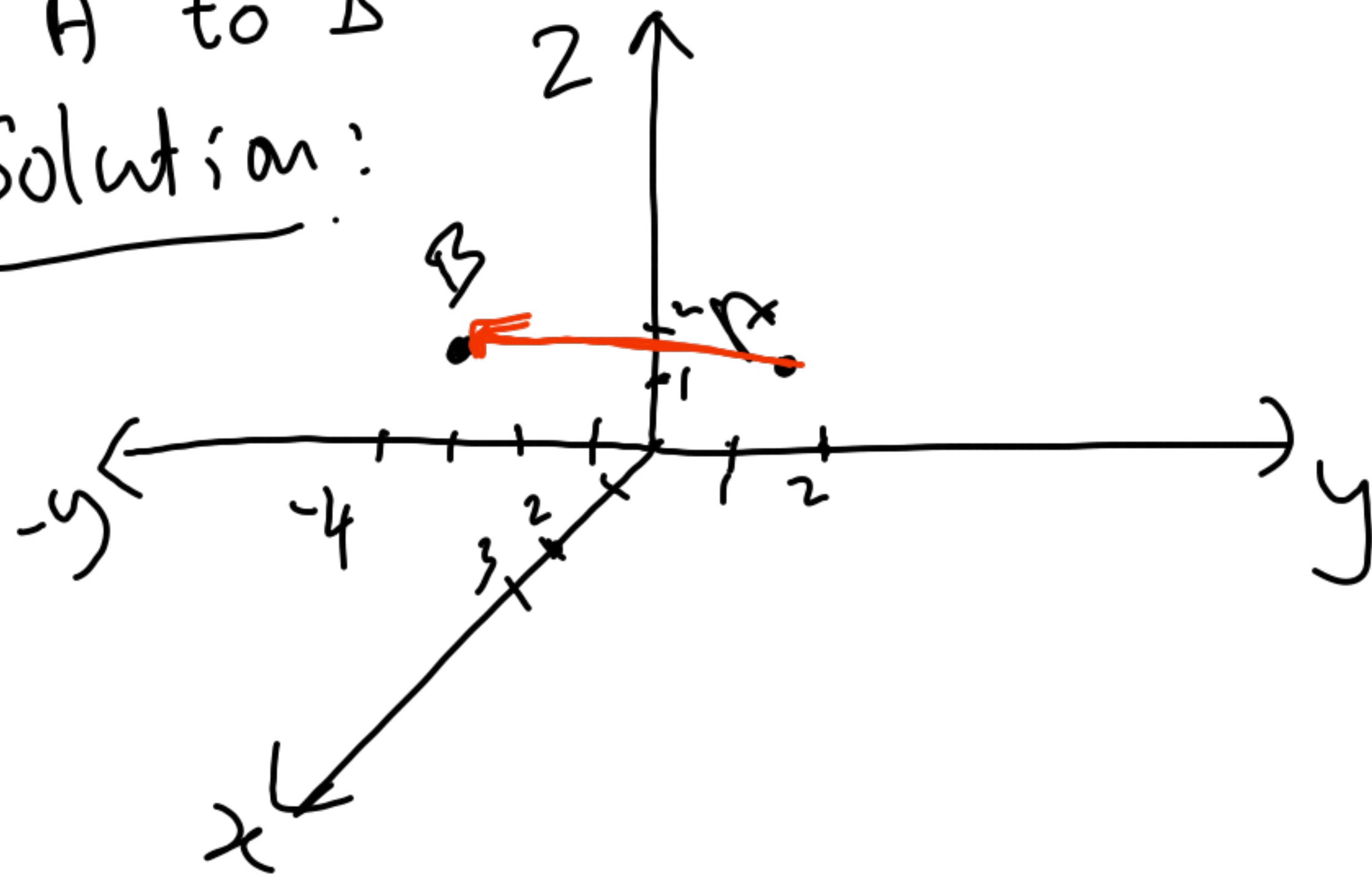


2) Two points $A(2, 2, 1)$ and $B(3, -4, 2)$ are given in the Cartesian System. Obtain the vector for A and B and a unit vector from

A to B

Solution:



$$A(2, 2, 1) \quad B(3, -4, 2)$$

$$x_1, y_1, z_1 \quad x_2, y_2, z_2$$

$$\vec{A} = x_1 \vec{a}_x + y_1 \vec{a}_y + z_1 \vec{a}_z$$

$$\vec{A} = 2\vec{a}_x + 2\vec{a}_y + 1\vec{a}_z$$

$$\vec{B} = x_2 \vec{a}_x + y_2 \vec{a}_y + z_2 \vec{a}_z$$

$$\vec{B} = 3\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z$$

Unit Vector:

$$\vec{a}_{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

$$\overrightarrow{AB} = (x_2 - x_1)\vec{a}_x + (y_2 - y_1)\vec{a}_y + (z_2 - z_1)\vec{a}_z$$

$$\overrightarrow{AB} = (3 - 2)\vec{a}_x + (-4 - 2)\vec{a}_y + (2 - 1)\vec{a}_z$$

$$\overrightarrow{AB} = 1\vec{a}_x - 6\vec{a}_y + 1\vec{a}_z$$

$$|\overrightarrow{AB}| = \sqrt{(1)^2 + (-6)^2 + (1)^2}$$

$$|\overrightarrow{AB}| = \sqrt{38} = 6.16$$

$$\vec{a}_{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{\vec{a}_x - 6\vec{a}_y + \vec{a}_z}{6.16}$$

$$a_{AB} = 0.16\vec{a}_x - 0.97\vec{a}_y + 0.16\vec{a}_z$$

$$|\vec{a}_{AB}| = \sqrt{(0.16)^2 + (-0.97)^2 + (0.16)^2}$$

$$|\vec{a}_{AB}| = 1$$

3) A laser emits light with a wavelength (λ) of 650 nanometers. Calculate the frequency (f) of the emitted light using the formula $f = \frac{c}{\lambda}$

$$f = \frac{3 \times 10^8}{650 \times 10^{-9}}$$

$$f = 4.615 \times 10^{14} \text{ Hz}$$

4) Given the Speed of light is $c = 3 \times 10^8 \text{ m/s}$. Calculate the refractive index of the medium, if the Speed of the light in that medium is $2.5 \times 10^8 \text{ m/s}$.

Soln.

$$c = 3 \times 10^8 \text{ m/s}$$

$$v = 2.5 \times 10^8 \text{ m/s}$$

$$n = ?$$

$$n = \frac{c}{v}$$

$$n = \frac{3 \times 10^8}{2.5 \times 10^8}$$

$$n = 1.2$$

5) A light is travelling from water to air. Calculate the critical angle for the light ray at the water-air interface.

Soln:

Refractive index of water (n_{water}) = 1.33

Refractive index of air (n_{air}) = 1

$$\theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$

$$n_{\text{water}} = n_1 = 1.33$$

$$n_{\text{air}} = n_2 = 1$$

$$\theta_c = \sin^{-1} \left(\frac{1}{1.33} \right)$$

$$\theta_c = 48.75^\circ$$

6) A light ray travelling in glass strikes enters the air at an angle of incidence 45° .

Calculate the critical angle for total internal reflection at the interface between air and glass.

Soln.

$$\theta_i = 45^\circ$$

$$\theta_c = ?$$

$$n_2 = n_{\text{air}} = 1$$

$$n_1 = n_{\text{glass}} = 1.5$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$= \sin^{-1}\left(\frac{1}{1.5}\right)$$

$$\theta_c = 41.8^\circ$$

An OFC System uses an optical fiber with a core refractive index of 1.5 and a cladding refractive index of 1.48. Calculate the acceptance angle of the fiber. Assume that the light enters the fiber from air.

Soln.

$$n_1 = 1.5$$

$$n_2 = 1.48$$

$$\theta_a = ?$$

$$n = 1$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = n \sin \theta_a$$

$$\theta_a = \sin^{-1} \left(\frac{NA}{n} \right)$$

$$NA = \sqrt{(1.5)^2 - (1.48)^2}$$

$$NA = 0.24$$

$$\theta_a = \sin^{-1}(0.24)$$

$$\theta_a = 14^\circ.1$$

8) An optic fiber has a NA of 0.48 and the refractive index of the cladding is 1.45. Calculate the refractive index of core.

Soln

$$NA = 0.48.$$

$$n_{\text{cladding}} = 1.45 = n_2.$$

$$n_1 = ?$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

Squaring both the sides.

$$(NA)^2 = n_1^2 - n_2^2$$

$$n_1^2 = NA^2 + n_2^2$$

$$n_1 = \sqrt{NA^2 + n_2^2}$$

$$n_1 = \sqrt{(0.48)^2 + (1.45)^2}$$

$$n_1 = 1.53$$

9) Show that $\vec{A} = 4a_x - 2a_y - a_z$.

and $\vec{B} = a_x + 4a_y - 4a_z$ are mutually

Perpendicular.

Soln:

$$\vec{A} = 4a_x - 2a_y - a_z$$

$$\vec{B} = a_x + 4a_y - 4a_z$$

$$\vec{A} \cdot \vec{B} = 0$$

$$\vec{A} \cdot \vec{B} = x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2$$

$$\vec{A} \cdot \vec{B} = 4 \times 1 + (-2) \times 4 + (-1) \times (-4)$$

$$\vec{A} \cdot \vec{B} = 4 - 8 + 4$$

$$\vec{A} \cdot \vec{B} = 0$$

Hence Both the Vectors are Perpendicular.

10) Given the Vectors $\vec{A} = 2\vec{a}_x - 5\vec{a}_y - 4\vec{a}_z$ and $\vec{B} = 3\vec{a}_x + 5\vec{a}_y + 2\vec{a}_z$. Find the dot product of $\vec{A}$ and $\vec{B}$ and the angle between both.

Soln.

$$\vec{A} = 2\vec{a}_x - 5\vec{a}_y - 4\vec{a}_z$$

$$\vec{B} = 3\vec{a}_x + 5\vec{a}_y + 2\vec{a}_z$$

$$\vec{A} \cdot \vec{B} = 2 \times 3 + (-5 \times 5) + (-4 \times 2)$$

$$= 6 - 25 - 8$$

$$\vec{A} \cdot \vec{B} = -27$$

$$\theta_{AB} = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|} \right)$$

$$|\vec{A}| = \sqrt{2^2 + (-5)^2 + (-4)^2} = 6.71$$

$$|\vec{B}| = \sqrt{3^2 + 5^2 + 2^2} = 6.16$$

$$\theta_{AB} = \cos^{-1} \left(\frac{-27}{6.71 \times 6.16} \right)$$

$$\theta_{AB} = 130.76$$