmatrix} = \begin{pmatrix} -1 \times 2 + -3 \times -1 + -5 \times 6 & -1 \times 0 + -3 \times 3 + -5 \times 0 & -1 \times 5 + -3 \times -1 + -5 \times -3 \end{pmatrix} \\ = \begin{pmatrix} -29 & -9 & 13 \end{pmatrix} \checkmark$$

$$x + 2y + 3z = -5$$

$$3x + y - 3z = 4$$

$$-3x + 4y + 7z = -7$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ -7 \end{pmatrix}$$

$$|A| = 1((1 \times 7) - (-3 \times 4)) - 2(3 \times 7 - -3 \times -3) + 3(3 \times 4 - 1 \times -3)$$

$$= 19 - 24 + 45$$

$$= 40$$

$$A_1 = \begin{pmatrix} -5 & 2 & 3 \\ 4 & 1 & -3 \\ -7 & 4 & 7 \end{pmatrix} \quad A_2 = \begin{pmatrix} 1 & -5 & 3 \\ 3 & 4 & -3 \\ -3 & -7 & 7 \end{pmatrix} \quad A_3 = \begin{pmatrix} 1 & 2 & -3 \\ 3 & 1 & 4 \\ -3 & 4 & -7 \end{pmatrix}$$

$$|A_1| = -5(1 \times 7 - -3 \times 4) - 2(4 \times 7 - -3 \times -7) + 3(4 \times 4 - 1 \times -7)$$

$$= -95 - 14 + 69 = -40$$

$$|A_2| = 1(4 \times 7 - -3 \times -7) - -3(3 \times 7 - -3 \times -3) + 3(3 \times -7 - 4 \times -3)$$

$$= 7 + 60 - 27 = 40$$

$$|A_3| = 1(1 \times -7 - 4 \times 4) - 2(3 \times -7 - 4 \times -3) + -5(3 \times 4 - 1 \times -3)$$

$$= -23 + 18 - 75 = -80$$

$$x = \frac{A_1}{A}$$

$$= \frac{-40}{40} = -1$$

$$y = \frac{A_2}{A}$$

$$= \frac{40}{40} = 1$$

$$z = \frac{A_3}{A}$$

$$= \frac{-80}{40} = -\frac{1}{2}$$