

ANALYSIS OF VARIANCE TECHNIQUES (ANOVA)

ANOVA

ANOVA is used when we want to know if two or more conditions or levels of the independent variable create significant mean differences on the dependent variable.

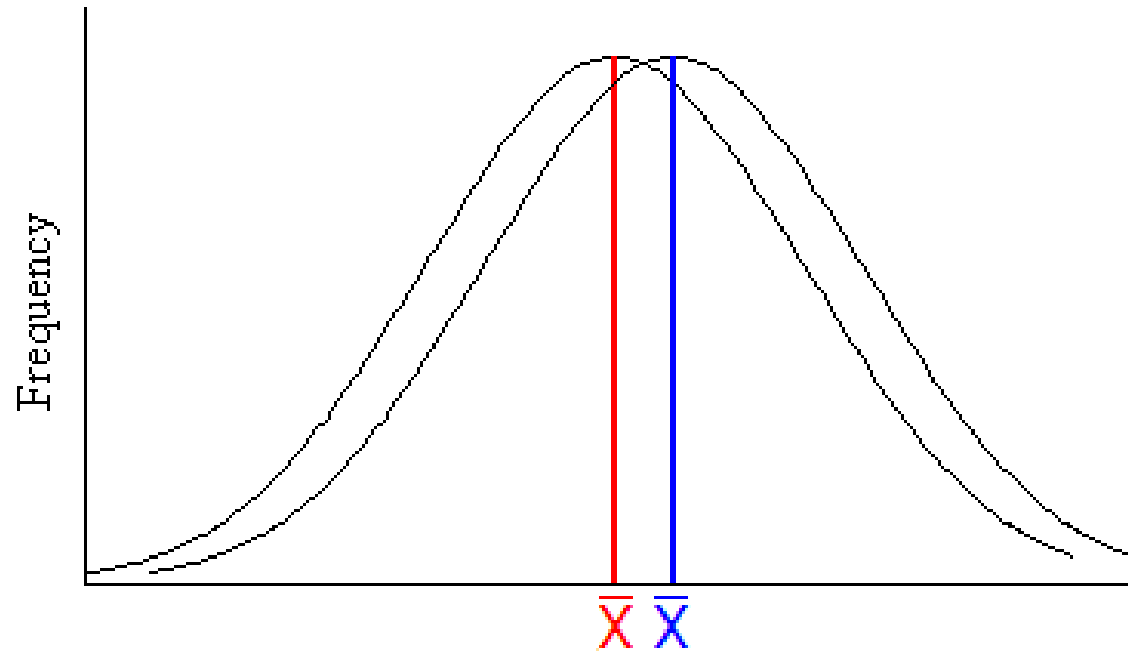
The IV must be a nominal or ordinal variable identifying conditions or levels, and we compare the DV mean scores of the different levels. The DV must be a scale variable.

The aim of the analysis is to identify the probability that any difference in the **DV sample means for different levels of the grouping variable** reflects a true difference in the **population means**.

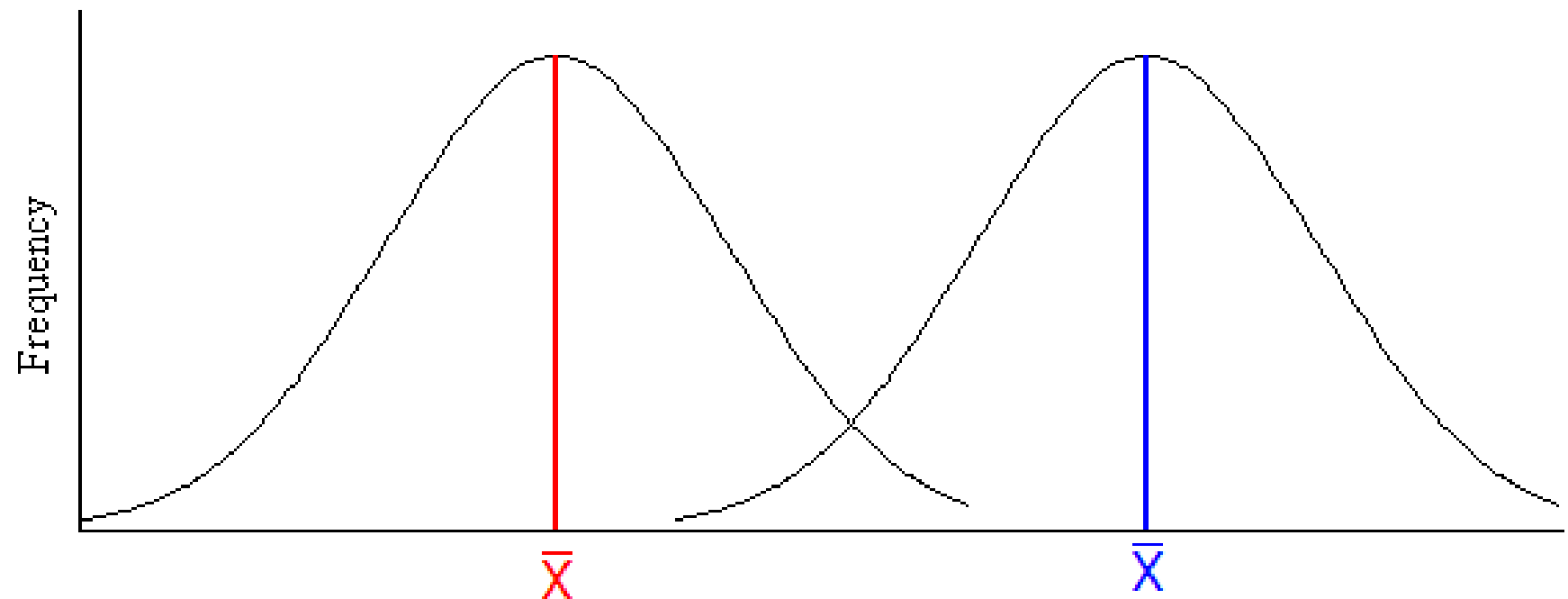
ANOVA

ANOVA compares two possible situations.

1. The **two or more samples of the IV** come from different populations of **normal distributions** which have **similar variances** (and, therefore, standard deviations) but have different population means or
2. The **two or more samples** come from the same population which is normally distributed. **Any difference in sample means is due to sampling error**. This is the default position. We should continue to believe that this is true unless we are 95% certain that it isn't true ($p < .05$).



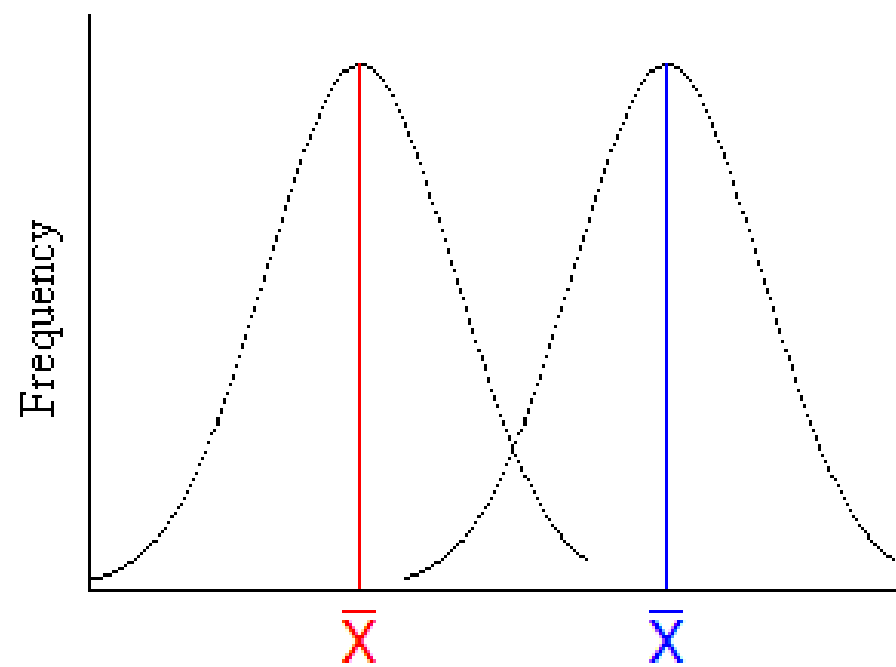
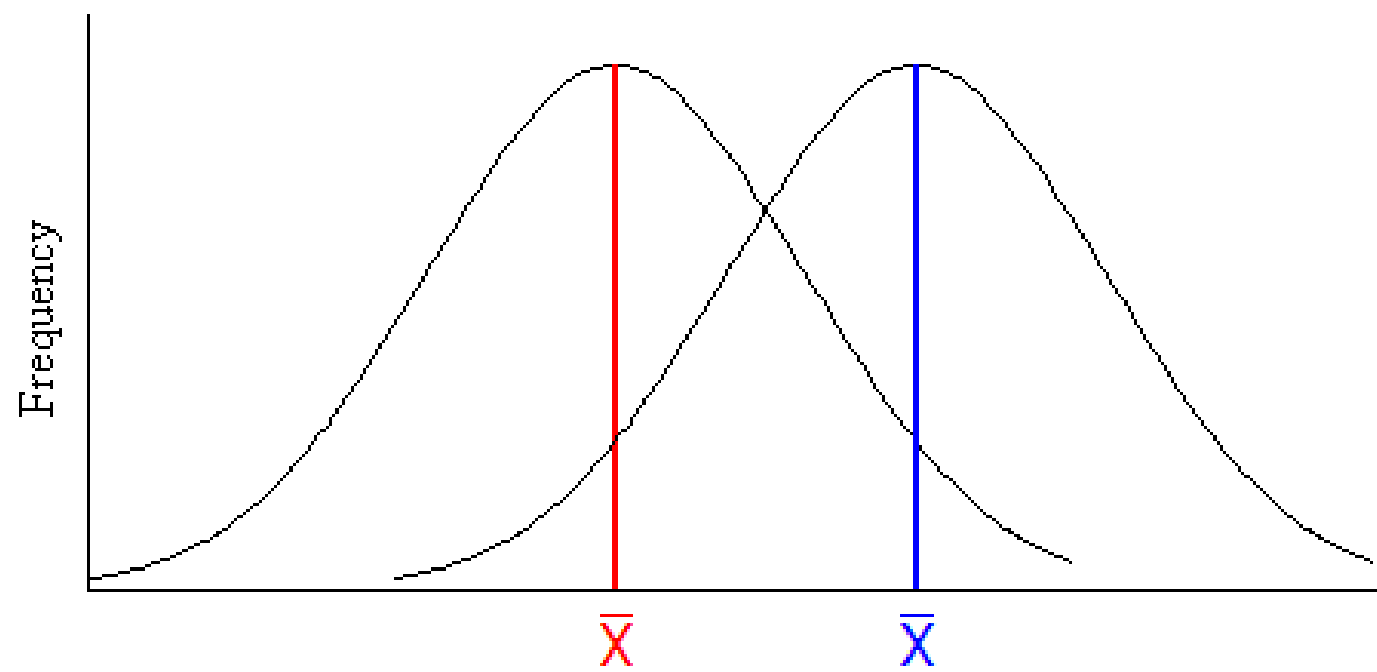
Two possible sets of
sample distributions



ANOVA

If we collected two samples, what would make us more certain that the two samples came from different populations?

1. The **bigger the difference** between two sample **means**, the **more likely** that the population means are **different**.
2. The smaller the variation within each group, the more likely that the population means are different.



THE LOGIC OF ANOVA

- ANOVA is built on comparing variances from two sources. These two estimates are:
 - **The Between Group variance** - a measure of the effect of the independent variable plus the error variance.
 - **The Within Groups variance** - the error variance alone.
- The **ratio** between them is called the **F ratio**

THE F RATIO OR STATISTIC

$$F = \frac{\text{Magnitude of variation between groups}}{\text{Magnitude of variation within groups}}$$

$$= \frac{\text{Between group variance}}{\text{Within group variance}}$$

So the bigger the value for F, the larger the between group variance is to the within group variance (the random variation).

PROBABILITY

- We use the F statistic to calculate the probability that the two sample means come from two different populations.
- Something else is also important when calculating the probability – **the larger the samples, the more likely the differences** also exist in the population.
- Thus the **significance** of F also depends on df
 - the larger the df the smaller the F for significance

F RATIO

- A **significant F ratio** informs us that the **group** or treatment means are **not all equal**
- When the variance between groups relative to within groups is large then the F ratio is large
- The **larger the F** ratio the more likely it is **significant** and that there are significant differences between some or all group means
- A **small F** ratio close to 1 implies that the two **variance** estimates are **very similar**

ASSUMPTIONS of ANOVA

- **Normality** - ANOVA is fairly robust for departures from normality as long as they are **not extreme**
- **Homogeneity of variance** - This similarity of variance in each group is needed in order to 'pool' the variances into one Within Group source of variance
- **Independence of errors** - Error here refers to the difference between each observation or score from its own group mean. In other words each score should be independent of any other score

ANOVA FAMILY

- **There are many types of ANOVA** just as there were statistics for measuring differences between two means.
- **One Way ANOVA'S** are the equivalent of the Independent t test for more than two levels of the IV
- **Repeated Measures ANOVA's** are the equivalent of the Repeated measures t test for more than two occasions
- **Factorial ANOVA's** look at interactions between two or more IV's

ONE WAY BETWEEN GROUPS ANOVA (INDEPENDENT GROUPS ANOVA)

- has only one IV which splits into two or more levels
- two basic variability components:
 - Between Groups or treatments (conditions) variability.
Variability due to the differences between groups and reflected in variability between sample means. The sources of this variability are:
 - treatment effects, i.e. the different conditions;
 - individual differences, i.e. the uniqueness of people
- and
 - experimental error, i.e. uncontrolled and unknown causes.
 - Within Groups or treatments (conditions) variability.
There is variability within each sample as each person within a sample produced different results from others in that sample. This is due to:
 - individual differences; and
 - experimental error.

F RATIO

- Once total variability is analysed into its basic components we simply compare them by computing *the F ratio*.
- For the independent measures ANOVA :

$$F = \frac{\text{variability between treatments}}{\text{variability within treatments}} \quad \text{or} \quad \frac{\text{between treatment variance}}{\text{within treatment variance}}$$

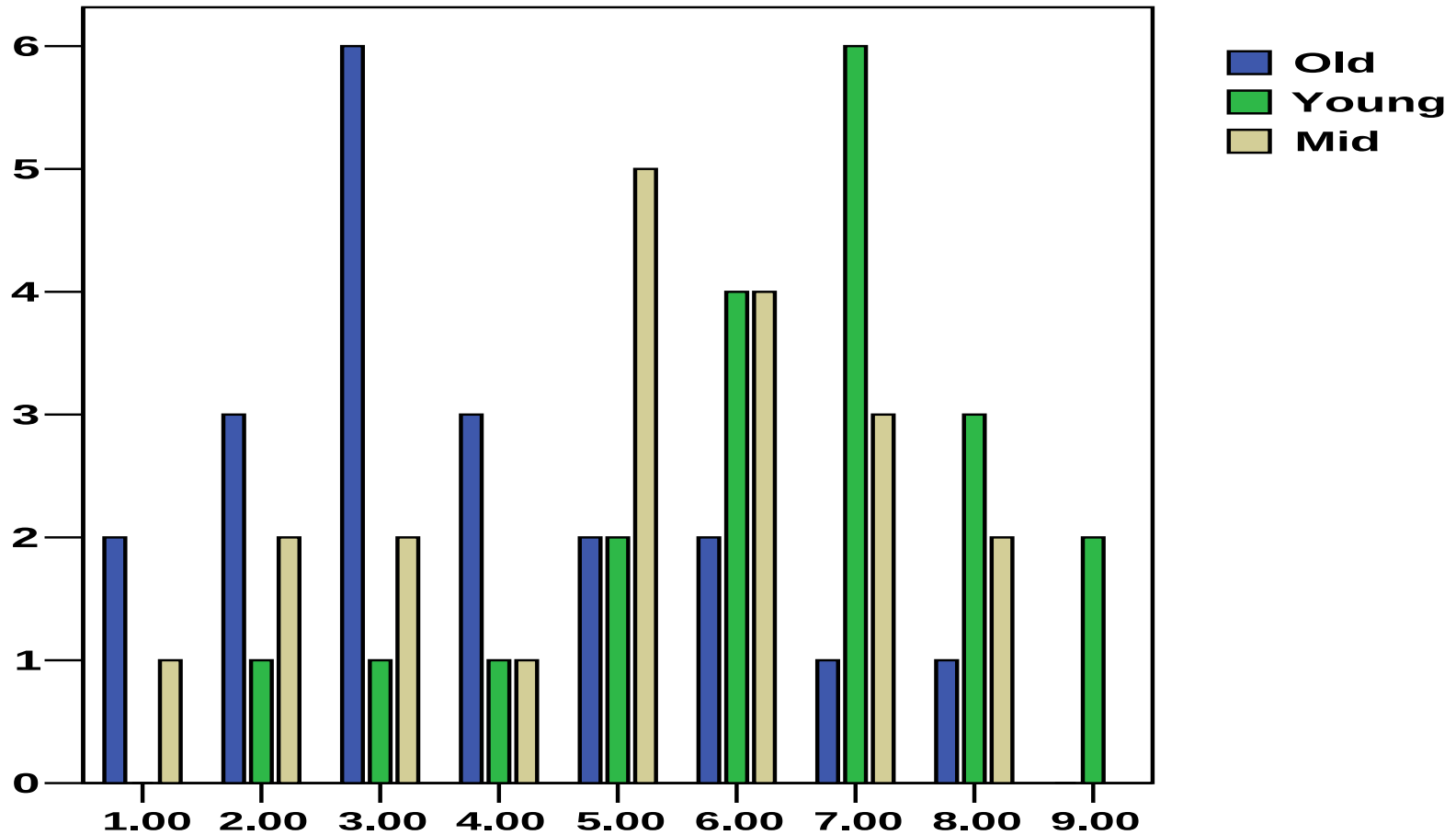
POST HOC TESTS

- **A Post Hoc Test** is conducted **after a significant F test** in order to identify where significant differences lie among **three or more treatments**
- Tukey's HSD, Scheffe, Bonferroni and the Games-Howell procedure are commonly used.
- When variances differ use the Games-Howell or Dunnett.

Example: One-Way ANOVA

- Test of attitude to income tax rates between three age groups (Old, Mid and Young)
- IV – AGE (three groups - nominal); DV – attitude to tax rates (score out of 10 - interval)
- $N = 60$
- One Way ANOVA with POST HOC Analysis

Histogram of test scores by Age Groups



Results

Descriptives

Test score

		N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum	Between-Component Variance
						Lower Bound	Upper Bound			
Old		20	3.7500	1.91600	.42843	2.8533	4.6467	1.00	8.00	1.5046
Young		20	6.3500	1.84320	.41215	5.4874	7.2126	2.00	9.00	
Mid		20	5.0500	2.01246	.45000	4.1081	5.9919	1.00	8.00	
Total		60	5.0500	2.17400	.28066	4.4884	5.6116	1.00	9.00	
Model	Fixed Effects			1.92513	.24853	4.5523	5.5477			
	Random Effects				.75056	1.8206	8.2794			

Results

Test of Homogeneity of Variance

Test Score

Levene Statistic	df1	df2	Sig.
.081	2	57	.922

Not significant
so homogeneity
of variances is
satisfied



Highly significant
F so significant
mean differences
exist



ANOVA

Test Score

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	67.600	2	33.800	9.120	.000
Within Groups	211.250	57	3.706		
Total	278.850	59			

Interpretation Of Table

- $F = 9.120$ and is highly significant ($p = .000$)
- But are there significant differences between all the groups?
- A Beronferroni Post Hoc test reveals the significant differences only lie between the young and old groups

Post Hoc

Multiple Comparisons

Dependent Variable: Attitude Score

Bonferroni

(I) Age Group	(J) Age Group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Old	Young	-2.6000 *	.60878	.000	-4.1017	-1.0983
	Mid	-1.3000	.60878	.111	-2.8017	.2017
Young	Old	2.6000 *	.60878	.000	1.0983	4.1017
	Mid	1.3000	.60878	.111	-.2017	2.8017
Mid	Old	1.3000	.60878	.111	-.2017	2.8017
	Young	-1.3000	.60878	.111	-2.8017	.2017

*. The mean difference is significant at the .05 level.

INTERPRETATION OF POST HOC TEST

The only significant difference on attitudes to income tax rates is between the young group and the old group ($p < .05$).

REPEATED MEASURES ANALYSIS OF VARIANCE

Analysis of Variance in which each individual is measured more than once so that the levels of the independent variable are the different times or types of observations for the same people

TWO FACTOR ANOVA or TWO WAY ANOVA or FACTORIAL ANOVA

- When **two factors** of interest are to be **examined** at the same time
- Tests a **null hypothesis for each** of the **independent** variables **and** also one for their **interaction**, the interaction effect
- **Interactions occur** when the effect of 1 IV on the DV is **not the same** under all the **conditions** of the other IV.

FACTORIAL ANOVA

Has three distinct hypotheses

1. The main effect of factor A (IV 'A'). The null hypothesis states that there are no statistically significant mean differences between levels of factor A
2. The main effect for factor B (IV 'B').
There is a similar null hypothesis for factor B
3. The A X B interaction. The null hypothesis states that there is no statistically significant interaction. That is the effect of either factor is independent of the levels of the other factor

EXAMPLE OF FACTORIAL OR TWO WAY ANOVA

Independent Variables:

- Alcohol use (Three groups: Little or No Alcohol Use; Moderate Alcohol Use; Heavy Alcohol Use)
- Gender

Dependent Variable:

- Self-Rated Health Status

N = 60 (20 per group)

Do alcohol use and gender interact in determining self health rating?

Descriptive Statistics

Dependent Variable: Health Rating

Alcohol Use	Gender	Mean	Std. Deviation	N
.00	1.00	1.8333	.75277	6
	2.00	3.8571	1.91581	14
	Total	3.2500	1.88833	20
1.00	1.00	6.8750	1.45774	8
	2.00	8.0833	2.06522	12
	Total	7.6000	1.90291	20
2.00	1.00	2.8235	1.50977	17
	2.00	2.0000	1.00000	3
	Total	2.7000	1.45458	20
Total	1.00	3.6774	2.37188	31
	2.00	5.4138	2.99425	29
	Total	4.5167	2.80732	60

Tests of Between-Subjects Effects

Dependent Variable: Health_Rating

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Observed Power ^a
Corrected Model	314.173 ^b	5	62.835	22.499	.000	.676	1.000
Intercept	773.732	1	773.732	277.048	.000	.837	1.000
Alcohol_Use	259.470	2	129.735	46.454	.000	.632	1.000
Gender	6.918	1	6.918	2.477	.121	.044	.340
Alcohol_Use*Gender	13.054	2	6.527	2.337	.106	.080	.453
Error	150.810	54	2.793				
Total	1689.000	60					
Corrected Total	464.983	59					

a. Computed using alpha = .05

b. R Squared = .676 (Adjusted R Squared = .646)

Interpretation of Previous Table

- The only significant main effects F was for levels of alcohol use on self rated health ($p < .000$).
- There were no significant differences for either gender or the interaction between gender and alcohol use.

INTERPRETATION OF PAIR-WISE COMPARISONS TABLE

- Since the level of alcohol consumption IV was significant it is necessary to determine whether the significance relates to all levels of alcohol consumption.
- The next table reveals that significant differences ($p < .05$) only lie between the moderate consumption group and the two other groups
- The nil and heavy groups did not differ significantly in their self rated health status

Pairwise Comparisons

Dependent Variable: Health_Rating

(I) Alcohol_Use (J) Alcohol_Use		Mean Difference (I-J)	Std. Error	Sig. ^a	95% Confidence Interval for Difference ^a	
					Lower Bound	Upper Bound
.00	1.00	-4.634 *	.558	.000	-5.753	-3.515
	2.00	.433	.663	.516	-.896	1.763
1.00	.00	4.634 *	.558	.000	3.515	5.753
	2.00	5.067 *	.648	.000	3.769	6.366
2.00	.00	-.433	.663	.516	-1.763	.896
	1.00	-5.067 *	.648	.000	-6.366	-3.769

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

ANALYSIS OF COVARIANCE (ANCOVA)

- a form of analysis that is based on a combination of regression and ANOVA
- do groups differ on a DV when you have partialled out another variable (the covariate) that has a relationship with the DV?
- ANCOVA answers the question '*What would the means of each group be on the DV if the means of the groups had the same mean on the co-variate*'?

NON-PARAMETRIC ALTERNATIVES FOR ANOVA

- ***Kruskal-Wallis One Way Non-Parametric ANOVA***
- ***Friedman Two way Non-Parametric ANOVA***
- ***Located under Non-Parametric tests in SPSS***