



12 Pages

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Rogers

Jack Nathan

1	Student Number	2017196
2	Title of Examination Paper	Control Engineering
3	Module Code	SGEE 5009
4	Date and Time of Examination	18/05/23 9:30 - 12:00 2.5 hrs
5	Examination Venue	
6	Seat Number	32
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
2	20
3	13
Total	32

Mark checked and
agreed
L Williams

Topic 2 - Formulas (not Questions)

$$\frac{G}{1 + H(s)G(s)}$$

$$T(s) = \frac{K\omega_n^2}{1 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \text{rad/s} \quad \text{Natural Frequency}$$

$$K\omega_n^2 = K = \text{Gain}$$

$$2\zeta\omega_n = 1 \quad \zeta = \frac{1}{2\omega_n} \quad \text{Damping Factor}$$

$$\omega = \omega_n \sqrt{1 - \zeta^2} = \text{rad/s} \quad \text{Damping Frequency}$$

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)}{\omega_n \sqrt{1 - \zeta^2}} \quad \text{Rise Time}$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \quad \text{Peak Time}$$

$$\text{OverShoot \%} = 100 \times e^{\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}}$$

$$t_s \approx \frac{4}{\zeta\omega_n} \quad \text{Settling time}$$

$$n = \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} \quad \text{number of Oscillations}$$

P.T.O.

Question 2

Question No.

Examiner Only

a) $\frac{G(s)}{1 + H(s) G(s)}$

$$G(s) = \frac{25}{s(s+1)}$$

$$H(s) = 1$$

$$\frac{\frac{25}{s(s+1)}}{1 + 1 \times \frac{25}{s(s+1)}}$$

multiply top and bottom by $s(s+1)$

$$\frac{25}{s(s+1)+25} = \frac{25}{s^2 + s + 25} \quad \checkmark$$

(4)

b) Use above equation with: $T(s) = \frac{K\omega_n^2}{1 + 2\zeta\omega_n s + \omega_n^2}$

(1)

(i) $\omega_n^2 = 25 \quad \therefore \omega_n = 5 \text{ rad/s (Natural Frequency)}$

(1)

$K\omega_n^2 = 25 \quad K \times 25 = 25 \quad \therefore K = 1 \text{ (Gain)}$

(ii) $2\zeta\omega_n = 1 \quad \zeta = \frac{1}{2\omega_n} = \frac{1}{2 \times 5}$

(2)

$\zeta = 0.1 \text{ (Damping Factor)}$

(iii) $\omega = \omega_n \sqrt{1 - \zeta^2}$
 $= 5 \sqrt{1 - 0.1^2}$
 $= 4.975 \text{ rad/s (Damped Frequency)}$

(2)

(iv) $t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)}{\omega_n \sqrt{1 - \zeta^2}} = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - 0.1^2}}{0.1}\right)}{4.975}$

(2)

$t_r = 0.336 \text{ Seconds (in radians on calculator) (Rise Time)}$

$t_r = -16.29 \text{ Seconds (in degrees on calculator)}$

$$(v) t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{4.975} = 0.63 \text{ seconds} \quad \uparrow \text{(Peak Time)} \checkmark$$

(2)

$$\begin{aligned} (vi) \text{ OverShoot \%} &= 100 \times e^{\frac{-3\pi}{\sqrt{1-\zeta^2}}} \\ &= 100 \times e^{\frac{-0.1\pi}{\sqrt{1-0.1^2}}} \\ &= 72.92\% \quad \checkmark \end{aligned}$$

(2)

$$(vii) t_s \approx \frac{4}{3\omega_n} = \frac{4}{0.1 \times 5} = 8 \text{ seconds} \quad \checkmark$$

(2)

$$(viii) n = \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} = \frac{2}{\pi} \sqrt{\frac{1}{0.1^2} - 1}$$

$$n = 6.33 \quad \checkmark$$

(2)

P.T.O

Question 3

a)

(i) $S^3 + 4s^2 + 3s + 5 \rightarrow$ All coefficients ^{numbers} are present.

$\rightarrow$ No negatives present

$\rightarrow$ This is a Stable System. ✓

(ii) $S^5 + 5s^4 + 3s^3 + s^2 - 4s + 1 \rightarrow$ All coefficients ^{numbers} are present.

$\rightarrow$ There is one negative.

$\rightarrow$ This is an Unstable System. ✓

3

(iii) $S^4 + 2s^3 + s^2 + 1 \rightarrow$ One coefficient number is missing.

$\rightarrow$ No Negatives present.

$\rightarrow$ This is a Critically Stable System. ✓

b) $G_o(s) = \frac{3}{s+2} \times \frac{1}{s(s+4)}$

$$= \frac{3}{s(s+6)(s+8)} = \frac{3}{s^2(s+6) + 8s}$$

$$= \frac{3}{s^3 + 6s^2 + 8s}$$

$$G(s) = \frac{G_o}{1 + H(s)G_o} = \frac{3}{s^3 + 6s + 8s + 6k}$$

$$H(s) = k$$

$$a_n = 1$$

$$a_{n-1} = 6$$

$$a_{n-2} = 8$$

$$a_{n-3} = 6k$$

2

1

P.T.O

$$b_1 = a_{n-2} - \left(\frac{a_n}{a_{n-1}}\right) \times a_{n-3} = 8 - \left(\frac{1}{6}\right) \times 6k$$

$$b_1 = 8 - k/2$$

$$b_2 = a_{n-4} - \left(\frac{a_n}{a_{n-1}}\right) \times a_{n-5} = 0 - \left(\frac{1}{6}\right) \times 0$$

$$b_2 = 0$$

$$c_1 = a_{n-3} - \left(\frac{a_{n-1}}{b_1}\right) \times b_2 = 6k - \left(\frac{6}{7k}\right) \times 0$$

$$c_1 = 6k$$

$$c_2 = a_{n-5} - \left(\frac{a_{n-1}}{b_1}\right) \times b_3 = 0 - \left(\frac{6}{7k}\right) \times 0$$

$$c_2 = 0$$

$$\begin{matrix} S^3 \\ S^2 \\ S^1 \\ S^0 \end{matrix} \begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \\ b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} = \begin{matrix} S^3 \\ S^2 \\ S^1 \\ S^0 \end{matrix} \begin{vmatrix} 1 & 8 \\ 6 & 6k \\ 8k/2 & 0 \\ 6k & 0 \end{vmatrix}$$

error
carried
through

Only change in column 1 is $8-k$.

It must be a positive, $\therefore k > 0$ and $k < 8$.

This is an underdamped system.

c) class or type 0.