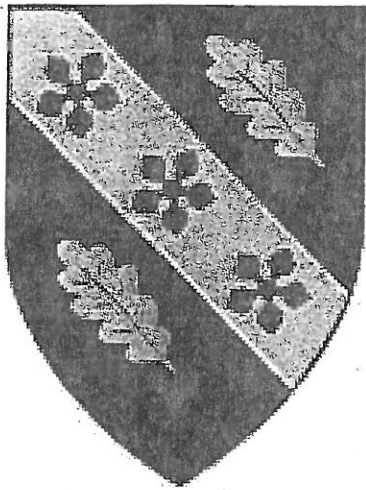




PRIFYSGOL CYMRU

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**FACULTY OF ARCHITECTURE
COMPUTING AND
ENGINEERING**

**MATHEMATICS
FORMULA BOOKLET**

LAWS OF INDICES

$$1. a^m \times a^n = a^{m+n}$$

$$2. \frac{a^m}{a^n} = a^{m-n}$$

$$3. (a^m)^n = a^{mn}$$

In addition:

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^0 = 1$$

LOGARITHMS

Definition:

If $y = a^x$, then $x = \log_a y$.

Laws:

$$1. \log(AB) = \log(A) + \log(B)$$

$$2. \log\left(\frac{A}{B}\right) = \log(A) - \log(B)$$

$$3. \log(A^B) = B\log(A)$$

Change of base:

$$\log_a y = \frac{\log_b y}{\log_b a}$$

QUADRATIC EQUATIONS

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PARTIAL FRACTIONS

Provided that the numerator $f(x)$ is of less degree than the relevant denominator, the following illustrate the form of partial fractions used:

Linear Factors:
$$\frac{f(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

Repeated Linear:
$$\frac{f(x)}{(x+a)^2} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2}$$

Quadratic:
$$\frac{f(x)}{(ax^2+bx+c)} = \frac{(Ax+B)}{(ax^2+bx+c)}$$

TRIGONOMETRY

IDENTITIES

$$\frac{\sin(x)}{\cos(x)} = \tan(x)$$

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)}$$

$$\cot(x) = \frac{1}{\tan(x)}$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sec^2(x) = 1 + \tan^2(x)$$

$$\operatorname{cosec}^2(x) = 1 + \cot^2(x)$$

$$\sin(A \pm B) = \sin(A) \cos(B) \pm \cos(A) \sin(B)$$

$$\cos(A \pm B) = \cos(A) \cos(B) \mp \sin(A) \sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A) \tan(B)}$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos(2x) = 2 \cos^2(x) - 1$$

$$\cos(2x) = 1 - 2 \sin^2(x)$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\sin(A) \cos(B) = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos(A) \sin(B) = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\cos(A) \cos(B) = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$\sin(A) \sin(B) = -\frac{1}{2} [\cos(A + B) - \cos(A - B)]$$

$$\sin(x) + \sin(y) = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) - \sin(y) = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\cos(x) + \cos(y) = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos(x) - \cos(y) = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

TRIANGLE FORMULAE

$$S \frac{O}{H}, C \frac{A}{H}, T \frac{O}{A}$$

Sine Rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Cosine Rule: $a^2 = b^2 + c^2 - 2bc \cos(A)$

Area of any triangle:

(i) $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$

(ii) $\frac{1}{2} ab \sin(C)$

(iii) $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

MATRICES AND DETERMINANTS

Matrices:

$$\text{if } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$\text{then } A + B = \begin{pmatrix} a + e & b + f \\ c + g & d + h \end{pmatrix}$$

$$A - B = \begin{pmatrix} a - e & b - f \\ c - g & d - h \end{pmatrix}$$

$$A \times B = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

$$\text{Inverse of } A = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Determinants:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

COMPLEX NUMBERS

$$z = a + jb = r(\cos\theta + j\sin\theta) = r \angle \theta = re^{j\theta}$$

$$\text{Where } j^2 = -1$$

Modulus

$$r = |z| = \sqrt{(a^2 + b^2)}$$

Argument

$$\theta = \arg z = \tan^{-1} \frac{b}{a}$$

Multiplication

$$z_1 z_2 = r_1 r_2 \angle (\theta_1 + \theta_2)$$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle (\theta_1 - \theta_2)$$

De Moivre's theorem

$$[r \angle \theta]^n = r^n \angle n\theta = r^n (\cos n\theta + j \sin n\theta)$$

$\cos^n \theta$ and $\sin^n \theta$ in terms of sines and cosines of multiples of θ :

$$\cos^n \theta = \frac{1}{2^n} \left(z + \frac{1}{z} \right)^n$$

$$\sin^n \theta = \frac{1}{j^n 2^n} \left(z - \frac{1}{z} \right)^n$$

$$\left(z^n + \frac{1}{z^n} \right) = 2 \cos n\theta$$

$$\left(z^n - \frac{1}{z^n} \right) = 2j \sin n\theta$$

DIFFERENTIAL CALCULUS

Product rule:

When $y = uv$, and u and v are functions of x :

$$\frac{dy}{dx} = u \left(\frac{dv}{dx} \right) + v \left(\frac{du}{dx} \right)$$

Quotient rule:

When $y = \frac{u}{v}$, and u and v are functions of x :

$$\frac{dy}{dx} = \frac{v \left(\frac{du}{dx} \right) - u \left(\frac{dv}{dx} \right)}{v^2}$$

Function of a function (i.e. Chain rule):

If u is a function of x :

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Partial Differentiation:

If $z = f(x, y, \dots)$ and $\partial x, \partial y, \dots$ denote small changes in $x, y, \dots$ respectively, then the corresponding change ∂z in z is given by:

$$\partial z \approx \frac{\partial z}{\partial x} \times \partial x + \frac{\partial z}{\partial y} \times \partial y + \dots$$

Standard Derivatives:

	y or $f(x)$	$\frac{dy}{dx}$ or $f'(x)$
1.	x^n	nx^{n-1}
2.	k (constant)	0
3.	$\sin(ax)$	$a \cos(ax)$
4.	$\cos(ax)$	$-a \sin(ax)$
5.	$\tan(ax)$	$a \sec^2(ax)$
6.	$\ln(x)$	$\frac{1}{x}$
7.	e^{ax}	ae^{ax}
8.	$\sec(ax)$	$a \sec(ax) \tan(ax)$
9.	$\operatorname{cosec}(ax)$	$-a \operatorname{cosec}(ax) \cot(ax)$
10.	$\cot(ax)$	$-a \operatorname{cosec}^2(ax)$
11.	$\sinh(ax)$	$a \cosh(ax)$
12.	$\cosh(ax)$	$a \sinh(ax)$
13.	$\tanh(ax)$	$a \operatorname{sech}^2(ax)$
14.	a^x	$a^x \ln(a)$
15.	$\sin^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}$
16.	$\cos^{-1}\left(\frac{x}{a}\right)$	$\frac{1}{\sqrt{a^2-x^2}}$
17.	$\tan^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{a^2+x^2}$
18.	$\sec^{-1}\left(\frac{x}{a}\right)$	$\frac{a}{x\sqrt{x^2-a^2}}$
19.	$\operatorname{cosec}^{-1}\left(\frac{x}{a}\right)$	$-\frac{a}{x\sqrt{x^2-a^2}}$
20.	$\cot^{-1}\left(\frac{x}{a}\right)$	$-\frac{a}{a^2+x^2}$

INTEGRAL CALCULUS

Integration by parts:

If u and v are both functions of x then:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Standard Integrals:

	y	$\int y dx$
1.	x^n	$\frac{x^{n+1}}{n+1} + c$ (except $n = -1$)
2.	$\cos(ax)$	$\frac{1}{a} \sin(ax) + c$
3.	$\sin(ax)$	$-\frac{1}{a} \cos(ax) + c$
4.	$\sec^2(ax)$	$\frac{1}{a} \tan(ax) + c$
5.	$\operatorname{cosec}^2(ax)$	$\frac{1}{a} \cot(ax) + c$
6.	$\operatorname{cosec}(ax) \cot(ax)$	$-\frac{1}{a} \operatorname{cosec}(ax) + c$
7.	$\sec(ax) \tan(ax)$	$\frac{1}{a} \sec(ax) + c$
8.	e^{ax}	$\frac{1}{a} e^{ax} + c$
9.	$\frac{1}{x}$	$\ln(x) + c$
10.	$\cos^2(x)$	$\frac{x}{2} + \frac{\sin(2x)}{4} + c$
11.	$\sin^2(x)$	$\frac{x}{2} - \frac{\sin(2x)}{4} + c$
12.	$\tan^2(x)$	$\tan(x) - x + c$
13.	$\cot^2(x)$	$-\cot(x) - x + c$
14.	$\frac{1}{x^2 - a^2}$	$\frac{1}{2a} \ln \left(\frac{x-a}{x+a} \right) + c$
15.	$\frac{1}{x^2 + a^2}$	$\frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$
16.	$\frac{1}{a^2 - x^2}$	$\frac{1}{2a} \ln \left(\frac{a+x}{a-x} \right) + c$

- | | | |
|-----|----------------------------|---|
| 17. | $\frac{1}{\sqrt{a^2-x^2}}$ | $\sin^{-1}\left(\frac{x}{a}\right) + c$ |
| 18. | $e^{ax} \cos(bx)$ | $\frac{e^{ax}(a \cos(bx) + b \sin(bx))}{a^2 + b^2}$ |
| 19. | $e^{ax} \sin(bx)$ | $\frac{e^{ax}(a \sin(bx) - b \cos(bx))}{a^2 + b^2}$ |

APPLICATIONS OF INTEGRATION

In the following, the function $y = f(x)$, between the ordinates $x = a$ and $x = b$ where $b > a$, lies entirely above the x axis.

1. Area under a curve: $\int_a^b y dx$
2. Mean value: $\frac{1}{b-a} \int_a^b y dx$
3. Volume: $\pi \int_a^b y^2 dx$
4. R.M.S value: $\sqrt{\left(\frac{1}{b-a}\right) \int_a^b y^2 dx}$
5. Centroids: $\bar{x} = \frac{\int_a^b xy dx}{\int_a^b y dx}$ $\bar{y} = \frac{\frac{1}{2} \int_a^b y^2 dx}{\int_a^b y dx}$

NUMERICAL INTEGRATION

The Trapezoidal rule:

$$\text{area} = (\text{interval width})[0.5(\text{first and last ordinates}) + \text{sum of remaining ordinates}]$$

Simpson's rule:

$$\text{area} = \frac{1}{3} (\text{interval width})[(\text{first and last ordinate}) + 4(\text{sum of evens}) + 2(\text{sum of odds})]$$

Mid-ordinate rule:

$$\text{area} = (\text{width interval})(\text{sum of mid - ordinates})$$

MENSURATION

Areas of plane figures

Rectangle: $Area = l \times b$ where l = length and b = breadth of rectangle.

Parallelogram: $Area = b \times h$ where b = length of one of the parallel sides and h = perpendicular distance between these two parallel sides.

Trapezium: $Area = \frac{1}{2}(a + b)h$ where $a + b$ = the sum of the parallel sides and h = perpendicular distance between these two parallel sides.

Triangle: $Area = \frac{1}{2}bh$ where b = length of the base of a triangle and h = perpendicular height.

Circle: $Area = \pi r^2$ where r = radius of the circle.
 $circumference = 2\pi r$ where r = radius of the circle.

Sector of a circle: $Area = \frac{1}{2}r^2\theta$ where r = radius of circle and θ = the angle subtended at the centre of the circle.

$length\ of\ arc, l = r\theta$ where r = radius of circle and θ = the angle subtended at the centre of this arc of length l .

Volumes of surface areas of regular solids

Rectangular prism (or cuboid):

Volume = $l \times b \times h$ where l = length, b = breadth and h = height.

$$\text{surface area} = 2(bh + hl + lb)$$

Cylinder:

Volume = $\pi r^2 h$ where h = length of cylinder and r = radius of its base.

Pyramid:

Volume = $\frac{1}{3} Ah$ where A = area of base and h = perpendicular height.

Total surface area = sum of areas of triangles forming sides plus area of base.

Cone:

Volume = $\frac{1}{3} \pi r^2 h$ where r = radius of base and h = vertical height.

Curved surface area = $\pi r l$ where l = slant height of cone.

$$\text{Total surface area} = \pi r l + \pi r^2$$

Sphere:

$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$\text{Surface area} = 4\pi r^2$$

STATISTICS

Mean:

$$\bar{x} = \frac{\sum xf}{f}$$

Standard deviation:

$$S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

Binomial distribution:

$$(q + p)^n = q^n + q^{n-1}np + \frac{q^{n-2}n(n-1)p^2}{2!} + \frac{q^{n-3}n(n-1)(n-2)p^3}{3!} + \dots$$

