

EXAMINATION 2013 / 2014

AHOLIADAU 2013 / 2014

EXAM

FACULTY OF APPLIED DESIGN AND ENGINEERING

NAME OF COURSE/PROGRAMME: MSc in Non-destructive Testing & Evaluation

YEAR OR LEVEL: Level 7

NAME OF MODULE/UNIT/EXAM PAPER: Electrical and Magnetic Methods

MODULE NO:

DATE OF EXAMINATION:

EXAMINATION TIMES:

START TIME: 10.00am

FINISH TIME: 12.00 am

TIME ALLOWED: 2hrs

INSTRUCTIONS TO CANDIDATES:

Answer all questions.

Where necessary refer to the magnetisation material curves provided.

Question 1. Total Marks - 12

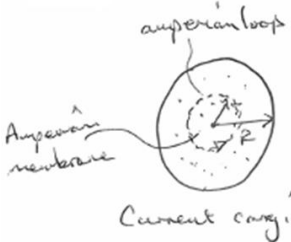
- a) Using Ampere's law and assuming a uniform current density show that the magnetic field within a current carrying conductor with a circular cross section can be given as:

$$B = \frac{\mu J r}{2}$$

(6 marks)

Answer:

Ampere's Law
Integral form:



$$\mu_0 \oint_S \vec{J} \cdot d\vec{s} = \oint_C \vec{B} \cdot d\vec{l}$$

Note: $\vec{J} = \int \vec{I} \cdot d\vec{A}$

$$\mu_0 \vec{I}_{AL} = \oint \vec{B}_{AL} \cdot d\vec{l}_{AL}$$

Big R is conductor radius, small r is amperian loop radius for which we want to calculate B.

Using current (I) enclosed within loop

$$I = J \times A \text{ where } A = \pi r^2$$

$$\mu_0 J \pi r^2 = \oint \vec{B} \cdot d\vec{l}$$

Since $\vec{B} \parallel d\vec{l}$ (parallel around loop) $\left[\vec{B} \cdot d\vec{l} = B dl \right]$ we can remove dot product

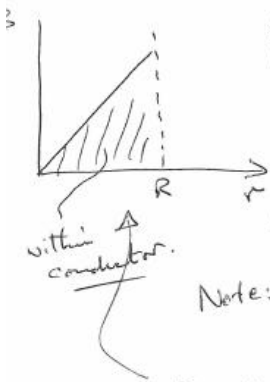
$$\mu_0 J \pi r^2 = \oint B dl$$

At radius, r B is uniform

$$\mu_0 J \pi r^2 = B \oint dl$$

Note: $\oint dl = 2\pi r$

$$\therefore \mu_0 J \pi r^2 = B 2\pi r$$

$$B = \frac{\mu_0 J \pi r^2}{2\pi r} \therefore B = \frac{\mu_0 J r}{2} \text{ --- Ans.}$$


- b) A magnetic particle inspection is to be carried out on a nickel-iron alloy component with a circular cross-section of diameter 20mm and length of 100mm. The required level of flux density is 1.2T. Using the direct current method calculate the necessary supply current.

(6 marks)

Answer:

Using $B = \frac{\mu J r}{2}$ and B required = 1T.

Re-arranging for J:

$$J = \frac{2 \times B}{\mu \times r}$$
$$H = \frac{B}{\mu}$$

From Nickel-Iron B-H Curve: $H = 225 \text{ A/m}$:

$$J = \frac{2 \times H}{r} = \frac{2 \times 225}{10 \times 10^{-3}} = 45000 \text{ A/m}^2$$

$$I = J \times A = J \times \frac{\pi d^2}{4} = 45000 \times \frac{\pi \times (20 \times 10^{-3})^2}{4} = 14.14 \text{ A}$$

Question 2. Total Marks - 14

- a) With reference to magnetic flux leakage testing, discuss the importance and methods of demagnetization of a ferromagnetic component.

(6 marks)

Answer:

After conducting a magnetic particle inspection, due to residual magnetism, it is usually necessary to demagnetize the component.

Remnant magnetic fields can:

Affect machining by causing cuttings to cling to a component.

Interfere with electronic equipment such as a compass.

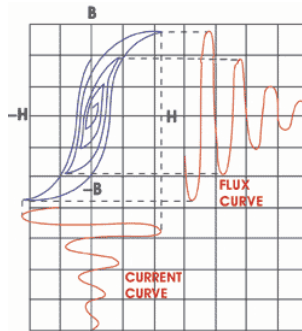
Create a condition known as "arc blow" in the welding process.

Arc blow may cause the weld arc to wander or filler metal to be repelled from the weld.

Cause abrasive particles to cling to bearing or facing surfaces and increase wear.

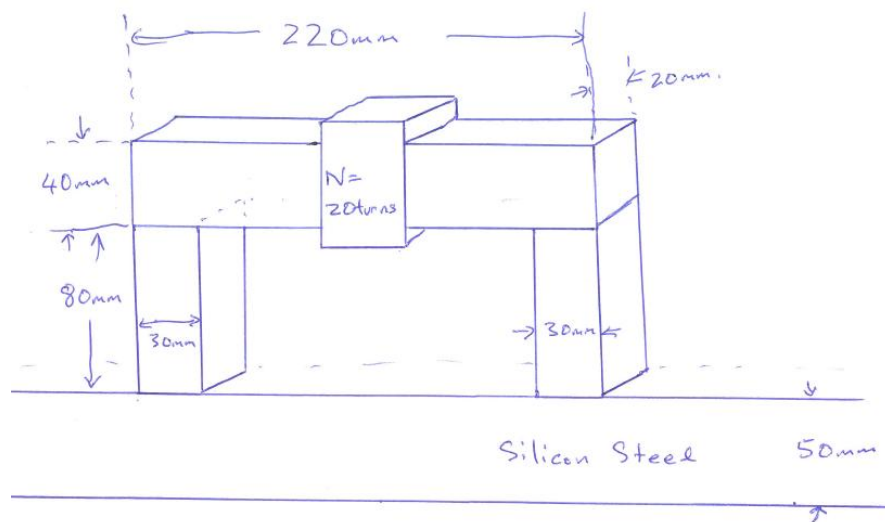
Removal of a field may be accomplished in several ways. Examples include:

Randomising the orientation of the magnetic domains can be achieved most effectively by heating the material above its curie temperature, or subjecting the component to a reversing and decreasing magnetic field (see the figure below) will return the dipoles to a nearly random orientation throughout the material. This can be accomplished by pulling a component out and away from a coil with AC passing through it, or alternately using an electromagnetic yoke with AC selected.



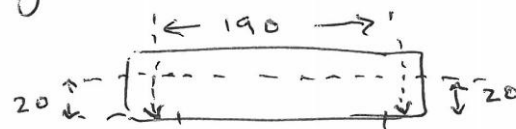
- b) Using the cast steel electromagnetic yoke as illustrated in the figure below, determine the current required to achieve a flux density in the silicon steel component of 0.9T. Note: The yoke is in direct contact with the component.

(8 marks)



Answer:

Determine mean path lengths
Magnet Bridge



$$l_B = 190 + 20 + 20 = 230 \text{ mm.}$$

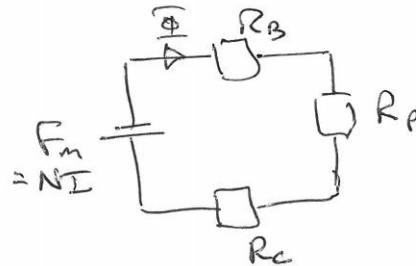
2x Pole lengths

$$l_p = 2 \times 80 \text{ mm} = 160 \text{ mm}$$

Component mean path length

$$l_c = 190 + 50 = 240 \text{ mm.}$$

Equivalent circuit



For $B_c = 0.9 \text{ T}$

$$\Phi \text{ in circuit} = B_c \times A_c = 0.9 (20 \times 10^{-3}) (50 \times 10^{-3}) \\ = 900 \times 10^{-6} \text{ Wb.}$$

From B/H curve $H_c = 140 \text{ A/m}$ [Silicon steel]

$$\therefore \text{mmf}_c = H_c l_c = 140 \times 240 \times 10^{-3} = \underline{\underline{33.6 \text{ A}\cdot\text{t}}}$$

$$B_p = \Phi / A_p = \frac{900 \times 10^{-6}}{30 \times 20 \times 10^{-6}} = 1.5 \text{ T}; \text{ From B/H curve } H_p = 2650 \text{ A/m}$$

$$\therefore \text{mmf}_p = H_p l_p = 2650 \times 160 \times 10^{-3} = \underline{\underline{424 \text{ A}\cdot\text{t}}}$$

$$B_B = \Phi / A = \frac{900 \times 10^{-6}}{40 \times 20 \times 10^{-6}} = 1.125 \text{ T, From B/H } H_B = 950 \text{ A/m.}$$

$$\therefore \text{mmf}_B = H_B l_B = 950 \times 230 \times 10^{-3} = 218.5 \text{ A}\cdot\text{t},$$

$$\therefore F_m = 33.6 + 424 + 218.5 \text{ A}\cdot\text{t} \\ = 676.1 \text{ A}\cdot\text{t}$$

For $N = 20$;

$$I = \frac{676.1}{20} = \underline{\underline{33.805 \text{ A}}} \quad \text{--- Ans.}$$

Question 3. Total Marks - 26

a) Using Ampere's law show that

- i) the magnetic flux density in the centre of a tightly wound solenoid with a non-conducting core can be given by:

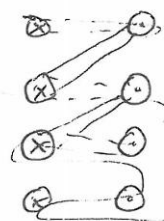
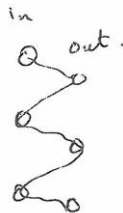
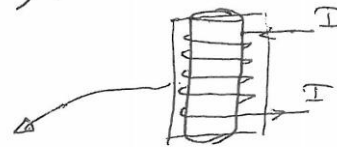
$$B = \frac{\mu_0 Ni}{h}$$

(10 marks)

Answer:

Magnetic field at the centre of a Solenoid (or inductor).

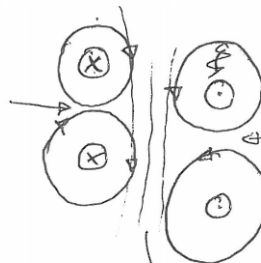
In cross section



In practice tightly wound.

spiral

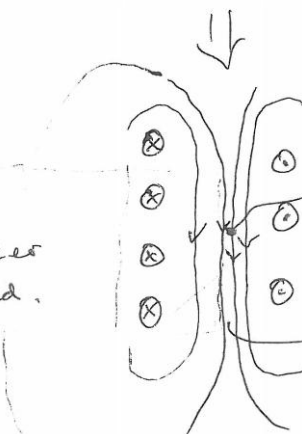
Note direction fields i.e. cancell out.



cancellation

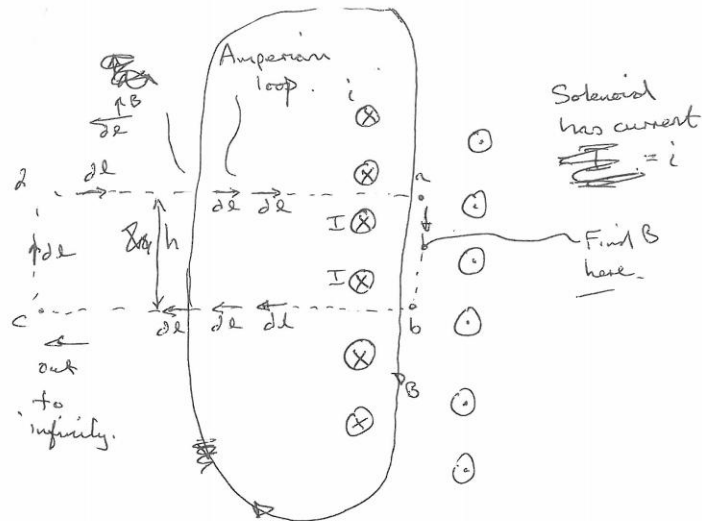
Summation

Weaker field.



want to calculate field at centre.

Strong magnetic field.



Apply Ampere's law

The current in the Amperian Loop is ~~$2 \times i$~~ $2 \times i$

$$\oint_{AL} \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} \quad (\text{Ampere's Law})$$

$$\mu_0 2i = \oint \vec{B} \cdot d\vec{l}$$

Split integral into 4 (a to b, b to c, c to d, and d to a)

$$\mu_0 2i = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$

Note the dot product of $d\vec{l}$ and $\vec{B}$ over b to c and d to a are perpendicular $\therefore$ this will go to zero

$$a \cdot b = ab \cos \theta$$

$$\cos 90^\circ = 0$$

$$\vec{B} \perp d\vec{l} \quad \vec{B} \cdot d\vec{l} = 0$$

c to d at α then $B = \Phi$

$\therefore$ only integral that matters is a to b.

$$\mu_0 I_i = \int_a^b \vec{B} \cdot d\vec{l} \quad \text{for most of a to b}$$

$\vec{B} \parallel d\vec{l}$

B is constant.

$\uparrow$

note: for only 2 turns.

$$\mu_0 I_i = \int_a^b B dl$$

remove dot product.

a to b is h [assume small]

B is approx. uniform

$$\mu_0 I_i = B \int dl$$

$$\mu_0 I_i = B h$$

$$B = \frac{\mu_0 I_i}{h}$$

Generic

$$B = \frac{\mu_0 N I_i}{h}$$

N : # of ~~loops~~ turns going through Amperian Loop.

$$B_{\text{science}} = \mu_0 n I$$

where $n = \frac{N}{h}$ [Number of loops / length]
or
turn density.

h = height of amperian loop.

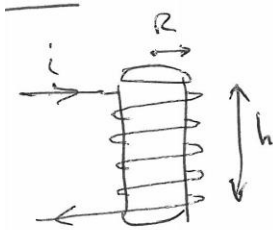
$\uparrow$
or length.

ii) and its inductance using Faraday's law can then be shown to be:

$$L = \frac{N^2 \mu_0 \pi R^2}{h}$$

(6 marks)

Answer:



From Faraday's Induction Law;

$$\mathcal{E}_{\text{ind}} = -N \frac{d\Phi}{dt} \text{ and } V = -L \frac{di}{dt}$$

Assuming B is uniform in solenoid core

$$\mathcal{E}_{\text{ind}} = -N \frac{d(BA)}{dt}$$

$$A = \pi R^2 \text{ and } B = \mu_0 n i$$

where n is turns density $\frac{N}{h}$

$$\mathcal{E}_{\text{ind}} = -N \frac{d[\mu_0 n i \pi R^2]}{dt} \quad \text{everything except } i \text{ is constant.}$$

$$\mathcal{E}_{\text{ind}} = -N \mu_0 n \pi R^2 \frac{di}{dt}$$

$$\therefore -N \mu_0 n \pi R^2 = L$$

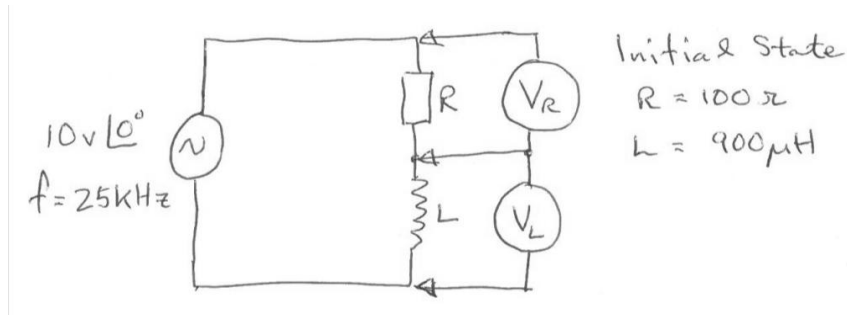
$$\text{or since } n = \frac{N}{h}$$

$$L = \frac{N^2 \mu_0 \pi R^2}{h}$$

Ans.

- b) The following circuit is used to determine and display the impedance variations in a conventional eddy current system. When the eddy current probe (L) is brought close to a conducting surface it's inductance value increases to 1mH. Determine the measure voltages, V_R and V_L for the initial and final values and illustrate how this is displayed on a complex impedance diagram.

(10 marks)



Answer:

Initial State

$$Z_L = 0 + jX_L = 0 + j\omega L$$

$$= 0 + j2\pi fL = 0 + j45\pi \approx 45\pi \angle 90^\circ \Omega$$

$$Z_R = 100 + j0$$

$$\therefore Z_T = 100 + j45\pi = 173.2 \angle 54.7^\circ \Omega$$

$$I = \frac{V_s}{Z_T} = \frac{10 \angle 0^\circ}{173.2 \angle 54.7^\circ} = 57.74 \angle -54.7^\circ \text{ mA}$$

$$V_R = I \cdot Z_R = 57.74 \times 10^{-3} \angle -54.7^\circ \times 100 \angle 0^\circ$$

$$= 5.77 \angle -54.7^\circ \text{ Volts}$$

$$V_L = I \cdot Z_L = 57.74 \times 10^{-3} \angle -54.7^\circ \times 45\pi \angle 90^\circ$$

$$= 8.163 \angle 35.3^\circ \text{ Volts}$$

When $L = 1 \text{ mH}$

$$Z_L = 0 + j\omega L \times 10^{-3}$$

$$= 0 + j50\pi = 50\pi \angle 90^\circ \Omega$$

$$Z_T = 100 + j50\pi = 186.21 \angle 57.52^\circ \Omega$$

$$I = \frac{V_s}{Z_T} = \frac{10 \angle 0^\circ}{186.21 \angle 57.52^\circ} = 53.7 \text{ mA} \angle -57.52^\circ$$

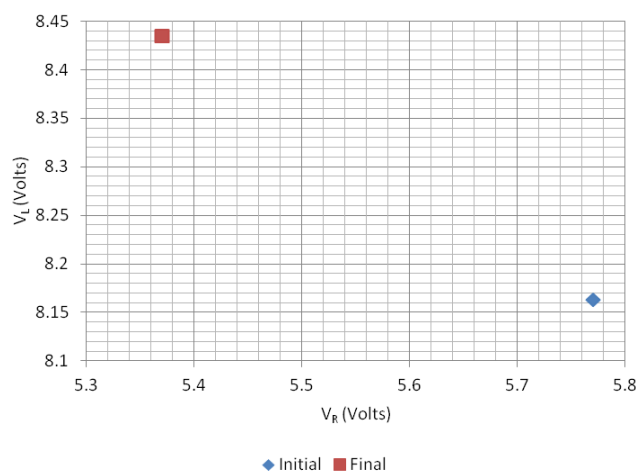
$$V_R = I \cdot Z_R = 53.7 \times 10^{-3} \angle -57.52^\circ \times 100 \angle 0^\circ$$

$$= 5.37 \angle -57.52^\circ \text{ Volts}$$

$$V_L = I \cdot Z_L = 53.7 \times 10^{-3} \angle -57.52^\circ \times 50\pi \angle 90^\circ$$

$$= 8.435 \angle 32.48^\circ \text{ Volts}$$

Complex Plane



Question 4. Total Marks - 8

An eddy current test coil is used to inspect a copper component 1mm in thickness. Assuming 0.5mm as the standard depth of penetration required and that the conductivity of copper is 98.6 %IACS:

- i) Calculate the required test frequency. **(2)**

Answer:

To determine the required test frequency, we need to re-arrange the depth of penetration formula:

$$\delta_{mm} = \frac{661}{\sqrt{f\mu_r\sigma}}$$

$$f = \frac{1}{\mu_r\sigma} \left(\frac{661}{\delta_{mm}} \right)^2$$

For copper, $\mu_r=1$

$$f = \frac{1}{98.6} \left(\frac{661}{0.5} \right)^2$$

$$f = 17.7\text{kHz to 3 dig. digits}$$

- ii) What would be the maximum phase difference between the induced near and far surface eddy currents? **(2)**

Answer:

Using the phase lag equation:

$$\theta = \frac{x}{\delta} \times 57.3$$

At the near surface there is 0° phase shift, since this is our reference.

At 1mm ($2 \times \delta$):

$$\theta = \frac{1}{0.5} \times 57.3$$

$$\theta = 114.6^\circ$$

Therefore the maximum phase difference between the near and far surface eddy currents would be 114.6° .

- iii) If we were to increase the test frequency by an order of 2, what would the effects be regarding the eddy current depth of penetration and near and far surface phase differences? **(4)**

Answer:

Using Test Frequency, $f = 2 \times 17.7\text{kHz} = 35.4\text{kHz}$,

$$\delta_{mm} = \frac{661}{\sqrt{f\mu_r\sigma}}$$

$$\delta_{mm} = \frac{661}{\sqrt{(35.4 \times 10^3) \times 98.6}}$$

$$\delta_{mm} = 0.354\text{mm to 3 dec. places}$$

At the near surface there is still a 0° phase shift, however, at the far surface the phase shift is calculated as:

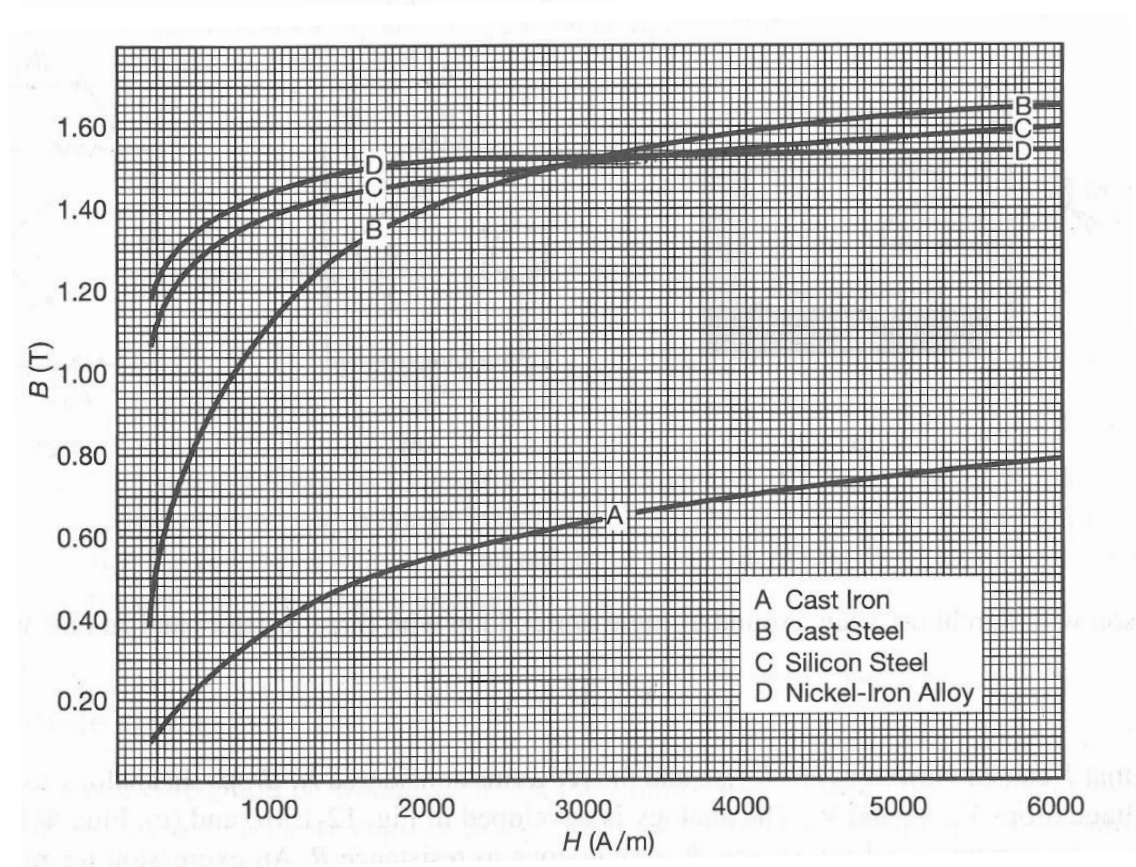
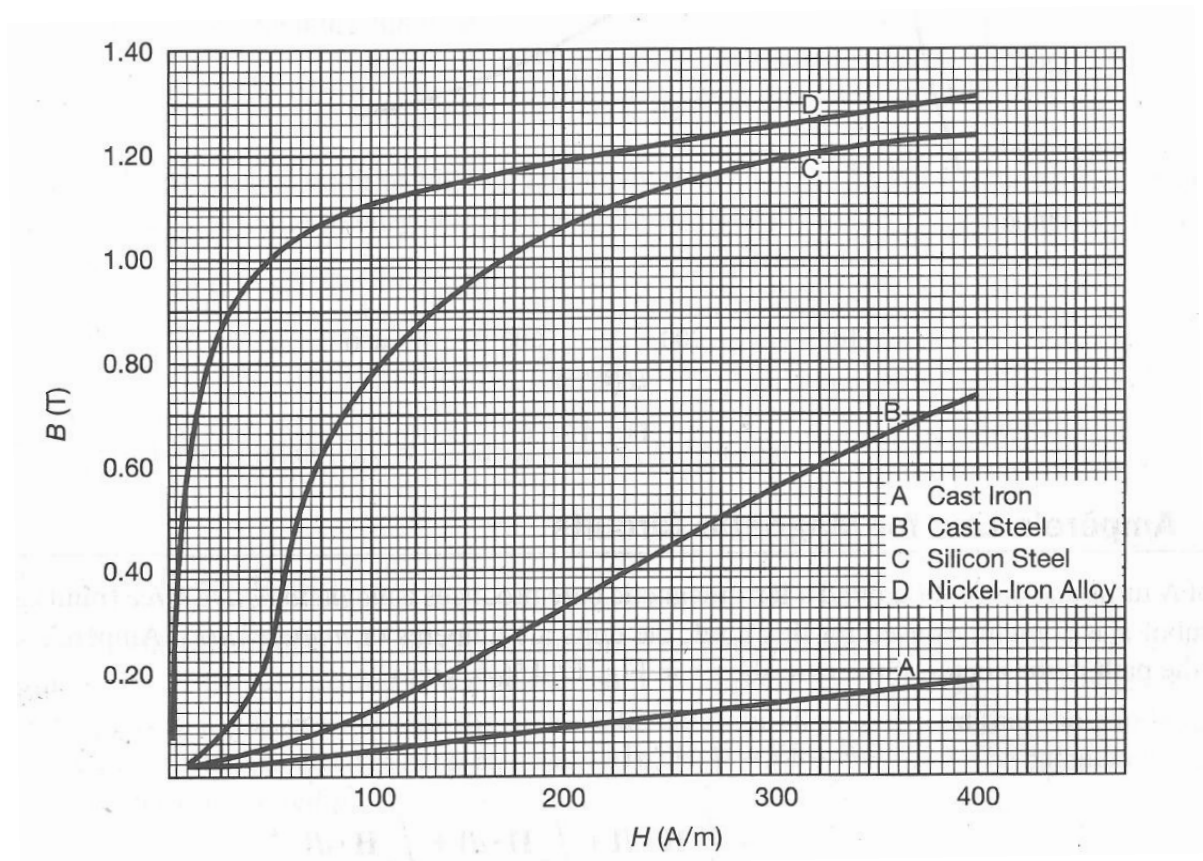
$$\theta = \frac{1}{0.354} \times 57.3$$

$$\theta = 162^\circ \text{ to 3 sig. figures}$$

By increasing the test frequency by 2, we have increased the phase difference from 114.6° to 162°. However, by reducing the depth of penetration from 0.5mm to 0.354mm we have reduced our sensitivity to far surface indications exponentially.

For example, for the first case where we have a test frequency of 17.7kHz, and $\delta=0.5\text{mm}$. Then 1mm is 2 x δ , and therefore the eddy currents at this depth are approx. 13.5% of those at the surface.

In the second case, where the test frequency is 35.4kHz, we have $\delta=0.354\text{mm}$. This very roughly $\frac{1}{3} \times 1\text{mm}$, and therefore the eddy currents at the far surface are approx. 5% of those at the surface.



General Formula and Constants

$$X_L = 2\pi fL$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\tan \theta = \frac{X_L}{R}$$

$$\delta_{mm} = \frac{661}{\sqrt{f\mu_r\sigma}}$$

$$\theta = \frac{x}{\delta} \times 57.3$$

$$B = \mu H$$

$$\mu_r = \frac{\mu}{\mu_o}$$

$$\mu_o = 4\pi \times 10^{-7}$$

Current Loop Centre Flux Density

$$B = \frac{\mu_r \mu_o I}{2R}$$

Solenoid Centre Flux Density

$$B = \mu n I$$

$$n = \frac{N}{h}$$

Magnetic Circuit Equations

$$\mu_o \iint J \cdot \partial s = \oint \vec{B} \cdot \vec{\partial l} \quad J = \int I \cdot \partial A$$

$$V_L = -L \frac{\partial I}{\partial t} \quad V_L = -N \frac{\partial \Phi}{\partial t}$$

$$mmf = H \cdot l \quad R_m = \frac{l}{\mu A} \quad F_m = N \cdot I \quad R_m = \frac{mmf}{\Phi} \quad B = \frac{\Phi}{A}$$