

6. Which of the following formulas are always true and which are sometimes false (all variables are positive)?

$$\begin{array}{ll} \text{(a)} \ln \frac{A+B}{C} = \ln A + \ln B - \ln C & \text{(b)} \ln \frac{A+B}{C} = \ln(A+B) - \ln C \\ \text{(c)} \ln \frac{A}{B} + \ln \frac{B}{A} = 0 & \text{(d)} p \ln(\ln A) = \ln(\ln A^p) \\ \text{(e)} p \ln(\ln A) = \ln(\ln A)^p & \text{(f)} \frac{\ln A}{\ln B + \ln C} = \ln A(BC)^{-1} \end{array}$$

7. Simplify the following expressions:

$$\text{(a)} \exp[\ln(x)] - \ln[\exp(x)] \quad \text{(b)} \ln[x^4 \exp(-x)] \quad \text{(c)} \exp[\ln(x^2) - 2 \ln y]$$

REVIEW PROBLEMS FOR CHAPTER 4

- (a) Let $f(x) = 3 - 27x^3$. Compute $f(0)$, $f(-1)$, $f(1/3)$, and $f(\sqrt[3]{2})$.

(b) Show that $f(x) + f(-x) = 6$ for all x .
- (a) Let $F(x) = 1 + \frac{4x}{x^2 + 4}$. Compute $F(0)$, $F(-2)$, $F(2)$, and $F(3)$.

(b) What happens to $F(x)$ when x becomes large positive or negative?

(c) Give a rough sketch of the graph of F .
- Figure A combines the graphs of a quadratic function f and a linear function g . Use the graphs to find those x where: (i) $f(x) \leq g(x)$ (ii) $f(x) \leq 0$ (iii) $g(x) \geq 0$.

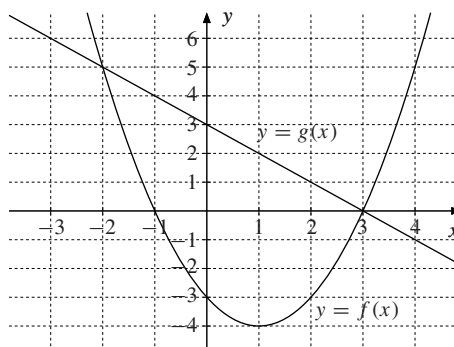


Figure A

- Find the domains of:
 - $f(x) = \sqrt{x^2 - 1}$
 - $g(x) = \frac{1}{\sqrt{x-4}}$
 - $h(x) = \sqrt{(x-3)(5-x)}$
- (a) The cost of producing x units of a commodity is given by $C(x) = 100 + 40x + 2x^2$. Find $C(0)$, $C(100)$, and $C(101) - C(100)$.

(b) Find $C(x+1) - C(x)$, and explain in words the meaning of the difference.

6. Find the slopes of the straight lines (a) $y = -4x + 8$ (b) $3x + 4y = 12$ (c) $\frac{x}{a} + \frac{y}{b} = 1$
7. Find equations for the following straight lines:
- (a) L_1 passes through $(-2, 3)$ and has a slope of -3 .
- (b) L_2 passes through $(-3, 5)$ and $(2, 7)$.
- (c) L_3 passes through (a, b) and $(2a, 3b)$ (suppose $a \neq 0$).
8. If $f(x) = ax + b$, $f(2) = 3$, and $f(-1) = -3$, then $f(-3) = ?$
9. Fill in the following table, then make a rough sketch of the graph of $y = x^2 e^x$.

x	-5	-4	-3	-2	-1	0	1
$y = x^2 e^x$							

10. Find the equation for the parabola $y = ax^2 + bx + c$ that passes through the three points $(1, -3)$, $(0, -6)$, and $(3, 15)$. (*Hint:* Determine a , b , and c .)
11. (a) If a firm sells Q tons of a product, the price P received per ton is $P = 1000 - \frac{1}{3}Q$. The price it has to pay per ton is $P = 800 + \frac{1}{5}Q$. In addition, it has transportation costs of 100 per ton. Express the firm's profit π as a function of Q , the number of tons sold, and find the profit-maximizing quantity.
- (b) Suppose the government imposes a tax on the firm's product of 10 per ton. Find the new expression for the firm's profits $\hat{\pi}$ and the new profit-maximizing quantity.
12. In Example 4.6.1, suppose a tax of t per unit produced is imposed. If $t < 100$, what production level now maximizes profits?
13. (a) A firm produces a commodity and receives \$100 for each unit sold. The cost of producing and selling x units is $20x + 0.25x^2$ dollars. Find the production level that maximizes profits.
- (b) A tax of \$10 per unit is imposed. What is now the optimal production level?
- (c) Answer the question in (b) if the sales price per unit is p , the total cost of producing and selling x units is $\alpha x + \beta x^2$, and the tax per unit is t .
- SM** 14. Write the following polynomials as products of linear factors:
- (a) $p(x) = x^3 + x^2 - 12x$ (b) $q(x) = 2x^3 + 3x^2 - 18x + 8$
15. Which of the following divisions leave no remainder? (a and b are constants; n is a natural number.)
- (a) $(x^3 - x - 1)/(x - 1)$ (b) $(2x^3 - x - 1)/(x - 1)$
- (c) $(x^3 - ax^2 + bx - ab)/(x - a)$ (d) $(x^{2n} - 1)/(x + 1)$
16. Find the values of k that make the polynomial $q(x)$ divide the polynomial $p(x)$:
- (a) $p(x) = x^2 - kx + 4$; $q(x) = x - 2$ (b) $p(x) = k^2 x^2 - kx - 6$; $q(x) = x + 2$
- (c) $p(x) = x^3 - 4x^2 + x + k$; $q(x) = x + 2$ (d) $p(x) = k^2 x^4 - 3kx^2 - 4$; $q(x) = x - 1$

- SM** 17. The cubic function $p(x) = \frac{1}{4}x^3 - x^2 - \frac{11}{4}x + \frac{15}{2}$ has three real zeros. Verify that $x = 2$ is one of them, and find the other two.
18. In 1964 a five-year plan was introduced in Tanzania. One objective was to double the real per capita income over the next 15 years. What is the average annual rate of growth of real income per capita required to achieve this objective?
- SM** 19. Figure B shows the graphs of two functions f and g . Check which of the constants a , b , c , p , q , and r are > 0 , $= 0$, or < 0 .

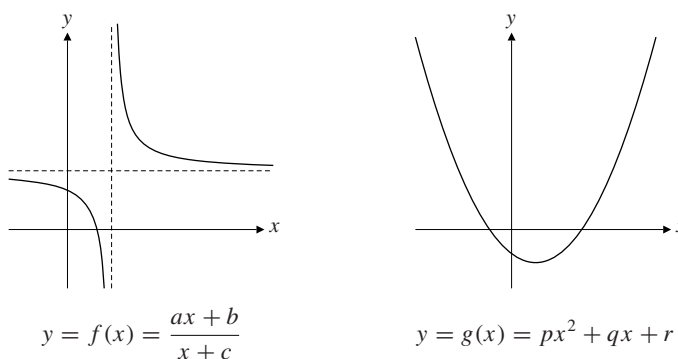


Figure B

20. (a) Determine the relationship between the Celsius (C) and Fahrenheit (F) temperature scales when you know that (i) the relation is linear; (ii) water freezes at 0°C and 32°F ; and (iii) water boils at 100°C and 212°F .
- (b) Which temperature is represented by the same number in both scales?
21. Solve for t : (a) $x = e^{at+b}$ (b) $e^{-at} = 1/2$ (c) $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2} = \frac{1}{8}$
- SM** 22. Prove the following equalities (with appropriate restrictions on the variables):
- (a) $\ln x - 2 = \ln(x/e^2)$ (b) $\ln x - \ln y + \ln z = \ln \frac{xz}{y}$
- (c) $3 + 2 \ln x = \ln(e^3 x^2)$ (d) $\frac{1}{2} \ln x - \frac{3}{2} \ln \frac{1}{x} - \ln(x+1) = \ln \frac{x^2}{x+1}$