



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

ANALYTICAL METHODS

COURSEWORK

Due - 12/01/2025

Programme and Module

BEng, HND & HNC Automotive, Motorcycle & Motorsport Engineering, Analytical Methods
Lecture – Richard Sutton
Module Code – ACHA4000

Robbie Sandford

Student Number – 2403962

Word Count – N/A

Table Of Contents

Contents

Question 1.....	2
Question 2.....	4
Question 3.....	6
Question 4.....	7
Question 5.....	9
Question 6.....	10
Question 7.....	12

Question 1

Q1

$$\text{i)} \quad (x+6)^2 + 2x = x^2 - 5$$

$$(x+6)(x+6) + 2x = x^2 - 5$$

$$x^2 + 12x + 36 + 2x = x^2 - 5$$

$$x^2 + 14x + 36 = x^2 - 5$$

$$14x + 41 = 0$$

$$x = \frac{-41}{14}$$

$$\text{ii)} \quad 2^{x+5} = 6^x$$

$$\ln 2^{x+5} = \ln 6^x$$

$$(x+5) \ln 2 = x \ln 6$$

$$x \ln 2 + 5 \ln 2 = x \ln 6$$

$$5 \ln 2 = x (\ln 6 - \ln 2)$$

$$x = \frac{5 \ln 2}{\ln 6 - \ln 2}$$

$$x = 3.15$$

$$\text{iii)} \quad 2 \log(x+3) = \log(x+5) + \log(x+8)$$

$$\log((x+3)^2) = \log(x+5) + \log(x+8)$$

$$\log((x+3)^2) = \log(x+5)(x+8)$$

$$\log(x^2 + 6x + 9) = \log(x^2 + 13x + 40)$$

$$x^2 + 6x + 9 = x^2 + 13x + 40$$

$$6x + 9 = 13x + 40$$

$$6x - 13x = 40 - 9$$

$$-7x = 31$$

$$x = -\frac{31}{7}$$

$$1V) \quad 1 - \frac{6x}{5} = 4 - \frac{x}{3} + \frac{5}{4}$$

(3)
(12)
(4)

60 is divisible by them all
times equation by 60

$$60x - 12(1 - 6x) = 5(4 - x) + 15$$

$$60x - 12 + 72x = 132x - 12 - \text{left}$$

$$20 - 5x + 15 = 35 - 5x - \text{Right}$$

$$132x - 12 = 35 - 5x$$

$$132x + 5x = 35 + 12$$

$$137x = 47$$

$$x = \frac{47}{137} \quad \text{or} \quad 0.34$$

Question 2

Q 2

$$\begin{aligned} \text{i) } 2x + 4y - 3z &= 120 & \textcircled{1} \\ 5x + 2y + z &= 140 & \textcircled{2} \\ x - 3y + 5z &= 100 & \textcircled{3} \end{aligned}$$

$$\begin{aligned} \textcircled{1} \times 3 &= 6x + 12y - 9z = 360 \\ \textcircled{3} \times 4 &= 4x - 12y + 20z = 400 \end{aligned}$$

$$10x + 11z = 760 \quad \textcircled{4}$$

$$\begin{aligned} \textcircled{2} \times 3 &= 15x + 6y + 3z = 420 \\ \textcircled{3} \times 2 &= 2x - 6y + 10z = 200 \end{aligned}$$

$$17x + 13z = 620 \quad \textcircled{5}$$

$$\begin{aligned} \textcircled{4} \quad 10x + 11z &= 760 \quad (\times 13) \\ 130x + 143z &= 9880 \quad \textcircled{6} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad 17x + 13z &= 620 \quad (\times 11) \\ 187x + 143z &= 6820 \quad \textcircled{7} \end{aligned}$$

$$\begin{aligned} \textcircled{6} - \textcircled{7} &= -130x + 143z = 9880 \\ 187x + 143z &= 6820 \end{aligned}$$

$$-57x = 3060$$

$$x = \frac{3060}{57}$$

$$x = -53.68$$

Sub into equation ⑥

$$130 \times (-53.68) + 143z = 9880$$

$$-6978.4 + 143z = 9880$$

$$143z = 16858.4$$

$$z = \frac{16858.4}{143}$$

$$z = 117.89$$

Sub into equation ①

$$2x + 4y - 3z = 120$$

$$2x(-53.68) + 4y - 3 \times (117.89) = 120$$

$$-107.36 + 4y - 353.67 = 120$$

$$-461.03 + 4y = 120$$

$$4y = 581.03$$

$$y = \frac{581.03}{4}$$

$$y = 145.26$$

4

Question 3

Q3

$$\begin{aligned} \text{i) } 0.6x^2 + 3.4x &= 7.4 \\ 0.6x^2 + 3.4x - 7.4 &= 0 \end{aligned}$$

$$ax^2 + bx + c = 0$$

$$a = 0.6$$

$$b = 3.4$$

$$c = -7.4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3.4 \pm \sqrt{3.4^2 - 4 \times (0.6) \times (-7.4)}}{2 \times (0.6)}$$

$$x = \frac{-3.4 \pm \sqrt{11.56 + 17.76}}{1.2}$$

$$x = \frac{-3.4 \pm \sqrt{29.32}}{1.2}$$

$$x = 1.68 \quad \text{or} \quad x = -7.38$$

Question 4

Q4

$$\text{i) } y = 5x^3 - 7x^2 + 3x - 1$$

$$\frac{dy}{dx} = 15x^2 - 14x + 3$$

$$\text{ii) } y = \frac{1}{x^5}$$

$$y = -x^{-5}$$

$$\frac{dy}{dx} = 5x^{-6} \quad \therefore \frac{5}{x^6}$$

$$\text{iii) } y = e^{\frac{x}{2}} + \cos \frac{x}{6} + \sin 2x$$

$$\frac{dy}{dx} = \frac{e^{\frac{x}{2}}}{2} - \frac{\sin \frac{x}{6}}{6} + 2 \cos 2x$$

$$\text{iv) } y = (x^7 + 4)^5 (3x^2 - 1)^4$$

$$\frac{dy}{dx} = 5(x^7 + 4)^4 \times (7x^6) \times 4(3x^2 - 1)^3 \times (6x)$$

$$\text{v) } y = \frac{(5x^2 + 3x + 1)^5}{e^x} \quad \begin{array}{l} \leftarrow u \\ \leftarrow v \end{array}$$

$$\frac{du}{dx} = 5(5x^2 + 3x + 1)^4 (10x + 3)$$

$$\frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = v \frac{du}{dx} - u \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{(e^x) \cdot 5(5x^2 + 3x + 1)^4 (10x + 3) - (5x^2 + 3x + 1)^5 (e^x)}{(e^x)^2}$$

(vi) $y = \sqrt{\frac{5x-1}{8x+5}} \quad \frac{du}{dx} = 5$

$y = \left(\frac{5x-1}{8x+5} \right)^{1/2} \leftarrow \begin{matrix} u \\ v \end{matrix} \quad \frac{dv}{dx} = 8$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{5x-1}{8x+5} \right)^{-1/2}$$

$$\left(\frac{(8x+5)5 - (5x-1)8}{(8x+5)^2} \right)$$

(vii) $y = (10x + \sqrt[3]{x^5+1})^5$

$$\frac{dy}{dx} = 5(10x + \sqrt[3]{x^5+1})^4 (10 + (5x^4)^{1/3})$$

Question 5

Q5

- 1) derivative $\therefore$ Velocity
- 2) derivative $\therefore$ acceleration
- 3) derivative $\therefore$ Jerk
- 4) derivative $\therefore$ Snap or Jounce
- 5) derivative $\therefore$ Crackle
- 6) derivative $\therefore$ Pop

Question 6

Q6

$$0^\circ \leq x \leq 360^\circ$$

i) $9 \sin x = 2$

$$\sin x = \frac{-2}{9}$$

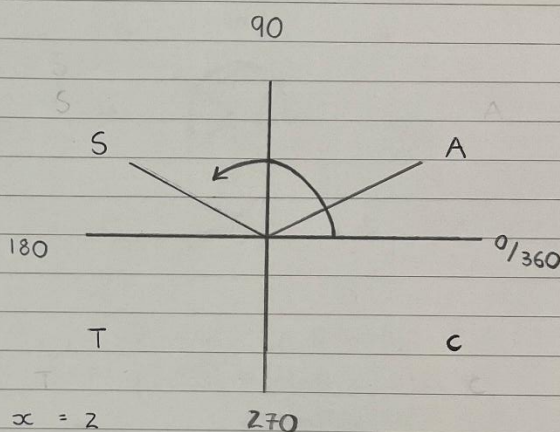
$$\sin^{-1} \left(\frac{2}{9} \right)$$

$$x = 12.84^\circ$$

$$180 - 12.84$$

$$= 167.16$$

$$x = 167.16^\circ$$



(ii) $5 \sin^2 x - 0.3 \cos x = 2$

$$\sin^2 x = 1 - \cos^2 x$$

$$5(1 - \cos^2 x) - 0.3 \cos x - 2 = 0$$

$$5 - 5 \cos^2 x - 0.3 \cos x - 2 = 0$$

$$-5 \cos^2 x - 0.3 \cos x + 3 = 0$$

$$5 \cos^2 x + 0.3 \cos x - 3 = 0$$

$$\cos x = \frac{-b \pm \sqrt{b^2 - 4AC}}{2a}$$

$$a = 5$$

$$b = 0.3$$

$$c = -3$$

$$\frac{-0.3 \pm \sqrt{60.09}}{10}$$

$$\cos x = 0.75$$

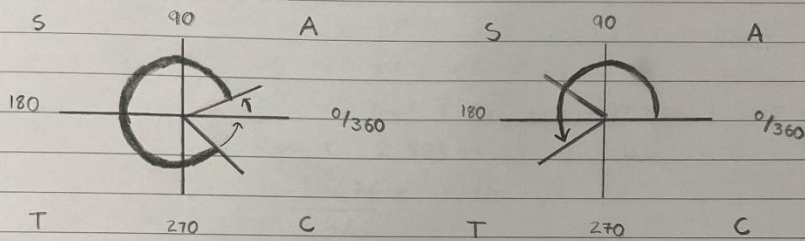
$$\cos^{-1}(0.75)$$

$$x = 41.4 \text{ } ^\circ$$

$$\cos x = -0.8$$

$$\cos^{-1}(-0.8)$$

$$x = 36.9 \text{ } ^\circ$$



$$360 - 41.4 = 318.6^\circ$$

$$x = 41.4^\circ$$

$$x = 318.6^\circ$$

$$180 - 36.9 = 143.1^\circ$$

$$180 + 36.9 = 216.9^\circ$$

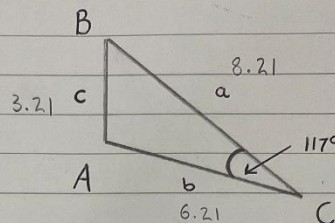
$$x = 143.1^\circ$$

$$x = 216.9^\circ$$

Question 7

Q 7

$$\begin{aligned} AB &= x \text{ cm} \\ AC &= (x+3) \text{ cm} \\ BC &= (x+5) \text{ cm} \\ \angle BAC &= 117^\circ \end{aligned}$$



$$\begin{aligned} (x+5)^2 &= x^2 + (x+3)^2 - 2x(x+3) \cos(117^\circ) \\ \cos(117^\circ) &= -\cos(63^\circ) = -0.454 \end{aligned}$$

$$(x+5)^2 = x^2 + (x+3)^2 - 2x(x+3)(-0.454)$$

$$x^2 + 10x + 25 = x^2 + x^2 + 6x + 9 + 0.908x(x+3)$$

$$x^2 + 10x + 25 = 2x^2 + 6x + 9 + 0.908x^2 + 2.724x$$

$$x^2 + 10x + 25 = 2.908x^2 + 8.724x + 9$$

$$0 = 1.908x^2 - 1.276x - 16$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1.908$$

$$b = -1.276$$

$$c = -16$$

$$\frac{-(-1.276) \pm \sqrt{120.48}}{3.816}$$

$$x = 3.21$$

$$\text{or } x = -2.54$$

$$AB = 3.21 \text{ cm (c)}$$

$$AC = 6.21 \text{ cm (b)}$$

$$BC = 8.21 \text{ cm (a)}$$

Area of triangle:

Heron's Formula:

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Semi perimeter} = \frac{a+b+c}{2}$$

(s)

$$\text{Area} = 8.82 \text{ cm}^2$$

$$\text{Semi perimeter} = 8.815$$

(s)