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(ACMA5001)
Control & Automation
exam

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Q 4(a)

$$7.5 \frac{d^2 \theta_0}{dt^2} + 40 \frac{d\theta_0}{dt} + 300 \theta_0 = 800 \theta_i$$

$$\frac{7.5}{7.5} \frac{d^2 \theta_0}{dt^2} + \frac{40}{7.5} 2\zeta \omega_n \frac{d\theta_0}{dt} + \omega_n^2 \frac{300}{7.5} = 800 \omega_n^2 \theta_i$$

$$\omega_n^2 \frac{300}{7.5} = \omega_n \sqrt{\frac{300}{7.5}} = 6.325 \text{ Hz}$$

$$2\zeta \omega_n \frac{40}{7.5} \therefore \zeta = \frac{40}{2\omega_n \times 7.5} = \frac{40}{2 \times 6.325 \times 7.5} = 0.4216$$

So $\zeta = 0.4216$ So, System is under damped.

$$(b) \exp\left(\frac{-\zeta \pi}{\sqrt{1-\zeta^2}}\right) \times 100\% = \exp\left(\frac{-0.4216 \times \pi}{\sqrt{1-0.4216^2}}\right) \times 100\%$$

= 0.232 or 23.2% peak overshoot

$$\text{Rise time} = t_r = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{6.325 \times \sqrt{1-0.4216^2}} = 0.2745 \text{ s}$$

$$\text{Peak time} = t_p = \frac{\pi \zeta}{\omega_n \sqrt{1-\zeta^2}} = \frac{0.4216 \times \pi}{6.325 \times \sqrt{1-0.4216^2}} = 0.548 \text{ s}$$

$$\text{Settling time } 2\% \quad t_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.4216 \times 6.325} = 1.5 \text{ s}$$