



Complete, fold and seal
(see note 2 below)

SURNAME:

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Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the box below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
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1	Student Number	2426155
2	Title of Examination Paper	Mathematics
3	Module Code	ACDB4003
4	Date and Time of Examination	12/05/26 9:30 AM
5	Examination Venue	Room 310
6	Seat Number	3
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
1	9
2	10
3	14
4	8
5	15
6	9
7	18
8	9
Total	92

92%

1

$$\begin{aligned}
 \text{i)} \quad & 3x + 4y - z - 2x - 3y + 2z + x + 4z \\
 & 3x - 2x + x + 4y - 3y - z + 2z + 4z \\
 & = 2x + y + 5z
 \end{aligned}$$

$$\begin{aligned}
 \text{ii)} \quad & x(x+4) - (x-3) \\
 & \cancel{x^2 + 4x} - \cancel{x} + 3 \\
 & x^2 + 4x - x + 3 \\
 & x^2 + 3x + 3
 \end{aligned}$$

$$\begin{aligned}
 \text{iii)} \quad & p^3qr \times 2pq^4 \times 4qr^2 \\
 & = 8p^4q^6r^3
 \end{aligned}$$

$$\begin{aligned}
 \text{iv)} \quad & \frac{48p^2q^3r}{12pq^2r^{-3}} \\
 & = 4p^2q^5r^4
 \end{aligned}$$

$$\begin{aligned}
 \text{v)} \quad & \frac{fg}{16h} \times \frac{gh^3}{4fh} \times \frac{32fh^5}{f^4} \\
 & = \frac{32f^2g^2h^8}{64f^5h^2} \\
 & = \frac{1}{2} f^{-3} g^2 h^6
 \end{aligned}$$

2

$$\begin{aligned} \text{i)} \quad x^2 + 7x &= -12 \\ x^2 + 7x + 12 &= 0 \\ (x+4)(x+3) &= 0 \end{aligned}$$

either

$$\begin{aligned} x+4 &= 0 & x+3 &= 0 \\ x &= -4 & \text{or } x &= -3 \end{aligned}$$

$$\text{ii)} \quad \frac{(x-3)}{2} = \frac{(2x+1)}{3} \times 6$$

$$\frac{6(x-3)}{2} = \frac{6(2x+1)}{3}$$

$$3(x-3) = 2(2x+1)$$

$$3x - 9 = 4x + 2$$

$$\begin{aligned} 4x - 3x &= -9 - 2 \\ x &= -11 \end{aligned}$$

5

5

3
a) i)

$$5^{x+1} = 6$$

$$\log 5^{x+1} = \log 6$$

$$(x+1) \log 5 = \log 6$$

$$x \log 5 + \log 5 = \log 6$$

$$x \log 5 = \log 6 - \log 5$$

$$x = \frac{\log 6 - \log 5}{\log 5}$$

$$x = 0.11 \text{ 2 d.p.'s}$$

ii) $2 \log x = \log(5x-6)$

$$\log x^2 = \log(5x-6)$$

Can't cancel logs!!

$$\Rightarrow x^2 = 5x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

either $x-3=0$

$x-2=0$

$x=3$

or

$x=2$

b) i) $\log(x^2 y z)$

$$= \log x^2 + \log y + \log z$$

$$= 2 \log x + \log y + \log z$$

ii) $\log\left(\frac{y^2 x}{z}\right)$

$$= \log y^2 + \log x - \log z$$

$$= 2 \log y + \log x - \log z$$

4.
i) $\underline{a} \cdot \underline{b} = (1 \times 1) + (2 \times 1) + (3 \times 1)$
 $= 1 + 2 + 3$
 $= 6$

ii) $\underline{a} \cdot \underline{b} = |\underline{a}| \cdot |\underline{b}| \cos \theta$

$$|\underline{a}| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$

$$= \sqrt{14}$$

$$|\underline{b}| = \sqrt{(1)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{3}$$

$$6 = \sqrt{14} \times \sqrt{3} \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{6}{\sqrt{14} \times \sqrt{3}} \right)$$

$$= 22.208 \text{ 3d.p's}$$

iii) $\underline{c} \cdot \underline{a} = (2 \times 1) + (-1 \times 2) + (0 \times 3)$
 $= 2 + (-2) + 0$
 $= 0$

$\therefore \underline{c}$ is perpendicular to $\underline{a}$

iv) $\underline{d} = \underline{a} \times \underline{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$ $+ - +$ determinant!

$$\underline{d} = +\mathbf{i} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= \mathbf{i} \left\{ \begin{matrix} (2 \times 1) - (1 \times 3) \\ 2 & -3 \\ -1 \end{matrix} \right\} - \mathbf{j} \left\{ \begin{matrix} (1 \times 1) - (1 \times 3) \\ 1 & -3 \\ -2 \end{matrix} \right\} + \mathbf{k} \left\{ \begin{matrix} (1 \times 1) - (1 \times 2) \\ 1 & -2 \\ -1 \end{matrix} \right\}$$

$$\underline{d} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

4
iv)

$$\hat{d} = \frac{d}{|d|}$$

$$= \frac{-i + 2j - k}{\sqrt{(-1)^2 + (2)^2 + (-1)^2}}$$

$$= \frac{-i + 2j - k}{\sqrt{6}}$$

②

$$5. \quad A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{bmatrix} \quad + \quad - \quad +$$

$$|A| = +3 \begin{vmatrix} -1 & 2 \\ -3 & -4 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix}$$

$$= +3 \left\{ \begin{matrix} (-1 \times -4) - (-3 \times 2) \\ 4 & -(-6) \\ 10 \end{matrix} \right\} - 2 \left\{ \begin{matrix} (2 \times -4) - (1 \times 2) \\ -8 & -2 \\ -10 \end{matrix} \right\} + 1 \left\{ \begin{matrix} (2 \times -3) - (1 \times -1) \\ -6 & -(-1) \\ -5 \end{matrix} \right\}$$

$$\begin{aligned} |A| &= 3(10) + 2(10) - 1(5) \\ &= 30 + 20 - 5 \\ &= 45 \end{aligned}$$

$$A^{-1} = \frac{1}{|A|} \times C^T$$

$$C = \begin{bmatrix} + \begin{vmatrix} -1 & 2 \\ -3 & -4 \end{vmatrix} & - \begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} & + \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} \\ - \begin{vmatrix} 2 & 1 \\ -3 & -4 \end{vmatrix} & + \begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix} & - \begin{vmatrix} 3 & 2 \\ 1 & -3 \end{vmatrix} \\ + \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} & - \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} & + \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} \end{bmatrix}$$

$$5. \quad + \left\{ \begin{array}{cc} (-1 \times -4) - (-3 \times 2) \\ 4 & - (-6) \\ 10 \end{array} \right\} - \left\{ \begin{array}{cc} (2 \times -4) - (1 \times 2) \\ -8 & - (2) \\ -10 \end{array} \right\} + \left\{ \begin{array}{cc} (2 \times -3) - (1 \times -1) \\ -6 & - (-1) \\ -5 \end{array} \right\}$$

$$- \left\{ \begin{array}{cc} (2 \times -4) - (-3 \times 1) \\ -8 & - (-3) \\ -5 \end{array} \right\} + \left\{ \begin{array}{cc} (3 \times -4) - (1 \times 1) \\ -12 & - (1) \\ -13 \end{array} \right\} - \left\{ \begin{array}{cc} (3 \times -3) - (1 \times 2) \\ -9 & - 2 \\ -11 \end{array} \right\}$$

$$+ \left\{ \begin{array}{cc} (2 \times 2) - (-1 \times 1) \\ 4 & - (-1) \\ 5 \end{array} \right\} - \left\{ \begin{array}{cc} (3 \times 2) - (2 \times 1) \\ 6 & - 2 \\ 4 \end{array} \right\} + \left\{ \begin{array}{cc} (3 \times -1) - (2 \times 2) \\ -3 & - 4 \\ -7 \end{array} \right\}$$

$$C = \begin{bmatrix} 10 & 10 & -5 \\ 5 & -13 & 11 \\ 5 & -4 & -7 \end{bmatrix}$$

$$C^T = \begin{bmatrix} 10 & 5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{45} \begin{bmatrix} 10 & 5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{bmatrix}$$

good

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6.

$$\begin{aligned} 3x + 2y + z &= 6 \\ 2x - y + 2z &= 3 \\ x - 3y - 4z &= -6 \end{aligned}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -6 \end{bmatrix}$$

Cancellers here!

$$A \underline{x} = \underline{b}$$

$$\underline{x} = A^{-1} \underline{b}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{45} \begin{bmatrix} 10 & 5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ -6 \end{bmatrix}$$

$$= \frac{1}{45} \begin{bmatrix} 10(6) + 5(3) + 5(-6) \\ 10(6) - 13(3) + 4(6) \\ -5(6) + 11(3) + 7(6) \end{bmatrix}$$

$$= \frac{1}{45} \begin{bmatrix} 60 + 15 - 30 \\ 60 - 39 + 24 \\ -30 + 33 + 42 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{45} \begin{bmatrix} 45 \\ 45 \\ 45 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

9

7.

i)

$$y = 4x$$

$$\frac{dy}{dx} = 4$$

2

ii)

$$y = \frac{1}{x} + 8x^{\frac{1}{2}}$$

$$= x^{-1} + 8x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -x^{-2} + 4x^{-\frac{1}{2}}$$

2

iii)

$$y = \frac{1}{x^4} + 7x^3$$

$$= x^{-4} + 7x^3$$

$$\frac{dy}{dx} = -4x^{-5} + 21x^2$$

2

iv)

$$y = \sin(2x)$$

$$\frac{dy}{dx} = 2\cos(2x)$$

2

v)

$$y = x^3 \cdot \cos(x)$$

$$u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$v = \cos(x)$$

$$\frac{dv}{dx} = -\sin(x)$$

$$\frac{dy}{dx} = x^3 \cdot (-\sin(x)) + \cos(x) \cdot 3x^2$$

$$= -x^3 \sin(x) + 3x^2 \cos(x)$$

$$= x^2(-x \sin x + 3 \cos x)$$

2

7.

vi)

$$y = (5x+3)^{10}$$

$$\text{Let } u = 5x+3$$

$$y = u^{10}$$

$$\frac{du}{dx} = 5$$

$$\frac{dy}{dx} = 10u^9$$

Sub in

$$\frac{dy}{dx} = 5 \times 10u^9$$

$$= 50(5x+3)^9$$

(2)

vii)

$$y = e^x + \ln(x) - 3$$

$$\frac{dy}{dx} = e^x + \frac{1}{x}$$

(8)

b)

$$y = \frac{(x-3)}{(1+x^2)}$$

$$\frac{du}{dx} = \frac{d}{dx}(x-3) = 1$$

$$\frac{dv}{dx} = \frac{d}{dx}(1+x^2) = 2x$$

$$\frac{dy}{dx} = \frac{1(1+x^2) - 2x(x-3)}{(1+x^2)^2}$$

$$= \frac{1+x^2-2x^2+6x}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{-x^2+6x+1}{(1+x^2)^2}$$

when $x = 0$

$$\frac{dy}{dx} = \frac{-(0)^2+6(0)+1}{(1+(0)^2)^2}$$

$$= \frac{1}{1} = 1$$

(3)

8.

$$i) \int (2x + 5x^4) dx$$

$$= \frac{2x^2}{2} + \frac{5x^5}{5} + C$$

$$= x^2 + x^5 + C$$

②

$$ii) \int (3x^2 + 5e^x) dx$$

$$= \frac{3x^3}{3} + 5e^x + C$$

$$= x^3 + 5e^x + C$$

②

$$iii) \int \left(\frac{1}{x} - 4\right) dx$$

$$= \ln(x) - 4x + C$$

②

$$iv) \int (x^2)(3x + 4) dx$$

$$= 3x^2 + 4x^2$$

$$= \frac{3x^3}{3} + \frac{4x^3}{3} + C$$

$$= x^3 + \frac{4}{3}x^3 + C$$

$$= \int (3x^3 + 4x^2) dx$$

$$= \frac{3x^4}{4} + \frac{4x^3}{3} + C$$

~~②~~

①

$$v) \int_0^1 (4x^3 + 2x) dx$$

$$= \left[\frac{4x^4}{4} + \frac{2x^2}{2} \right]_0^1$$

(Next page →)

$$\begin{aligned} 8 \quad v) &= \left[x^4 + x^2 \right]_0^1 \\ &= \left[(1)^4 + (1)^2 \right] - \left[(0)^4 + (0)^2 \right] \\ &= [2] \end{aligned}$$

This is a very good effort!
Very well done!

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