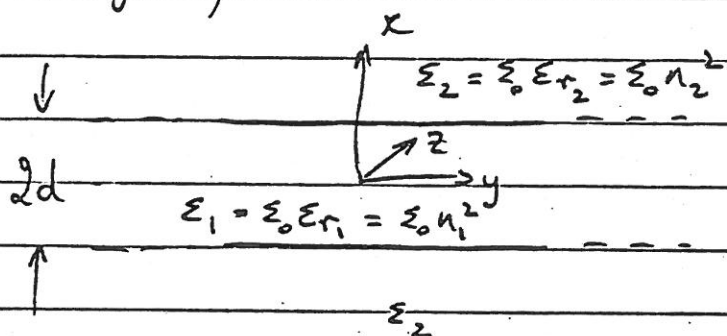


Guided modes of the Slab Waveguide

Consider guided waves propagating in the z -direction in the infinite dielectric slab, thickness $2d$, lying in the yz plane i.e.



For waves travelling in the z -direction we can expect a phase constant of the form $\exp j(\omega t - \beta z)$. Also, because of uniformity in the y -direction, the derivative $\partial/\partial y$ of all variables must be zero.

Consider Maxwell's Eqs.

$$\text{curl } \underline{H} = \epsilon_0 \mu^2 \frac{\partial \underline{E}}{\partial t}$$

$$\text{curl } \underline{E} = -\mu_0 \frac{\partial \underline{H}}{\partial t}$$

$$\text{Since } \text{curl } \underline{H} = \underline{i} \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) + \underline{j} \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) + \underline{k} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)$$

$$\text{and } \frac{\partial}{\partial y} = 0 \text{ and } \frac{\partial}{\partial z} = -j\beta$$

we have

$$\text{curl } \underline{H} = \epsilon_0 \mu^2 j\omega \underline{E} = \underline{i} j\beta H_y - \underline{j} \left(j\beta H_x + \frac{\partial H_z}{\partial x} \right) + \underline{k} \frac{\partial H_y}{\partial x}$$

Hence

$$\epsilon_0 \mu^2 j\omega E_x = j\beta H_y \quad (1)$$

$$\epsilon_0 \mu^2 j\omega E_y = -j\beta H_x - \frac{\partial H_z}{\partial x} \quad (2)$$

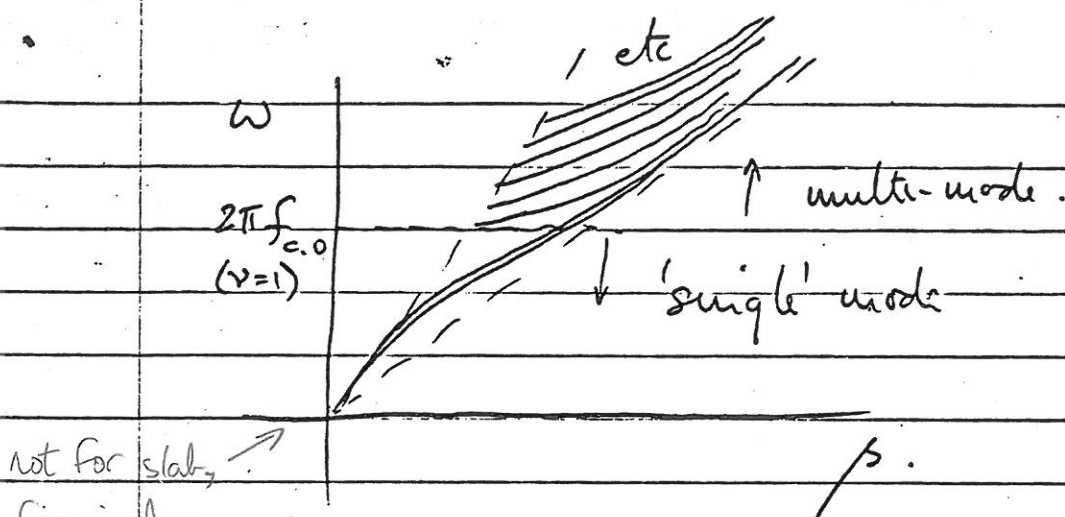
$$\epsilon_0 \mu^2 j\omega E_z = \frac{\partial H_y}{\partial x} \quad (3)$$

Similarly from $\text{curl } \underline{E} = -\mu_0 \frac{\partial \underline{H}}{\partial t}$

$$j\omega \mu_0 H_x = -j\beta E_y \quad (4)$$

$$j\omega \mu_0 H_y = \frac{\partial E_z}{\partial x} + j\beta E_x \quad (5)$$

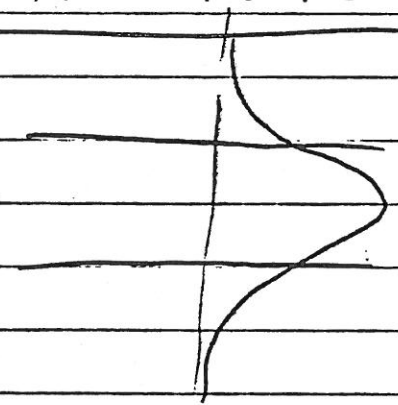
$$j\omega \mu_0 H_z = -\frac{\partial E_y}{\partial x} \quad (6)$$



(Note: $v_{\text{phase}} = \frac{\omega}{\beta}$ $v_{\text{group}} = \frac{d\omega}{d\beta}$)

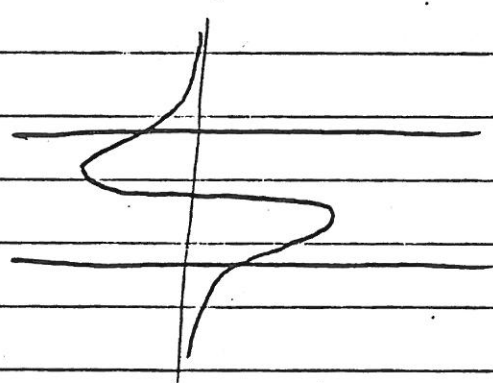
Shapes of Transverse Fields.

$\nu = 0$



E_y (TE_0) (even)
 H_y (TM_0)

$\nu = 1$



E_y (TE_1) (odd)
 H_y (TM_1)

etc.

These two sets of equations are consistent with the solutions

(i) $E_x = H_y = E_z = 0; E_y; H_x; H_z \neq 0$ (TE modes)

(ii) $E_y = H_x = H_z = 0; E_x; H_y; E_z \neq 0$ (TM modes)
In real case, $\epsilon_y \neq 0$, so get other components.

For case (i), TE modes, we can write eqn 2 as

$$\epsilon_0 \mu^2 j\omega E_y = -\frac{\partial H_z}{\partial x} + j\beta \left(\frac{\beta}{\omega\mu_0} \right) E_y \quad (\text{using 4})$$

$$\text{hence } j \left(\epsilon_0 \mu^2 \omega - \frac{\beta^2}{\omega\mu_0} \right) E_y = -\frac{\partial H_z}{\partial x}$$

Differentiating w.r.t. x and substituting from 6 leads to

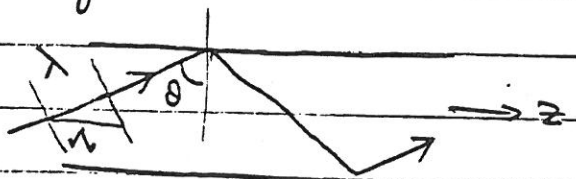
$$\frac{\partial^2 H_z}{\partial x^2} + \left(\epsilon_0 \mu_0 \mu^2 \omega^2 - \beta^2 \right) H_z = 0$$

Since $\mu_0 \epsilon_0 = \frac{1}{c^2}$ and $\frac{\omega^2}{c^2} = k_0^2$ ($k_0 = \frac{2\pi}{\lambda_0}$ in free space)
then

$$\frac{\partial^2 H_z}{\partial x^2} + (\mu^2 k_0^2 - \beta^2) H_z = 0 \quad (7)$$

where for $-d < x < d$ $\mu = \mu_1$
and $-d > x > d$ $\mu = \mu_2$

If we consider the physical situation;



We can expect waves to propagate by total internal reflection, for angles $\theta >$ than the critical angle θ_c . The phase periodicity in z -direction, $\lambda > \lambda_c$

From 8, 11 $\tan(Kd)_{c.o} = \infty$

$$\therefore (Kd)_{c.o} = \pi/2, 3\pi/2, \dots \quad (\text{odd})$$

and from 9, 10 $\tan(Kd)_{c.o} = 0$

$$(Kd)_{c.o} = 0, \pi, 2\pi, \dots \quad (\text{even})$$

The cut-off conditions taken together yield

$$(Kd)_{c.o} = v \cdot \frac{\pi}{2}$$

where v is an integer and describes the mode number.

Since $K = k_0 (u_1^2 - \beta^2/k_0^2)^{1/2}$
 and at cut-off $\gamma = 0 = k_0 (\beta^2/k_0^2 - u_2^2)^{1/2}$
 then $K_{c.o} = k_0 (u_1^2 - u_2^2)^{1/2}$

$$\text{and } (Kd)_{c.o} = k_0 d (u_1^2 - u_2^2)^{1/2} = V.$$

The $V=0$ cut-off condition means that $k_0 = 0$ i.e. $\lambda = \infty$, $f \rightarrow 0$. $\therefore$ Two modes, called TE_0 , TM_0 have zero cut-off frequency.

For $V = \pi/2 = \frac{2\pi}{c} f d (u_1^2 - u_2^2)^{1/2}$
 i.e. $f_{c.o} = \frac{c}{4d (u_1^2 - u_2^2)^{1/2}}$

For all frequencies of operation $f < f_{c.o}$, only the TE_0 , TM_0 modes can propagate.

The eqns 8-11 establish the way in which β varies with ω . These are called dispersion relations and, if sketched, are approximately

($k_x = \frac{2\pi}{\lambda}$). $\therefore \rho$ decreases from $n_1 k_0$ as θ decreases from $\pi/2$. At, or just beyond $\theta = \theta_c$, the wave will propagate in the cladding at near grazing incidence, hence with $\rho = n_2 k_0$. Therefore we can expect that for guided waves $n_2 k_0 < \rho < n_1 k_0$.

We can now solve eqn 7 for guided modes;
 for $u = u_1$, $(u_1^2 k_0^2 - \rho^2) > 0$ ($= k^2$)
 $u = u_2$, $(\rho^2 - u_2^2 k_0^2) > 0$ ($= \gamma^2$).

Hence, given the symmetry of the structure with respect to the plane $x=0$, we can write

$$H_z = A \cos Kx \quad \text{or} \quad H_z = A' \sin Kx$$

for $-d < x < d$

and $H_z = B \exp -\gamma x$ ($x > d$) $H_z = B' \exp -\gamma x$
 $= B \exp +\gamma x$ ($x < -d$) $= -B' \exp +\gamma x$

Hence we have a complete solution for H_z for guided TE modes.

For the transverse field components

$$E_y = -j \frac{\omega \mu_0}{K} A \sin Kx \quad (\text{from 6}) \quad -d < x < d \quad \text{or} \quad E_y = +j \frac{\omega \mu_0}{K} A' \cos Kx$$

(called 'odd' mode)

(even' mode)

and $E_y = +j \frac{\omega \mu_0}{\gamma} B \exp -\gamma x$ $x > d$ $E_y = j \frac{\omega \mu_0}{\gamma} B' \exp -\gamma x$
 $= -j \frac{\omega \mu_0}{\gamma} B \exp \gamma x$ $x < -d$ $E_y = j \frac{\omega \mu_0}{\gamma} B' \exp \gamma x$

From eqn (4)

$$H_x = \frac{\rho}{\omega \mu_0} E_y$$

(Note $\frac{H_x}{H_z} = \frac{u}{(u_1^2 - u_2^2)^{1/2}} \gg 1$; $\therefore$ wave nearly transverse)

We can now apply boundary conditions at the interfaces between n_1 and n_2 .
Hence at $x=d$, H_z , E_y are continuous.

$\therefore$ (For odd modes)

$$A \cos kd = B \exp -\gamma d$$

$$-\frac{j\omega\mu_0}{k} A \sin kd = +\frac{j\omega\mu_0}{\gamma} B \exp -\gamma d.$$

For a non-trivial solution

$$\begin{vmatrix} \cos kd & -\exp -\gamma d \\ \frac{1}{k} \sin kd & \frac{1}{\gamma} \exp -\gamma d \end{vmatrix} = 0$$

Hence $\tan kd = -\frac{k}{\gamma}$ (8)

TE modes

[Alternatively, for even modes,

$$\tan kd = \frac{\gamma}{k} \quad (9)$$

Eqs 8, 9 are functions of β , but are transcendental (i.e. you cannot write $\beta = \dots$)

A similar analysis for TM modes would give

$$\tan kd = \frac{n_1^2}{n_2^2} \frac{\gamma}{k} \quad (\text{even}) \quad (10)$$

$$\tan kd = -\frac{n_1^2}{n_2^2} \frac{k}{\gamma} \quad (\text{odd}) \quad (11)$$

Particular guided modes cease to propagate when $\theta = \theta_c$, i.e. $\beta \rightarrow n_2 k_0$ or $\gamma \rightarrow 0$. This condition is called 'cut-off'.

$$\text{TE odd} \quad \tan Kd = -\frac{K}{\gamma}$$

$$\text{TM odd} \quad \tan Kd = -\frac{n_2^2}{n_1^2} \frac{K}{\gamma}$$

$$\text{TE even} \quad \tan Kd = \frac{\gamma}{K}$$

$$\text{TM even} \quad \tan Kd = \frac{n_1^2}{n_2^2} \frac{\gamma}{K}$$

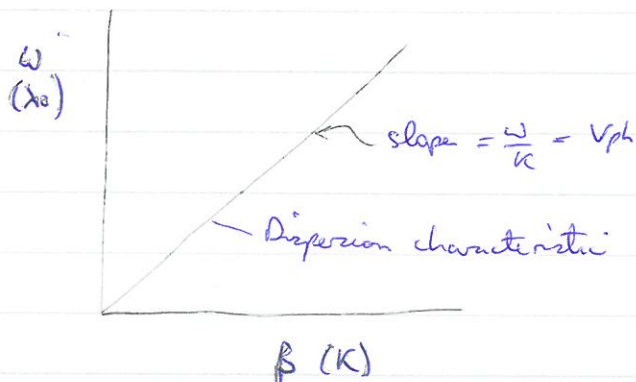
$$K = (n_1^2 k_0^2 - \beta^2)^{1/2}$$

$$\gamma = (\beta^2 - n_2^2 k_0^2)^{1/2}$$

$$n_2 k_0 < \beta < n_1 k_0$$



$$k_0 = \frac{2\pi}{\lambda_0}$$



All these guided modes just cease to propagate when $\theta = \theta_c$ i.e. as $\beta \rightarrow n_2 k_0$, i.e. $\gamma = 0$

$$K_{c.o.} \rightarrow (n_1^2 k_0^2 - n_2^2 k_0^2)^{1/2} = k_0 (n_1^2 - n_2^2)^{1/2}$$

$$\therefore \text{Even modes} \quad (Kd)_{c.o.} = 0, \pi, 2\pi, \dots$$

$$\text{Odd modes} \quad (Kd)_{c.o.} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

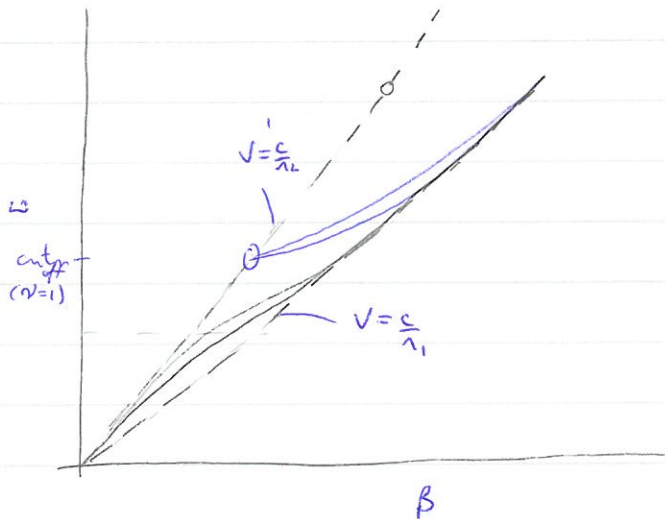
$$i.e. (Kd)_{co} = v \cdot \frac{\pi}{2}$$

$$v = 0, 1, 2, 3, \dots$$

= mode number.

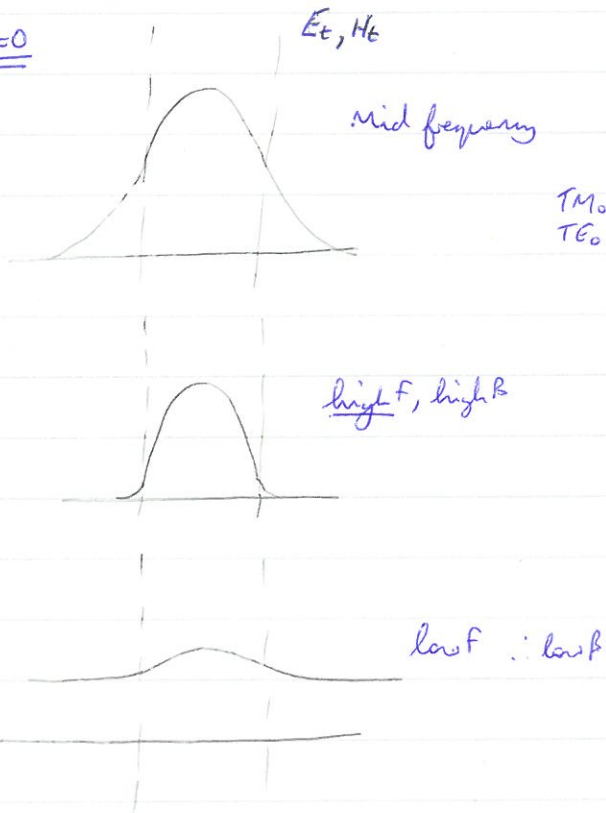
$$= K_0 d (\bar{n}_1^2 - \bar{n}_2^2)^{1/2}$$

$$\frac{v\pi}{2} = \left(\frac{2\pi f_0}{c} \right) d (\bar{n}_1^2 - \bar{n}_2^2)^{1/2}$$



v, v' are the limiting factors. Propagating mode must always lie between these bound.
for slab, cut off frequencies are evenly spaced.

$v=0$

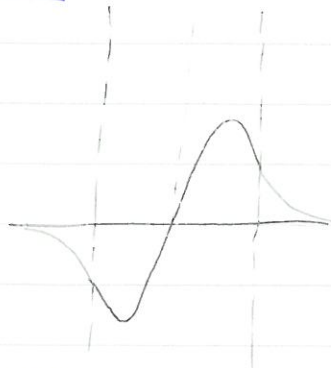


$v=0$ has cut off frequency 0.

at low f , is similar to a plane wave propagating in the cladding.

As increase frequency, more & more modes will start propagating.

$V=1$

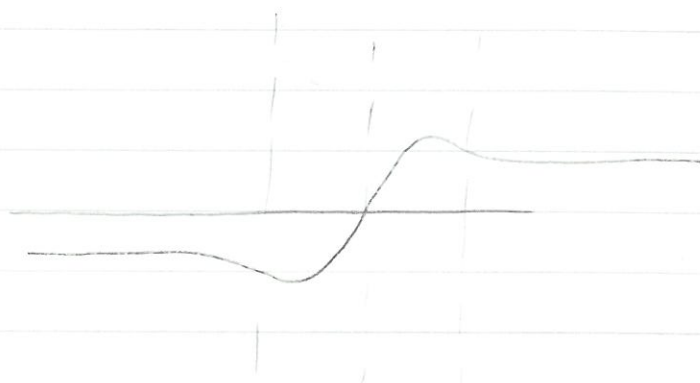


TM_1
 TE_1

high f



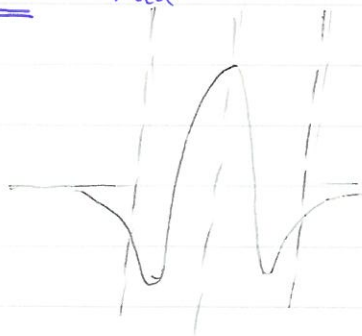
mid f



low f

$V=2$

mid f



TM_2
 TE_2

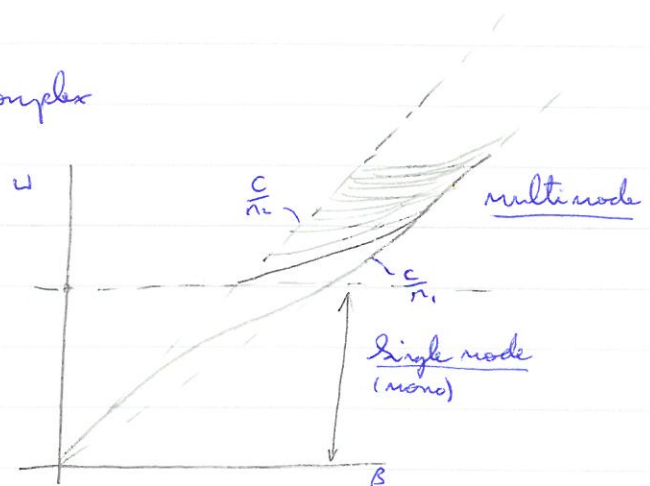
$V=3$

mid f



TM_3
 TE_3

Red - characteristic eqn. more complex



Obviously would expect
bounds to be $\frac{c}{n_2}$, $\frac{c}{n_1}$.

Now orthogonal modes are
degenerate i.e. the same
if symmetrical

would expect

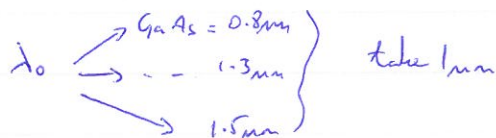
$$k_0 a (n_1^2 - n_2^2)^{1/2} = 2.405 \quad \text{1st zero of zeroth order Bessel } J_0$$

a = radius of rod

Red modes are not evenly spaced where they cut off - they tend to start very close together. So basically have a choice between one mode or many hundreds.

low dispersion associated with single mode operation - spreading only due to fact the v_{ph} is not a straight line.

high dispersion with multimode - each mode has different v_{ph} , so get high dispersion.



$\therefore$ for single mode

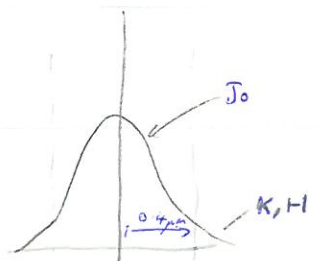
$$a < \frac{2.405}{2\pi(n_1^2 - n_2^2)^{1/2}}$$

ray air-cored
 $n_1 = 1.5$
 $n_2 = 1.0$

$$\therefore a \approx \frac{2.4}{6 \times 1.1}$$

ie less than $0.4 \mu m$

Mechanically almost impossible to achieve the focus and stability. So not air-cored.

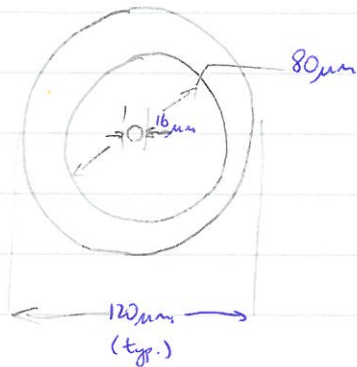


So for single mode, generally choose $(n_1 - n_2) \approx 10^{-3}$

$$\text{then } a \leq \frac{2.4}{6 \times 3 \times 10^{-2} \times 1.7} = \frac{2.4 \times 10}{3} \approx 8 \mu\text{m}$$

Easier to form into, easier to join together, but still not easy. Not truly practicable for widespread use. Only very low efficiency with non-coherent source. Need a good laser source.

For multimode, core may be 80 μm diameter. Can use LED source or laser.



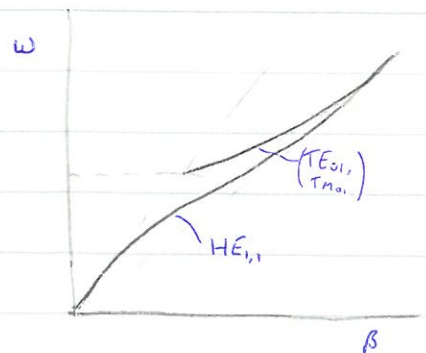
Nomenclature

Slab

TE_v, TM_v
 TM_v

Rod

$HE_{v,m}$
 $EH_{v,m}$ } $LP_{v,m}$ — hybrid modes
linearly polarized
 $TM_{0,m}$
 $TE_{0,m}$



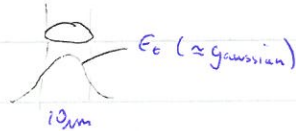
$$LP_{v,m} \text{ mode} = HE_{v+1,m} + EH_{v-1,m}$$

HE_{11} already linearly polarized

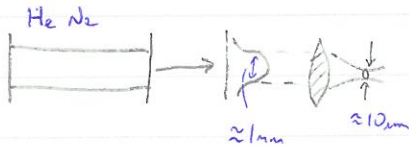
$HE_{11} \rightarrow LP_{0,1}$

Launching

Monomode



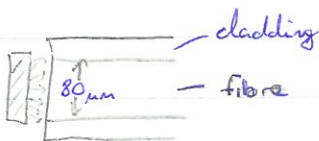
eg HeNe laser has approx gaussian output



Not used in communication,
but in lab can get $> 90\%$
efficiency.

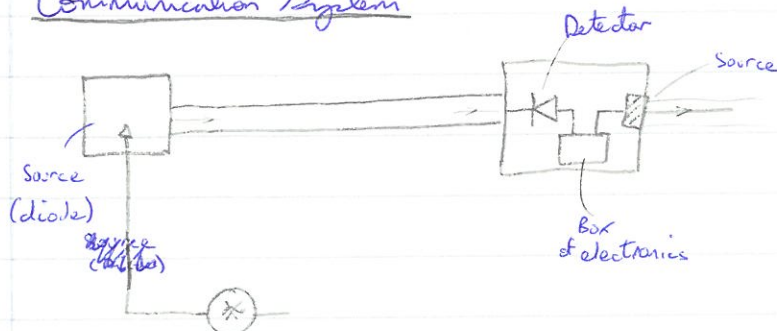
In communication, generally use a GaAs slice with an emitting area $\approx 3-5\mu m$, and try to place the optical input over the source.

Multimode



Use LED throwing out light in all directions. Multimode fibre picks up the energy required by its modes.

Communication System



Dispersion (loss)

$$\text{phase vel}^y = \frac{\omega}{\beta}$$

$$\text{group vel}^y, v_g = \frac{d\omega}{d\beta} = c \frac{dk_0}{d\beta}$$

$$\omega = ck_0$$

$$\text{Time delay } \tau = \frac{l}{v_g} = \frac{l}{c} \left(\frac{d\beta}{dk_0} \right)$$

Function of material refractive index.

Different modes have different v_g 's.

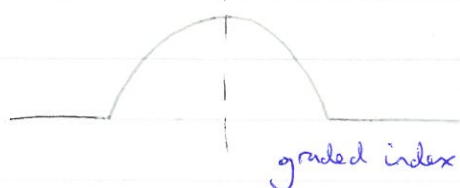
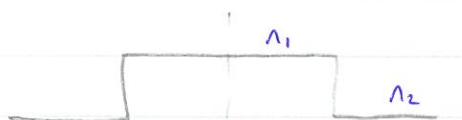
This results in a fact: if a pulse goes in one end, a wider pulse comes out the other

$$\frac{d\beta}{dk_0} \approx \frac{d}{dk_0} (n_2 k_0) + \frac{d}{dk_0} (n_2 k_0 F)$$

(at 1 Mb/s $\approx 1 \text{ km}$)

Because of high dispersion with step index guide, graded index

fibres are used. For these, the group velocities of various modes are approximately uniform



140 Mbit/s - 10 km

cf
co-axial

140 Mb/s < 2 km

For single mode, only problem is change in refractive index for frequency

$$T_s = \frac{L}{c} \frac{dB}{dK_0} = \frac{L}{c} \frac{d}{dK_0} (n_2 K_0)$$

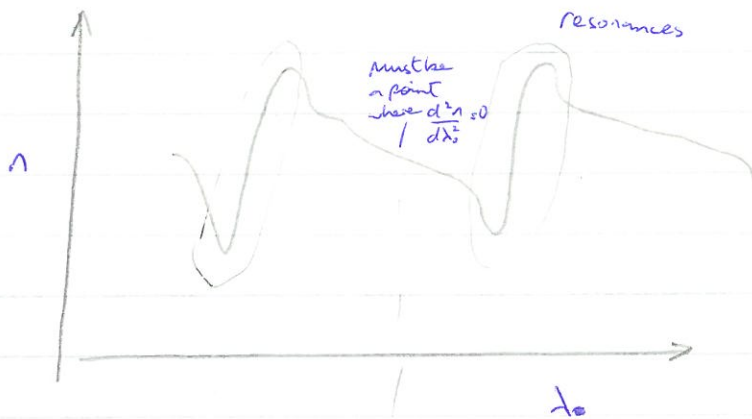
$$\uparrow$$

$$f(\lambda) \rightarrow K_0$$

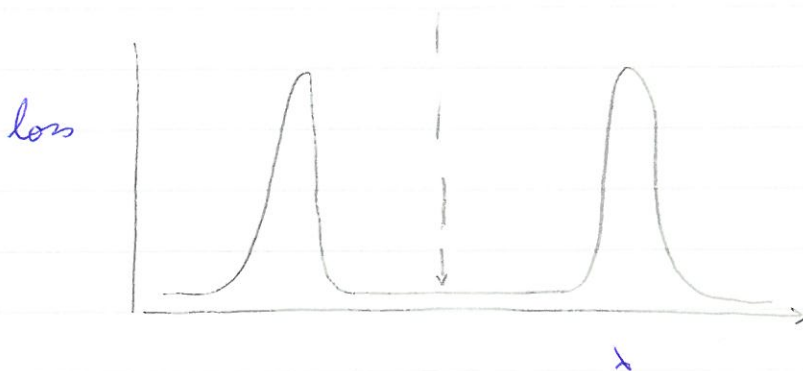
$$\delta T_s = \frac{d^2}{dK_0^2} (n_2 K_0) \delta K_0$$

change from K_0 to λ_0

$$\delta T_s = \frac{L}{c} \lambda_0 \left(\frac{d^2 n_2}{d\lambda_0^2} \right) \delta \lambda$$

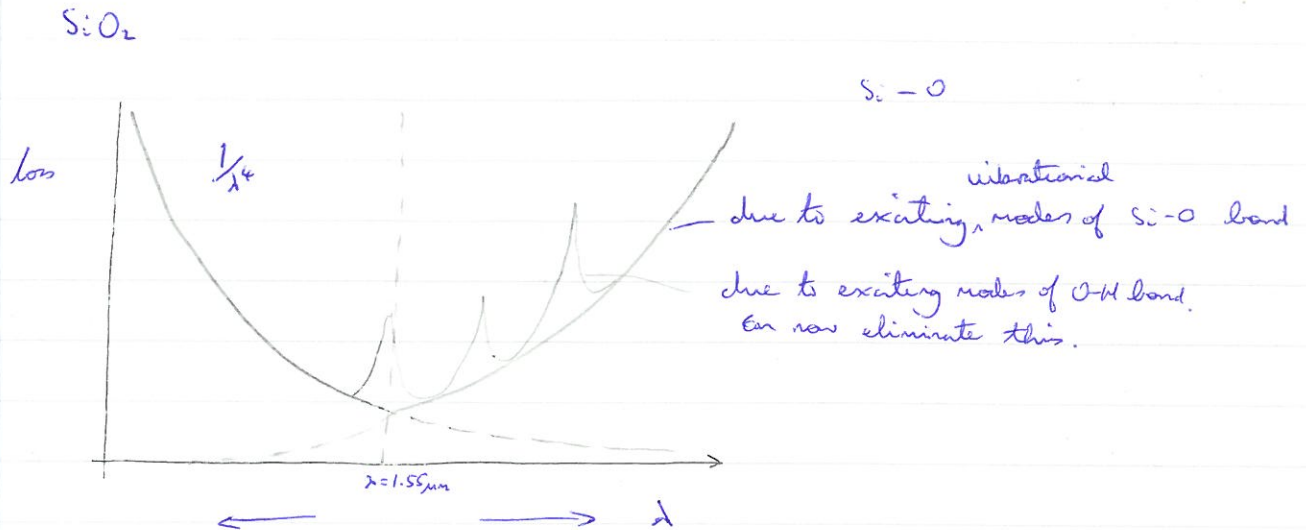


Kinks are due to absorption in material. Heavy losses occur around absorption lines.



$$\underline{\text{SiO}_2} \rightarrow \frac{d^2 n}{d\lambda^2} = 0 \quad \text{at} \quad \underline{\lambda = 1.3 \mu\text{m}}$$

Because of dispersion, want as coherent a source as possible.



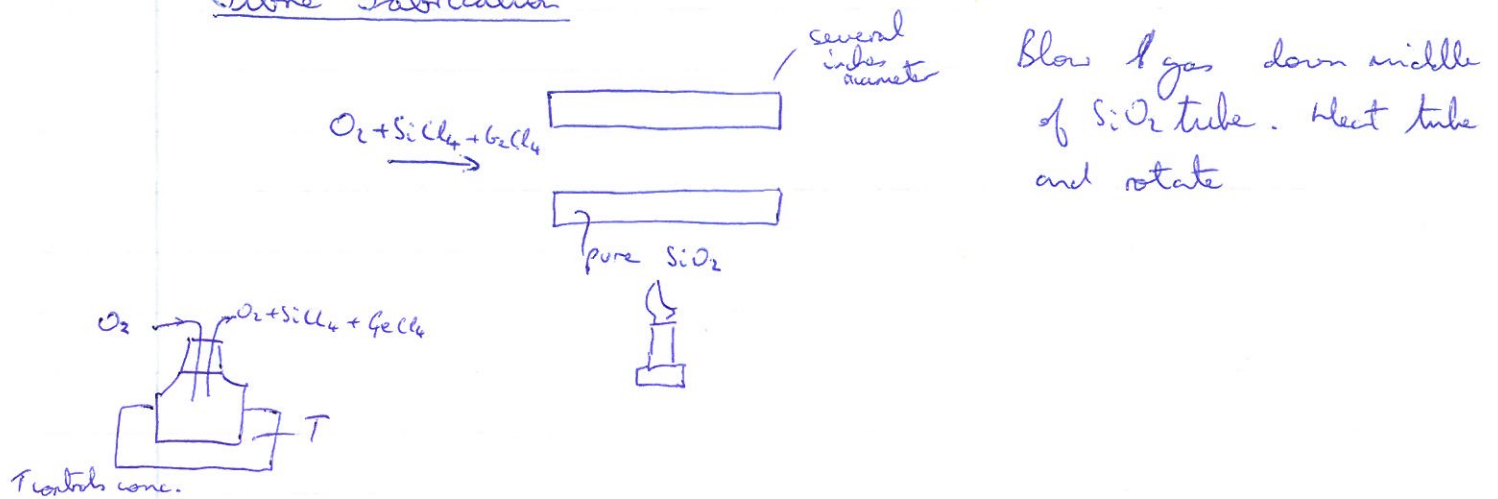
low λ (high f) dominant loss mechanism is Rayleigh scattering.

Minimum loss occurs at $\lambda = 1.55 \mu\text{m}$ in silica. (SiO₂)

May get down to $\approx 0.5 \text{ dB/km}$ at $\lambda = 1.55 \mu\text{m}$. loss is very wavelength dependent.

Obviously if shift vibrational mode loss further up in wavelength, can get less loss (as $1/\lambda^4$ must obviously fall off). This work is being considered at 3-5 μm

Fibre Fabrication



GeO_2 , SiO_2 dep. deposits on the inner wall of the tube in a monolike deposit. When sufficient deposition has occurred, the heat is increased till the tube collapses. Now get a 1"-2" diameter rod. This is transferred to a pulling machine and stretched. One rod of this type may give several kilometers of fibre.

Signal Processing - wavefront is a function of $x, y (t)$.

For coherent radiation,

Far ~~field~~ field amplitude distribution - F.T. of near field amplitude distribution

For incoherent systems,

Image plane intensity distribution is proportional to input plane intensity distribution. (object)

Coherent System



Interesting but unsatisfactory. If $z = 1m$, get fourier transform 6m away. To reduce the distance, lenses are used.