

Odysseas Andreatsos

St. N: 2005263

Time: 12.20

Date: 10/1/2022

Module Code: ACHA4000

Page 1 of 10

Q1

$$i) \frac{3x-1}{5} = \frac{6x+5}{9}$$

$$9(3x-1) = 5(6x+5)$$

$$27x-9 = 30x+25$$

$$-9-25 = 30x-27x$$

$$-34 = 3x$$

$$\frac{-34}{3} = x$$

$$x = \frac{-34}{3}$$

$$ii) 1 + \frac{5}{x} = \frac{-6}{x^2}$$

$$x^2 + 5x = -6$$

$$x \quad 2$$

$$x^2 + 5x + 6 = 0$$

$$x \quad 3$$

$$(x+2)(x+3) = 0$$

$$x+2=0 \Rightarrow x_1 = -2$$

$$x+3=0 \Rightarrow x_2 = -3$$

$$iii) 2 \log(3x+1) = \log(x+3) + \log(x+7)$$

$$\log(3x+1)^2 = \log(x+3)(x+7)$$

$$(3x+1)^2 = (x+3)(x+7)$$

$$9x^2 + 6x + 1 = x^2 + 7x + 3x + 21$$

$$9x^2 - x^2 + 6x - 10x + 1 - 21 = 0$$

$$8x^2 - 4x - 20 = 0$$

$$x_{1,2} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(8)(-20)}}{2 \times 8} = \frac{4 \pm \sqrt{656}}{16}$$

$$x_1 = \frac{4 + \sqrt{656}}{16} = 1.851$$

$$x_2 = \frac{4 - \sqrt{656}}{16} = -1.351$$

Odysseas Androutsos

St.N: 2005263

Time: 12:35

Date: 10/1/2022

Module code: ACHA4000

Page 2 of 10

(b)
(i) $8P^3Q^3 - 4P^2Q^3 + 2P^3Q =$

$$= 2P^2Q(4PQ - 2Q^2 + P)$$

(ii) $x^2 - y^2 =$
 $(x+y)(x-y)$

(c)

(i) $x^2 + 12x + 20 = 0$
 $(x+10)(x+2)$

$$x+10=0 \Rightarrow x_1 = -10$$

$$x+2=0 \Rightarrow x_2 = -2$$

(ii) $2x^2 + x - 15 = 0$
 $(2x-5)(x+3) = 0$

$$2x-5=0 \Rightarrow x_1 = \frac{5}{2}$$

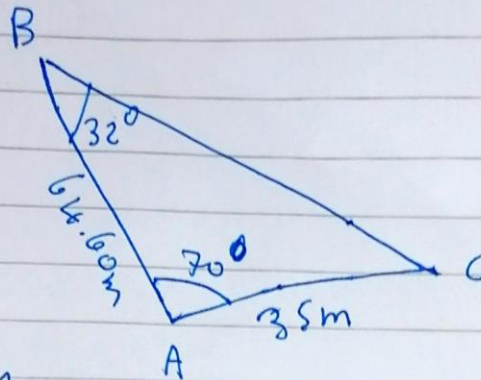
$$x+3=0 \Rightarrow x_2 = -3$$

(iii) $x^2 - 400 = 0$
 $(x-20)(x+20)$

$$x-20=0 \Rightarrow x_1 = 20$$

$$x+20=0 \Rightarrow x_2 = -20$$

(Q2)



(i) $\angle C = 180^\circ - 32^\circ - 70^\circ = 78^\circ$
 $\angle C = 78^\circ$

(ii) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{35}{\sin 32} = \frac{AB}{\sin 78} \Rightarrow AB = \frac{35 \times \sin 78}{\sin 32}$

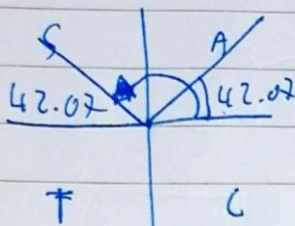
$AB = 64.60 \text{ m}$

(iii) $BC = \frac{AC \cdot \sin A}{\sin B} = \frac{35 \times \sin 70}{\sin 32}$

$BC = 62.06 \text{ m}$

(Q3)

$$\begin{aligned} \text{(i)} \quad \sin x &= 0.67 \\ x &= \sin^{-1} 0.67 \\ x &= 42.07^\circ \end{aligned}$$



$$\begin{aligned} x &= 180^\circ - 42.07 = 137.93^\circ \\ x &= 137.93^\circ \end{aligned}$$

$$\text{(ii)} \quad \tan^2 x = 7 \tan x + 10 = 0$$

$$\tan x = A$$

$$A^2 - 7A + 10 = 0$$

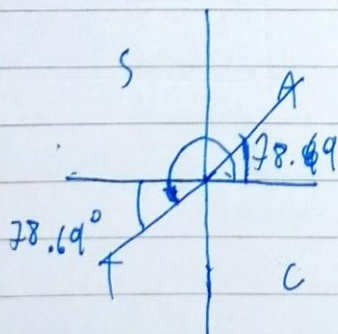
$$(A - 5)(A - 2) = 0$$

$$(A - 5) = 0 \Rightarrow A = 5$$

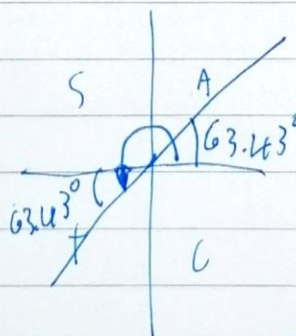
$$(A - 2) = 0 \Rightarrow A = 2$$

$$\begin{aligned} \tan x &= 5 \\ x &= \tan^{-1} 5 \\ x_1 &= 78.69^\circ \end{aligned}$$

$$\begin{aligned} \tan x &= 2 \\ x &= \tan^{-1} 2 \\ x_2 &= 63.43^\circ \end{aligned}$$



$$\begin{aligned} x &= 180 + 78.69 \\ x &= 258.69^\circ \end{aligned}$$



$$\begin{aligned} x &= 180^\circ + 63.43^\circ \\ x_2 &= 243.43^\circ \end{aligned}$$

Olysses Andrausos

st.N: 2005263

Time: 13:10

Date: 10/1/2022

Module Code: ACHA4000

Page 4 of 10

Q4

$$(i) \begin{aligned} y &= 5x^2 + 12x - 16 \\ y &= 4x^2 + 3x - 4 \end{aligned}$$

$$5x^2 + 12x - 16 = 4x^2 + 3x - 4$$

$$5x^2 - 4x^2 + 12x - 3x - 16 + 4 = 0$$

$$x^2 + 9x - 12 = 0$$

$$(x+10)(x-1) =$$

$$x+10=0$$

$$x = -10$$

$$x-1=0$$

$$x = 1$$

$$\begin{aligned} y_1 &= 5x^2 + 12x - 16 \\ &= 5(-10)^2 + 12(-10) - 16 \\ &= 500 - 120 - 16 \end{aligned}$$

$$\boxed{y_1 = 364}$$

$$\begin{aligned} y_2 &= 5x^2 + 12x - 16 = \\ &= 5(1)^2 + 12(1) - 16 = \end{aligned}$$

$$\boxed{y_2 = 1}$$

Solutions when $x = -10$
when $x = 1$

$$\begin{aligned} y &= 364 \\ y &= 1 \end{aligned}$$

Olyseas Andradas

St. N: 2005263

Time: 13:10

Date: 10/1/2022

Module Code: ACH4000

Page 5 of 10

Q5

$$Z_1 = 3 + 7j$$

$$Z_2 = 2 - 10j$$

Odysseas Androutsos

St. N: 2005263

Time: 13:20

Date: 10/1/2022

Module code: ACH1400

Page 6 of 10

$$\begin{aligned} \text{(i)} \quad Z_1 + Z_2 &= (3 + 7j) + (2 - 10j) \\ &= 3 + 7j + 2 - 10j \\ &= 5 - 3j \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad Z_1 - Z_2 &= (3 + 7j) - (2 - 10j) \\ &= 3 + 7j - 2 + 10j \\ &= 1 + 17j \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \frac{Z_1}{Z_2} &= \frac{3 + 7j}{2 - 10j} \\ &= \frac{3 + 7j}{2 - 10j} \times \frac{2 + 10j}{2 + 10j} \\ &= \frac{(3 + 7j)(2 + 10j)}{2^2 - (10j)^2} \\ &= \frac{6 + 30j + 14j + 70j^2}{4 - 100j^2} = \frac{6 + 34j + 70(-1)}{4 - 100(-1)} \\ &= \frac{-64 + 34j}{4 + 100} = -\frac{64}{100} + \frac{34}{100}j \\ &= -0.64 + 0.34j \end{aligned}$$

(b)

$$7x^2 = -2x - 11$$

$$7x^2 + 2x + 11 = 0$$

$$x_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 7 \cdot 11}}{2 \cdot 7}$$

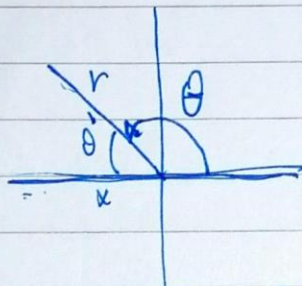
$$= \frac{-2 \pm \sqrt{-304}}{14}$$

$$x_1 = \frac{-2 + \sqrt{-304}}{14} = \frac{1}{7} + 1.245j$$

$$= 0.143 + 1.245j$$

$$x_2 = \frac{-2 - \sqrt{-304}}{14} = -\frac{1}{7} - 1.245j$$

$$= -0.143 - 1.245j$$



$$r = \sqrt{0.143^2 + 1.245^2} = 1.57$$

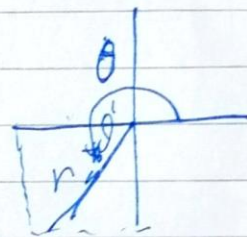
$$\tan \theta' = \frac{1.245}{0.143} = 8.706$$

$$\theta' = \tan^{-1} 8.706 = 83.45^\circ$$

$$\theta = 180 - 83.45 = 96.55^\circ$$

Solution (1) in Polar Form
 $1.57 (\cos 96.55^\circ + j \sin 96.55^\circ)$

Solution (2)
 $r = 1.57$ (as before)



$$\tan \theta' = \frac{1.245}{0.143} = 8.706$$

$$\theta' = \tan^{-1} 8.706 = 83.45^\circ$$

$$\theta = 180 + 83.45 = 263.45^\circ$$

Solution 2 in Polar form
 $1.57 (\cos 263.45^\circ + j \sin 263.45^\circ)$

Odysseas Andrioutsos

St. N 2005263

Time: 13:00

Date: 10/1/2022

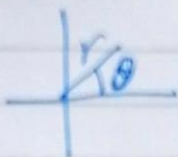
Module Code: ACHA4000

Page 8 of 10

(c) $(5+3j)^4$

$$r = \sqrt{5^2 + 3^2} \Rightarrow r = 5.83$$

$$\tan \theta = \frac{3}{5}$$



$$\theta = \tan^{-1}\left(\frac{3}{5}\right)$$

$$\theta = 30.96^\circ$$

we apply the de Moivre's theorem

$$r^n (\cos n\theta + j \sin n\theta)$$

$$5.83^4 (\cos (4 \times 30.96^\circ) + j \sin (4 \times 30.96^\circ))$$

$$1155.25 (\cos 123.84^\circ + j \sin 123.84^\circ)$$

(Q6)

(i) $y = 9x^3 + 7x^2 - x + 4$
 $\frac{dy}{dx} = 27x^2 + 14x - 1$

(ii) $y = (2x+3)(7x-5)$
 $= 6x^2 - 10x + 9x - 15$
 $= 6x^2 - x - 15$
 $\frac{dy}{dx} = 12x - 1$

(iii) $y = 2 \ln x + \frac{5}{x}$
 $= 2 \ln x + 5x^{-1}$

$$\frac{dy}{dx} = \frac{2}{x} - 5x^{-2} = \frac{2}{x} - \frac{5}{x^2}$$

(iv) $y = 7 \sin x + 3 \cos 2x + \tan \frac{x}{2}$
 $\frac{dy}{dx} = 7 \cos x - 6 \sin x + \frac{1}{2} \sec^2 \frac{x}{2}$

$$(V) y = x^5 \sin 2x$$

$$\frac{dy}{dx} = x^2 \times 2 \cos 2x + 2x \sin 2x$$

$$\frac{dy}{dx} = 2x^2 \cos 2x + 2x \sin 2x$$

$$(vi) y = \frac{e^{3x}}{x^5} = x^{-5} e^{3x}$$

$$\frac{dy}{dx} = -5x^{-6} e^{3x} + x^{-5} 3e^{3x}$$

$$\frac{dy}{dx} = \frac{-3e^3}{x^4} + \frac{3e^{3x}}{x^5}$$

$$(vii) y = (5x^2 - 2x + 1)^5$$

$$\frac{dy}{dx} = 5(5x^2 - 2x + 1)^4 (10x - 2)$$

(Q7)

$$(i) \int x^5 + 2x^2 + x - 10 \, dx$$

$$= \frac{x^6}{6} + \frac{2x^3}{3} + \frac{x^2}{2} - 10x + C$$

$$(ii) \int (2x^2 + 5)(x^2 - 1) \, dx$$

$$= \frac{2x^4}{4} + \frac{5x^3}{3} - \frac{2x^2}{2} - 5x + C$$

$$= \frac{x^4}{2} + \frac{5x^3}{3} - x^2 - 5x + C$$

(iii)

$$\int_2^5 2x^2(x^2 + 3) \, dx$$

$$= \left[\frac{2x^5}{5} + \frac{6x^3}{3} \right]_2^5$$

$$= \frac{2 \times 5^5}{5} + \frac{6 \times 5^3}{3} - \frac{2 \times 2^5}{5} - \frac{6 \times 2^3}{3}$$

$$= 1250 + 250 - 12.8 - 16$$

$$= 1496.8$$

Odysseas Andraoutsos

STN: 2005263

Time: 13:47

Date: 10/1/2022

Module code: ACHAK000

Page 9 of 10

(11)

$$\int 5 \sin 3x - \cos \frac{x}{3} dx$$

$$= \frac{-5}{3} \cos 3x - \frac{\sin \frac{x}{3}}{\frac{1}{3}} + C$$

$$= -\frac{5}{3} \cos 3x - 3 \sin \frac{x}{3} + C$$

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St N: 2005263

Time: 13:52

Date: 10/1/2022

Module Code: ACAA40000

Page 10 of 10