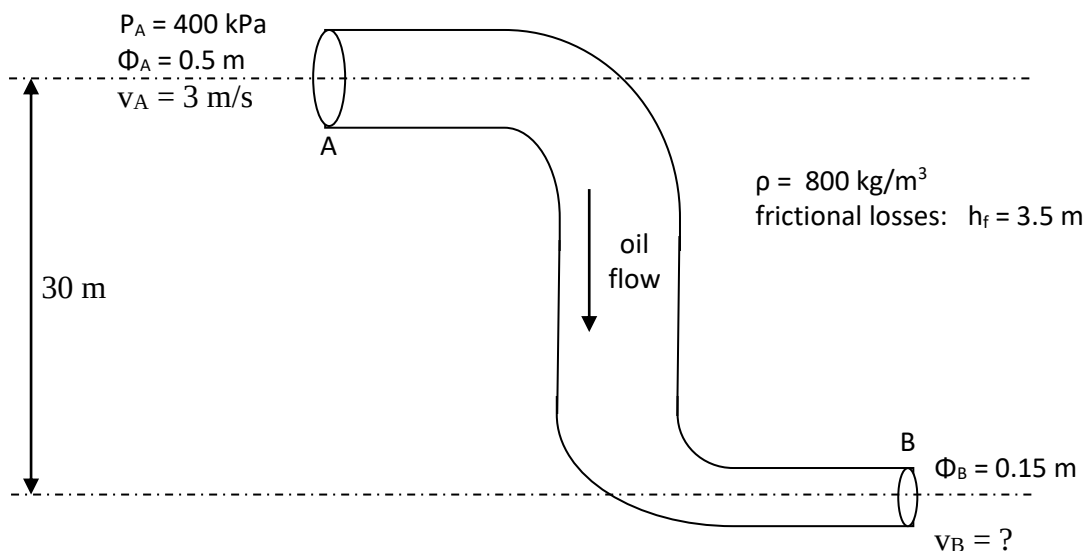


APPLICATIONS OF BERNOULLI'S EQUATION

Problem 1

Oil of density 800 kg/m^3 enters a 500 mm diameter pipeline with a velocity of 3 m/s and pressure 400 kPa, and is discharged through an orifice 150 mm in diameter and 30 m below the entry point. If the frictional losses are equivalent to a head of 3.5 m of the oil, determine the velocity and pressure of the oil at the point of discharge.

SOLUTION



From continuity equation: $Q = Area_A \cdot v_A = Area_B \cdot v_B \Rightarrow A_A \cdot v_A = A_B \cdot v_B$

Therefore:

$$v_B = \frac{A_A \cdot v_A}{A_B} = \frac{\pi \frac{0.5^2}{4} \times 3}{\pi \frac{0.15^2}{4}} = \frac{0.5^2 \times 3}{0.15^2} = 33.3 \text{ m/s}$$

Applying Bernoulli's equation:

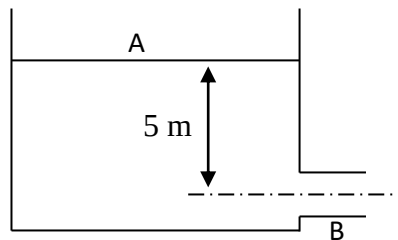
$$\begin{aligned}\frac{P_A}{\rho \cdot g} + \frac{v_A^2}{2 \cdot g} + h_A &= \frac{P_B}{\rho \cdot g} + \frac{v_B^2}{2 \cdot g} + h_B + h_f \\ \Rightarrow \frac{400 \times 10^3}{800 \times 9.81} + \frac{3^2}{2 \times 9.81} + 30 &= \frac{P_B}{800 \times 9.81} + \frac{33.3^2}{2 \times 9.81} + 3.5 \\ \Rightarrow \frac{P_B}{800 \times 9.81} &= \frac{400 \times 10^3}{800 \times 9.81} + \frac{3^2}{2 \times 9.81} + 30 - \frac{33.3^2}{2 \times 9.81} - 3.5 \\ \Rightarrow P_B &= \left(\frac{400 \times 10^3}{800 \times 9.81} + \frac{3^2 - 33.3^2}{2 \times 9.81} + 26.5 \right) \times 800 \times 9.81 \\ \Rightarrow \boxed{P_B = 167.13 \text{ kN/m}^2} \\ \Rightarrow P_B &= 1.67 \text{ bar}\end{aligned}$$

Problem 2

An open water tank discharges water to the atmosphere through a pipe at its base. The height of the water in the tank above the discharge pipe is 5 m.

- With what velocity will the water emerge through the discharge pipe?
- What will be the volume flow rate from the outlet pipe if the discharge pipe has a diameter of 100 mm?

SOLUTION



The pressures at the open surface in the tank and at the outlet are both the same as they are open to the atmosphere, $P_A = P_B = P_{atm}$. Assuming the height drops very slowly due to the leak, we may set the velocity at the upper surface $v_A = 0$.

Bernoulli's equation becomes:

$$\begin{aligned}\frac{P_{atm}}{\rho \cdot g} + \frac{0}{2 \cdot g} + h_A &= \frac{P_{atm}}{\rho \cdot g} + \frac{v_B^2}{2 \cdot g} + h_B \\ \Rightarrow v_B^2 &= 2 \cdot g(h_A - h_B) \\ \Rightarrow v_B &= \sqrt{2 \times 9.81 \times 5} = 9.9 \text{ m/s}\end{aligned}$$

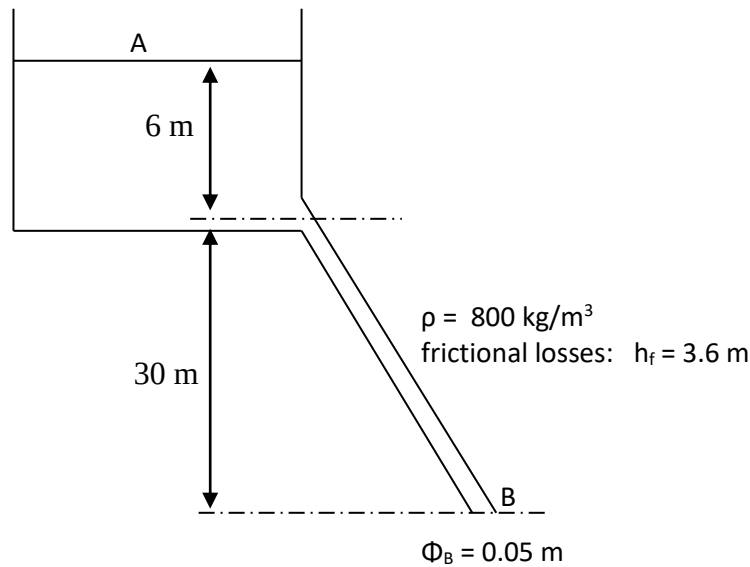
And:

$$Q = A_B \cdot v_B = \pi \frac{0.1^2}{4} \times 9.9 = 77.8 \times 10^{-3} \text{ m}^3/\text{s}$$

Problem 3

A 50 mm diameter pipe-line falls a vertical distance of 30 m from an open oil reservoir and discharges into an open tank. The head of oil above the pipe entrance is 6 m and the loss of head due to pipe friction is 3.6 m. Calculate the discharge in the atmosphere in litres per second and in tonnes per hour. Relative density is 0.8. What is the loss of power due to friction, in kilowatts?

SOLUTION



Applying Bernoulli's equation:

$$\begin{aligned} \frac{P_A}{\rho \cdot g} + \frac{v_A^2}{2 \cdot g} + h_A &= \frac{P_B}{\rho \cdot g} + \frac{v_B^2}{2 \cdot g} + h_B + h_f \\ \Rightarrow \frac{P_{atm}}{\rho \cdot g} + \frac{0}{2 \cdot g} + 36 &= \frac{P_{atm}}{\rho \cdot g} + \frac{v_B^2}{2 \cdot g} + 3.6 \\ \Rightarrow v_B &= \sqrt{(36 - 3.6) \times 2 \times 9.81} = 25.2 \text{ m/s} \end{aligned}$$

Therefore:

$$Q = A_B \cdot v_B = \pi \frac{0.05^2}{4} \times 25.2 = 0.0495 \text{ m}^3/\text{s}$$

And:

$$\dot{m} = Q \cdot \rho = 0.0495 \times 800 = 39.6 \text{ kg/s}$$

$$= \frac{39.6 \times 3600}{1000} \text{ tonne/h}$$

$$= 142.58 \text{ tonne/h}$$

The energy lost in friction, i.e. the loss of power, is given by:

$\text{loss of power} = \text{mass flow rate} \times g \times h_f$
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J/s or Watt

kg/s

m/s^2

m

Therefore:

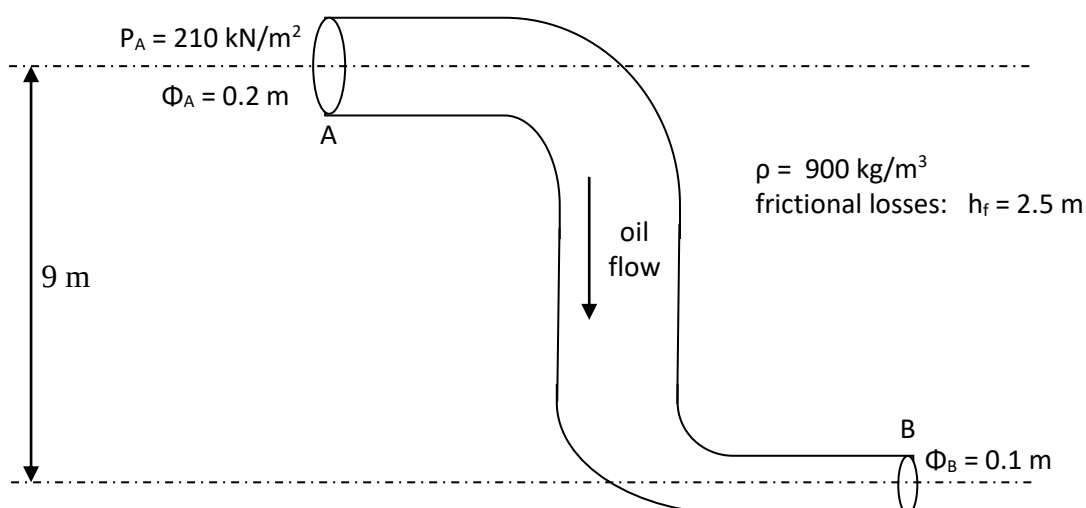
$$\text{loss of power} = 39.6 \times 9.81 \times 3.6$$

$$\therefore \text{loss of power} = 1,398.7 \text{ W}$$

Problem 4

The diameter of a pipe tapers gradually in the direction of oil flow as the level drops 9 m from point A to point B. The relative density of oil is 0.9. At A the gauge pressure is 210 kN/m² and the pipe diameter is 200 mm; at B the diameter is 100 mm. The frictional losses are equivalent to a head of 2.5 m of oil. What is the pressure at B when the flow rate is 226.8 tonnes per hour?

SOLUTION



The mass flow rate $\dot{m}$ is given:

$$\dot{m} = Q \times \rho$$

$$\begin{aligned}
&= 226.8 \text{ tonnes/hr} \\
&= \frac{226.8 \times 1000}{3600} \text{ kg/s} \\
&= 63 \text{ kg/s}
\end{aligned}$$

The density of oil is 900 kg/m^3 , therefore the volumetric flow rate Q is:

$$\begin{aligned}
Q &= \frac{\dot{m}}{\rho} \\
&= \frac{63}{900} \text{ m}^3/\text{s} \\
&= 0.07 \text{ m}^3/\text{s}
\end{aligned}$$

From continuity equation: $Q = \text{Area}_A \cdot v_A = \text{Area}_B \cdot v_B \Rightarrow 0.07 = A_A \cdot v_A = A_B \cdot v_B$

Therefore:

$$\begin{aligned}
v_A &= \frac{Q}{A_A} = \frac{0.07}{\frac{\pi}{4} \times 0.2^2} = 2.2 \text{ m/s} \\
v_B &= \frac{Q}{A_B} = \frac{0.07}{\frac{\pi}{4} \times 0.1^2} = 8.9 \text{ m/s}
\end{aligned}$$

Applying Bernoulli's equation:

$$\begin{aligned}
\frac{P_A}{\rho \cdot g} + \frac{v_A^2}{2 \cdot g} + h_A &= \frac{P_B}{\rho \cdot g} + \frac{v_B^2}{2 \cdot g} + h_B + h_f \\
\Rightarrow \frac{210 \times 10^3}{900 \times 9.81} + \frac{2.2^2}{2 \times 9.81} + 9 &= \frac{P_B}{900 \times 9.81} + \frac{8.9^2}{2 \times 9.81} + 2.5 \\
\Rightarrow \frac{P_B}{900 \times 9.81} &= \frac{210 \times 10^3}{900 \times 9.81} + \frac{2.2^2}{2 \times 9.81} + 9 - \frac{8.9^2}{2 \times 9.81} - 2.5 \\
\Rightarrow P_B &= \left(\frac{210 \times 10^3}{900 \times 9.81} + \frac{2.2^2 - 8.9^2}{2 \times 9.81} + 6.5 \right) \times 900 \times 9.81 \\
\Rightarrow \boxed{P_B = 233.9 \text{ kN/m}^2}
\end{aligned}$$

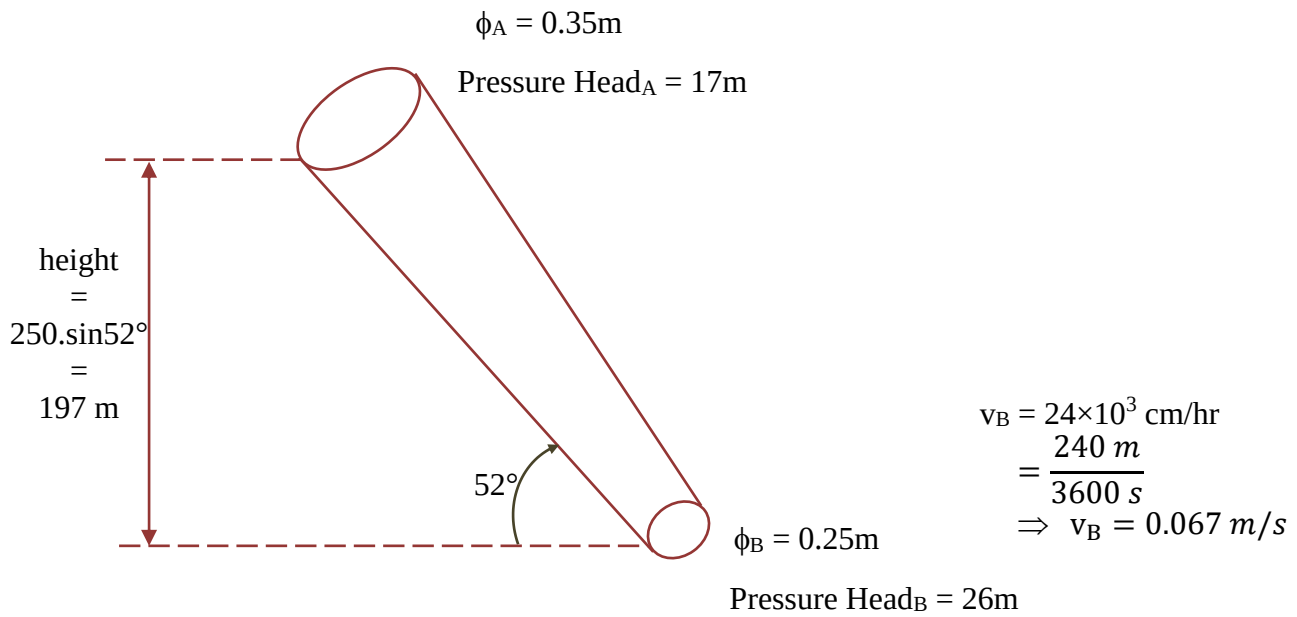
Problem 5

Water flows down a pipe from a level A to a level B. The pipe is 250 m long and is inclined at 52° to the horizontal. The pipe is tapered from a diameter of 35 cm at A to a diameter of 25 cm at B. Knowing:

- the pressure head at A is 17 m,
- the pressure head at B is 26 m,
- the velocity of water at B is $24 \times 10^3 \text{ cm/hr}$.

Find the loss of power due to friction in Watts.

SOLUTION



- **Applying the continuity equation:**

$$\begin{aligned}A_A \cdot v_A &= A_B \cdot v_B \\ \Rightarrow \frac{\pi \phi_A^2}{4} v_A &= \frac{\pi \phi_B^2}{4} v_B \\ \Rightarrow v_A &= \left(\frac{\phi_B}{\phi_A} \right)^2 v_B \\ \Rightarrow v_A &= \left(\frac{0.25}{0.35} \right)^2 \times 0.067 \\ \Rightarrow v_A &= 0.034 \text{ m/s}\end{aligned}$$

- **Applying Bernoulli's equation:**

$$\begin{aligned}\frac{P_A}{\rho \cdot g} + \frac{v_A^2}{2 \cdot g} + h_A &= \frac{P_B}{\rho \cdot g} + \frac{v_B^2}{2 \cdot g} + h_B + h_f \\ \Rightarrow 17 + \frac{0.034^2}{2 \times 9.81} + 197 &= 26 + \frac{0.067^2}{2 \times 9.81} + 0 + h_f \\ \Rightarrow h_f &= 17 + \frac{0.034^2}{2 \times 9.81} + 197 - 26 - \frac{0.067^2}{2 \times 9.81} \\ \Rightarrow h_f &= 17 + \frac{0.034^2}{2 \times 9.81} + 197 - 26 - \frac{0.067^2}{2 \times 9.81} \\ \Rightarrow h_f &= 188 \text{ m}\end{aligned}$$

- **The mass flow rate $\dot{m}$ is given by:**

$$\begin{aligned}
 \dot{m} &= Q \times \rho \\
 &= A_A \cdot v_A \times \rho \\
 &= \frac{\pi \phi_A^2}{4} v_A \times \rho \\
 &= \frac{\pi \times 0.35^2}{4} \times 0.034 \times 1000 \\
 &= 3.27 \text{ kg/s}
 \end{aligned}$$

- **The energy lost in friction, i.e. the loss of power, is given by:**

loss of power = mass flow rate $\times g \times h_f$			
J/s <u>or</u> Watt	kg/s	m/s ²	m

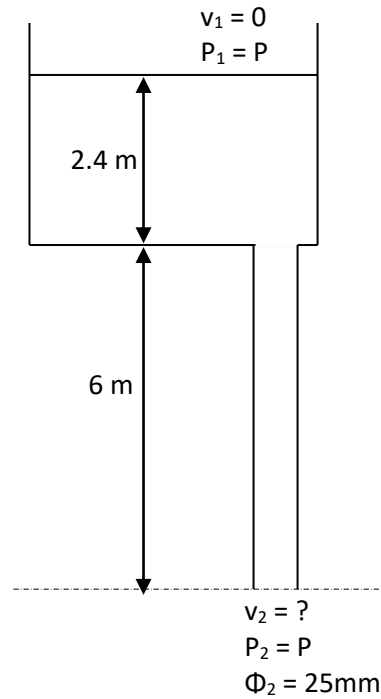
Therefore:

$$\begin{aligned}
 \text{loss of power} &= 3.27 \times 9.81 \times 188 \\
 \Rightarrow \text{loss of power} &= 6,035.4 \text{ W}
 \end{aligned}$$

Problem 6

An open tank contains oil of relative density 0.85 to a depth of 2.4 m. It discharges the oil to the atmosphere through a 25 mm diameter straight pipe at a point 6 m below the bottom of the tank. Ignoring losses, determine the discharge in litres per second and tonnes per hour.

SOLUTION



Applying Bernoulli's equation gives:

$$\frac{P_1}{\rho} + \frac{v_1^2}{2} + g \cdot h_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2} + g \cdot h_2$$

The pressure at the open surface in the tank and at the outlet are both the same as they are open to the atmosphere. The velocity at the upper water surface in the tank is assumed to be zero. Hence:

$$\frac{P}{\rho} + 0 + g \cdot (6 + 2.4) = \frac{P}{\rho} + \frac{v_2^2}{2}$$

$$\Rightarrow \frac{v_2^2}{2} = g \times 8.4$$

$$\Rightarrow v_2^2 = 2 \times 10 \times 8.4$$

$$\Rightarrow v_2 = \sqrt{2 \times 10 \times 8.4}$$

$$\Rightarrow v_2 = 12.96 \text{ m/s}$$

The volumetric flow rate Q is given by:

$$Q = A_2 \cdot v_2$$

$$= \frac{\pi}{4} \times 0.025^2 \times 12.96$$

$$= 6.4 \times 10^{-3} \text{ m}^3/\text{s}$$

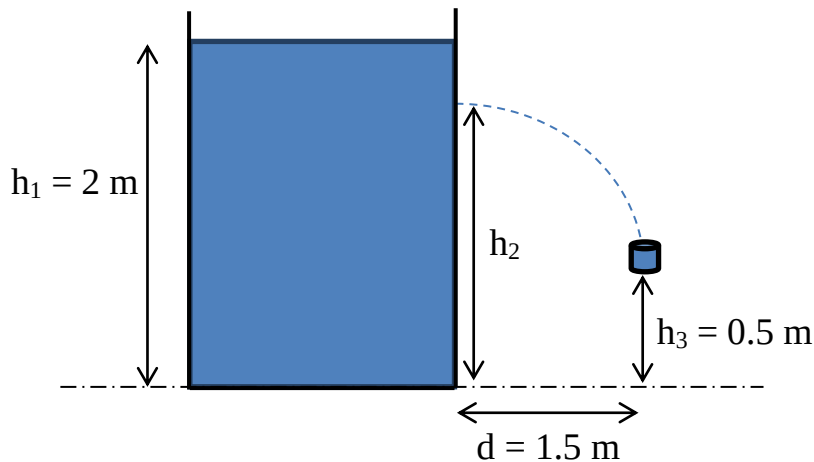
$$\Rightarrow \boxed{Q = 6.4 \text{ litres/s}} \quad (\text{as } 1 \text{ m}^3 = 1,000 \text{ litres})$$

The mass flow rate $\dot{m}$ is given by:

$$\begin{aligned}\dot{m} &= Q \times \rho \\ &= (6.4 \times 0.85) \text{ kg/s} \\ &= 5.4 \text{ kg/s} \\ &= \frac{5.4 \times 3600}{1000} \text{ tonne/h} \\ \Rightarrow \dot{m} &= 19.4 \text{ tonne/h}\end{aligned}$$

Problem 7

A tank of water is filled to a height h_1 of 2m. Water sprays out from a small opening at a height h_2 on the tank's side, and lands in a glass located at a horizontal distance of 1.5m from the tank. Determine the height h_2 of the opening if the glass stands at a height h_3 of 0.5 m as shown in the following figure. Neglect frictions and glass' dimensions.



SOLUTION:

Applying Bernoulli's equation gives:

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

The pressure at the open surface in the tank and at the outlet are both the same as they are open to the atmosphere. The velocity at the upper water surface in the tank is assumed to be zero. Hence:

$$\frac{P}{\rho g} + 0 + 2 = \frac{P}{\rho g} + \frac{v_2^2}{2g} + h_2$$

$$\therefore \frac{v_2^2}{2g} = 2 - h_2$$

$$\therefore v_2^2 = 2g(2 - h_2)$$

$$\therefore v_2^2 = 39.24 - 19.62h_2$$

If the jet of water travels for a time t before hitting the glass, then from linear motion law,

$$s = v_0 t + \frac{1}{2} a t^2$$

- Motion along x axis:

$$d = v_2 t \quad \therefore \quad t = \frac{d}{v_2}$$

- Motion along y-axis:

$$h_2 - h_3 = \frac{1}{2} g t^2$$

$$\therefore h_2 - h_3 = \frac{g d^2}{2 v_2^2} = \frac{g d^2}{2(2g(2 - h_2))}$$

$$\therefore h_2 - 0.5 = \frac{d^2}{4(2 - h_2)}$$

$$\therefore (h_2 - 0.5)(8 - 4h_2) = 1.5^2$$

$$\therefore 8h_2 - 4h_2^2 - 4 + 2h_2 = 1.5^2$$

$$\therefore 4h_2^2 - 10h_2 + 6.25 = 0$$

$$\therefore h_2 = \frac{10 \mp \sqrt{(-10)^2 - 4 \times 4 \times 6.25}}{2 \times 4} = \frac{10}{8} = 1.25m$$