



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

Complete, fold and seal
(see note 2 below)

SURNAME: WILLIAMS

FORENAME(S): MATTHEW

1	Student Number	1809220/1
2	Title of Examination Paper	MATHEMATICS (TEST 2)
3	Module Code	M4X00391
4	Date and Time of Examination	10/5/2019 PM
5	Examination Venue	HALL
6	Seat Number	20
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
2	
3	
Q2	
a	12 ✓
b	10 ✓
c	12 ✓
d	1 ✓
e	5 ✓
Q3	
a	20 ✓
b	5 ✓
c	7 ✓
d	8 ✓
~~e~~	
Total	80/80 ✓ 100% ✓

2. Question 2 - Complex numbers

$$\text{a) i) } z_1 + z_2 = 2 + 3j + 5 - 2j$$

$$= 7 + j \quad \checkmark \text{ ①}$$

$$\text{ii) } z_1 - z_3 = (2 + 3j) - (4 + j) \quad \checkmark \text{ ①}$$

$$= -2 + 2j \quad \checkmark \text{ ①}$$

$$\text{iii) } \frac{z_1}{z_2} = \frac{2+3j}{5-2j} \times \frac{5+2j}{5+2j} \quad \checkmark \text{ ①}$$

$$= \frac{(2+3j)(5+2j)}{(5-2j)(5+2j)}$$

$$= \frac{10 + 4j + 15j + 6j^2}{25 + 10j - 10j - 4j^2} \quad \checkmark \text{ ①}$$

$$= \frac{10 + 19j - 6}{25 + 4} \quad \checkmark \text{ ①}$$

$$= \frac{4 + 19j}{29}$$

$$= \frac{4}{29} + \frac{19j}{29} \quad \checkmark \text{ ①}$$

$$\text{iv) } \overline{z_1} + \overline{z_3} \quad \overline{z_1} = 2 - 3j \quad \overline{z_3} = 4 - j$$

$$= (2 - 3j) + (4 - j) \quad \checkmark$$

$$= \cancel{6 - 3j} \quad 6 - 4j \quad \checkmark \text{ ①}$$

$$v) (z_1)^2 + z_2 + z_3$$

$$(2+3j)(2+3j) + (5-2j) + (4+j) \quad \checkmark \textcircled{1}$$

$$(4+6j+6j+9j^2) + (9-j) \quad \checkmark$$

$$(4+12j-9) + (9-j) \quad \checkmark \textcircled{1}$$

$$(-5+12j) + (9-j) \quad \checkmark$$

$$4+11j \quad \checkmark \quad \checkmark \quad \textcircled{1} \quad \textcircled{1}$$

$$b) 14x^2 = -3x - 9$$

$$14x^2 + 3x + 9 = 0 \quad \checkmark \textcircled{1}$$

$$a=14 \quad b=3 \quad c=9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \checkmark$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(14)(9)}}{2(14)} \quad \checkmark \textcircled{1}$$

$$x = \frac{-3 \pm \sqrt{-495}}{28} \quad \checkmark \textcircled{1}$$

$$x = \frac{-3 \pm \sqrt{495} \times \sqrt{-1}}{28} \quad \checkmark$$

$$x = \frac{-3 \pm \sqrt{495}j}{28} \quad \checkmark \textcircled{1}$$

$$x = \frac{-3 + \sqrt{495}j}{28}$$

$$\text{or} \quad x = \frac{-3 - \sqrt{495}j}{28}$$

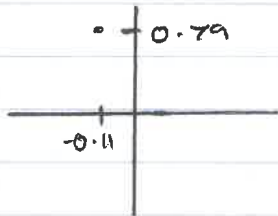
$$x = -0.11 + 0.79j \quad \checkmark \quad \textcircled{1}$$

$$x = -0.11 - 0.79j \quad \checkmark \quad \textcircled{1}$$

$$r = \sqrt{(-0.11)^2 + (0.79)^2}$$

$$r = 0.8 \quad \checkmark \quad \textcircled{1}$$

$$\theta = \tan^{-1}\left(\frac{0.79}{0.11}\right) = 82.07^\circ$$



$$\theta = 180 - 82.07$$

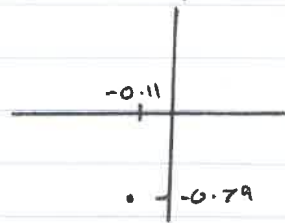
$$= 97.93^\circ \quad \checkmark$$

$$0.8 (\cos(97.93) + j \sin(97.93)) \quad \checkmark \quad \textcircled{1}$$

$$r = \sqrt{(-0.11)^2 + (-0.79)^2}$$

$$r = 0.8 \quad \checkmark \quad \textcircled{1}$$

$$\theta = \tan^{-1}\left(\frac{0.79}{0.11}\right) = 82.07^\circ$$



$$\theta = 180 + 82.07$$

$$= 262.07^\circ \quad \checkmark$$

$$0.8 (\cos(262.07) + j \sin(262.07)) \quad \checkmark \quad \textcircled{1}$$

c) i) $(6 + j5)^5$

$$r = \sqrt{6^2 + 5^2}$$

$$r = \sqrt{61} \quad \checkmark \quad \textcircled{1}$$

$$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.81^\circ \quad \checkmark \quad \textcircled{1}$$

$$(\sqrt{61})^5 (\cos(5 \times 39.81) + j \sin(5 \times 39.81)) \quad \checkmark \quad \textcircled{1}$$

$$29061.94 (-0.95 - 0.33j) \quad \checkmark \quad \textcircled{1}$$

$$-27608.84 - 9590.44j \quad \checkmark$$

$\textcircled{1} \quad \checkmark \quad \textcircled{1}$

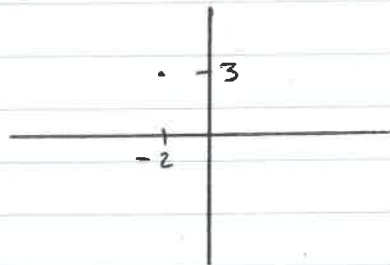
6

$$\text{ii) } (-2 + 3j)^7$$

$$r = \sqrt{(-2)^2 + (3)^2}$$

$$r = \sqrt{13} \quad \checkmark \textcircled{1}$$

$$\theta = \tan^{-1}\left(\frac{3}{-2}\right) = 56.31^\circ$$



$$\theta = 180 - 56.31$$

$$= 123.69^\circ \quad \checkmark \textcircled{1}$$

$$(\sqrt{13})^7 (\cos(7 \times 123.69) + j \sin(7 \times 123.69)) \quad \checkmark \textcircled{1}$$

$$7921.4 (-0.83 + 0.56j) \quad \checkmark \textcircled{1}$$

$$-6574.76 + 4435.98j$$

$$\textcircled{1} \quad \checkmark \quad \checkmark \quad \textcircled{1}$$

$$\text{d) Complex conjugate of } 1 + 8j \text{ is } 1 - 8j \quad \checkmark \textcircled{1}$$

$$\text{e) } z_1 z_2 z_3$$

$$(6 + j)(2 - 5j)(1 - j)$$

$$(12 - 30j + 2j - 5j^2)(1 - j) \quad \checkmark \textcircled{1}$$

$$(12 - 28j + 5)(1 - j)$$

$$(17 - 28j)(1 - j)$$

$$17 - 17j - 28j + 28j^2 \quad \checkmark$$

$$17 - 45j - 28$$

$$-11 - 45j \quad \checkmark$$

$$\textcircled{1} \quad \textcircled{1}$$

$$r = \sqrt{(-11)^2 + (-45)^2}$$

$$r = 46.32 \quad \checkmark$$

$$\theta = \tan^{-1}\left(\frac{45}{11}\right) = 76.26^\circ$$



$$\theta = 180 + 76.26 = 256.26^\circ \quad \checkmark \quad \textcircled{1}$$

$$46.32 (\cos(256.26) + j\sin(256.26)) \quad \checkmark \quad \textcircled{1}$$

5/11

3. Question 3 - Matrices

a) i) AB

$$\begin{aligned}
 5(3) - 1(2) + 6(-3) &= -5 \checkmark \\
 5(1) - 1(-5) + 6(1) &= 16 \checkmark \\
 5(1) - 1(4) + 6(1) &= 7 \checkmark \\
 7(3) + 5(2) - 1(-3) &= 34 \checkmark \\
 7(1) + 5(-5) - 1(1) &= -19 \checkmark \\
 7(1) + 5(4) - 1(1) &= 26 \checkmark \\
 1(3) + 0(2) + 2(-3) &= -3 \checkmark \\
 1(1) + 0(-5) + 2(1) &= 3 \checkmark \\
 1(1) + 0(4) + 2(1) &= 3 \checkmark
 \end{aligned}$$

$$AB = \begin{vmatrix} -5 & 16 & 7 \\ 34 & -19 & 26 \\ -3 & 3 & 3 \end{vmatrix}$$

ii) BA

$$\begin{aligned}
 3(5) + 1(7) + 1(1) &= 23 \checkmark \\
 3(-1) + 1(5) + 1(0) &= 2 \checkmark \\
 3(6) + 1(-1) + 1(2) &= 19 \checkmark \\
 2(5) - 5(7) + 4(1) &= -21 \checkmark \\
 2(-1) - 5(5) + 4(0) &= -27 \checkmark \\
 2(6) - 5(-1) + 4(2) &= 25 \checkmark \\
 -3(5) + 1(7) + 1(1) &= -7 \checkmark \\
 -3(-1) + 1(5) + 1(0) &= 8 \checkmark \\
 -3(6) + 1(-1) + 1(2) &= -17 \checkmark
 \end{aligned}$$

$$BA = \begin{vmatrix} 23 & 2 & 19 \\ -21 & -27 & 25 \\ -7 & 8 & -17 \end{vmatrix}$$

iii) AC

$$\begin{aligned}
 5(6) - 1(4) + 6(2) &= 38 \checkmark \\
 5(5) - 1(3) + 6(1) &= 28 \checkmark \\
 7(6) + 5(4) - 1(2) &= 60 \checkmark \\
 7(5) + 5(3) - 1(1) &= 49 \checkmark \\
 1(6) + 0(4) + 2(2) &= 10 \checkmark \\
 1(5) + 0(3) + 2(1) &= 7 \checkmark
 \end{aligned}$$

$$AC = \begin{vmatrix} 38 & 28 \\ 60 & 49 \\ 10 & 7 \end{vmatrix}$$

iv) Cannot calculate CA (1)

v) |A|

$$\begin{vmatrix} 5 & -1 & 6 \\ 7 & 5 & -1 \\ 1 & 0 & 2 \end{vmatrix} \begin{vmatrix} 5 & -1 \\ 7 & 5 \\ 1 & 0 \end{vmatrix}$$

$\begin{vmatrix} 5 & -1 \\ 7 & 5 \end{vmatrix} = 30 - 0 = 16$
 $\begin{vmatrix} 5 & -1 \\ 1 & 0 \end{vmatrix} = 50 - 1 = 51$

$$51 - 16 = 35$$

vi) |B|

$$\begin{vmatrix} 3 & 1 & 1 \\ 2 & -5 & 4 \\ -3 & 1 & 1 \end{vmatrix} \begin{vmatrix} 3 & 1 \\ 2 & -5 \\ -3 & 1 \end{vmatrix}$$

$\begin{vmatrix} 3 & 1 \\ 2 & -5 \end{vmatrix} = 15 - 2 = 29$
 $\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix} = -15 - 12 = -27$

$$-27 - 29 = -56$$

$$vii) A+B \quad \begin{vmatrix} 5+3 & -1+1 & 6+1 \\ 7+2 & 5-5 & -1+4 \\ 1-3 & 0+1 & 2+1 \end{vmatrix} = \begin{vmatrix} 8 & 0 & 7 \\ 9 & 0 & 3 \\ -2 & 1 & 3 \end{vmatrix}$$

viii) Cannot calculate A+D

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(4)(-152)}}{2(4)}$$

$$x = \frac{14 \pm \sqrt{2628}}{8}$$

$$x = \frac{14 + \sqrt{2628}}{8}$$

or

$$x = \frac{14 - \sqrt{2628}}{8}$$

$$x = 8.16 \text{ (2dp)}$$

$$x = -4.66 \text{ (2dp)}$$

d)
$$\begin{cases} 5x + 3y + z = 25 \\ 3x - 2y + 3z = 26 \\ 2x + y - 5z = -6 \end{cases}$$

$$\begin{bmatrix} 5 & 3 & 1 \\ 3 & -2 & 3 \\ 2 & 1 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 25 \\ 26 \\ -6 \end{bmatrix}$$

A

$$\begin{vmatrix} 5 & 3 & 1 \\ 3 & -2 & 3 \\ 2 & 1 & -5 \end{vmatrix} = 5 \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} - 3 \begin{vmatrix} 3 & 3 \\ 2 & -5 \end{vmatrix} + 1 \begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix}$$

$$= 5(9 - 2) - 3(-15 - 6) + 1(3 - 2)$$

$$= 5(7) - 3(-21) + 1(1) = 35 + 63 + 1 = 105$$

$$|A| = 71 + 34 = 105$$

A₁

$$\begin{vmatrix} 25 & 3 & 1 \\ 26 & -2 & 3 \\ -6 & 1 & -5 \end{vmatrix} = 25 \begin{vmatrix} -2 & 3 \\ 1 & -5 \end{vmatrix} - 3 \begin{vmatrix} 26 & 3 \\ -6 & -5 \end{vmatrix} + 1 \begin{vmatrix} 26 & -2 \\ -6 & 1 \end{vmatrix}$$

$$= 25(10 - 3) - 3(-130 - 18) + 1(26 - 12)$$

$$= 25(7) - 3(-148) + 1(14) = 175 + 444 + 14 = 633$$

$$|A_1| = 222 + 303 = 525$$

8/11

$$A_2 \left| \begin{array}{ccc|ccc} 5 & 25 & 1 & 5 & 25 & \\ 3 & 26 & 3 & 3 & 26 & \\ 2 & -6 & -5 & 2 & -6 & \end{array} \right|$$

$\begin{array}{l} \nearrow 52 \quad \nearrow -90 \quad \nearrow -375 = -413 \\ \searrow -650 \quad \searrow 150 \quad \searrow -18 = -518 \end{array}$

✓

$$|A_2| = -518 + 413$$

$$= -105 \quad \checkmark \quad \textcircled{1}$$

$$A_3 \left| \begin{array}{ccc|ccc} 5 & 3 & 25 & 5 & 3 & \\ 3 & -2 & 26 & 3 & -2 & \\ 2 & 1 & -6 & 2 & 1 & \end{array} \right|$$

$\begin{array}{l} \nearrow -100 \quad \nearrow 130 \quad \nearrow -54 = -24 \\ \searrow 60 \quad \searrow 156 \quad \searrow 75 = 291 \end{array}$

✓

$$|A_3| = 291 + 24$$

$$= 315 \quad \checkmark \quad \textcircled{1}$$

$$x = \frac{525}{105}$$

$$y = \frac{-105}{105}$$

$$z = \frac{315}{105}$$

$$x = 5 \quad \checkmark$$

①

$$y = -1 \quad \checkmark$$

①

$$z = 3 \quad \checkmark$$

①