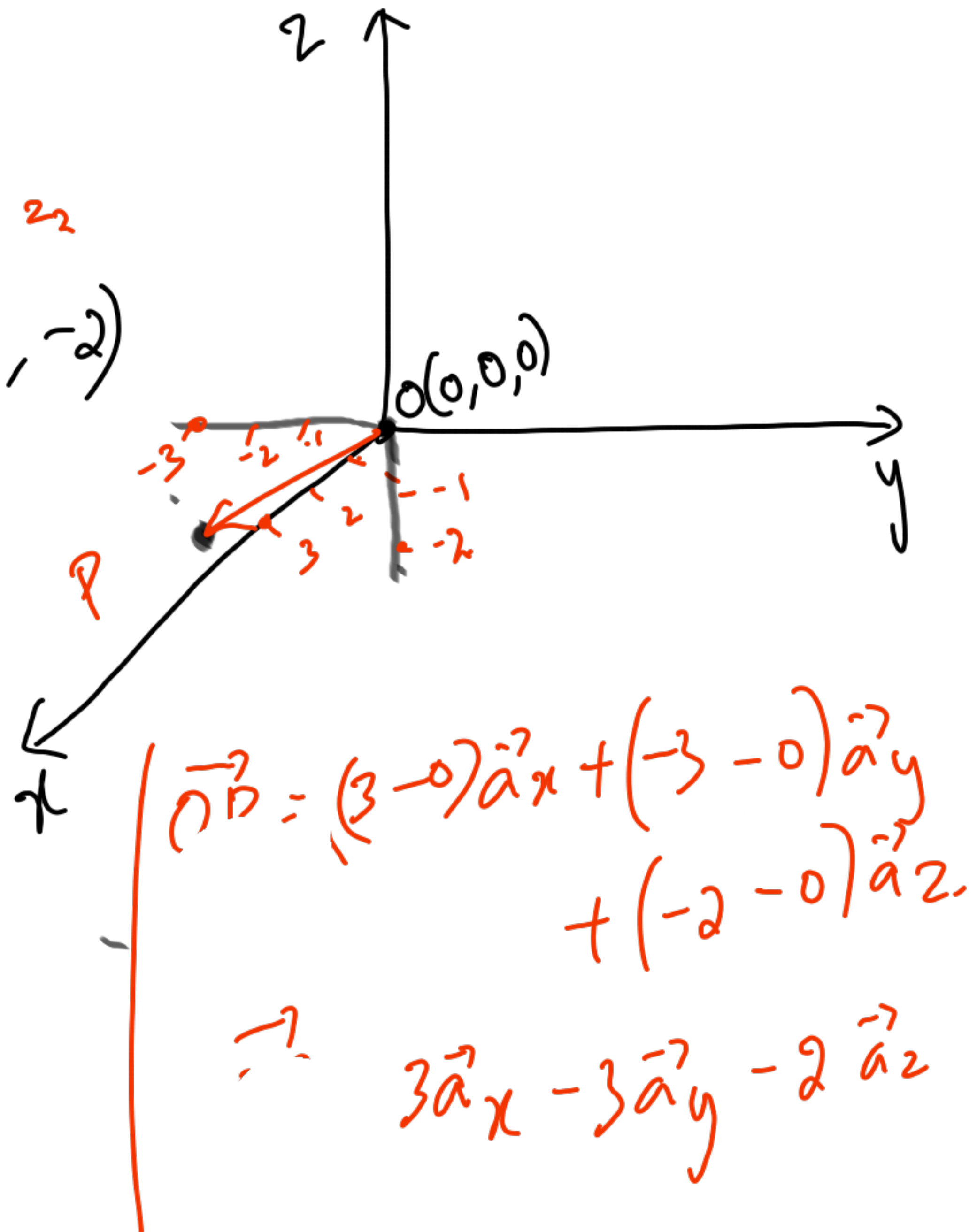


i) Obtain the Unit vector in the direction from the origin towards the point $(3, -3, -2)$

Soln: at origin $O(0, 0, 0)$, $P(3, -3, -2)$

$$\vec{P} = 3\vec{a}_x - 3\vec{a}_y - 2\vec{a}_z$$

$$\vec{O} = 0\vec{a}_x + 0\vec{a}_y + 0\vec{a}_z$$



$$|\vec{OP}| = \sqrt{3^2 + (-3)^2 + (-2)^2}$$

$$|\vec{OP}| = 4.69$$

$$\vec{a}_{op} = \frac{\vec{OP}}{|\vec{OP}|}$$

$$= \frac{3\vec{a}_x - 3\vec{a}_y - 2\vec{a}_z}{4.69}$$

$$\vec{a}_{op} = 0.639\vec{a}_x - 0.639\vec{a}_y - 0.426\vec{a}_z$$

$$= \frac{3}{4.69}\vec{a}_x - \frac{3}{4.69}\vec{a}_y - \frac{2}{4.69}\vec{a}_z$$

$$|\vec{a}_{op}| = \sqrt{(0.639)^2 + (-0.639)^2 + (-0.426)^2}$$

2) Two points $A(2, 2, 1)$ and $B(3, -4, 1)$ are given in the Cartesian System. Obtain the Vector from A to B and the Unit

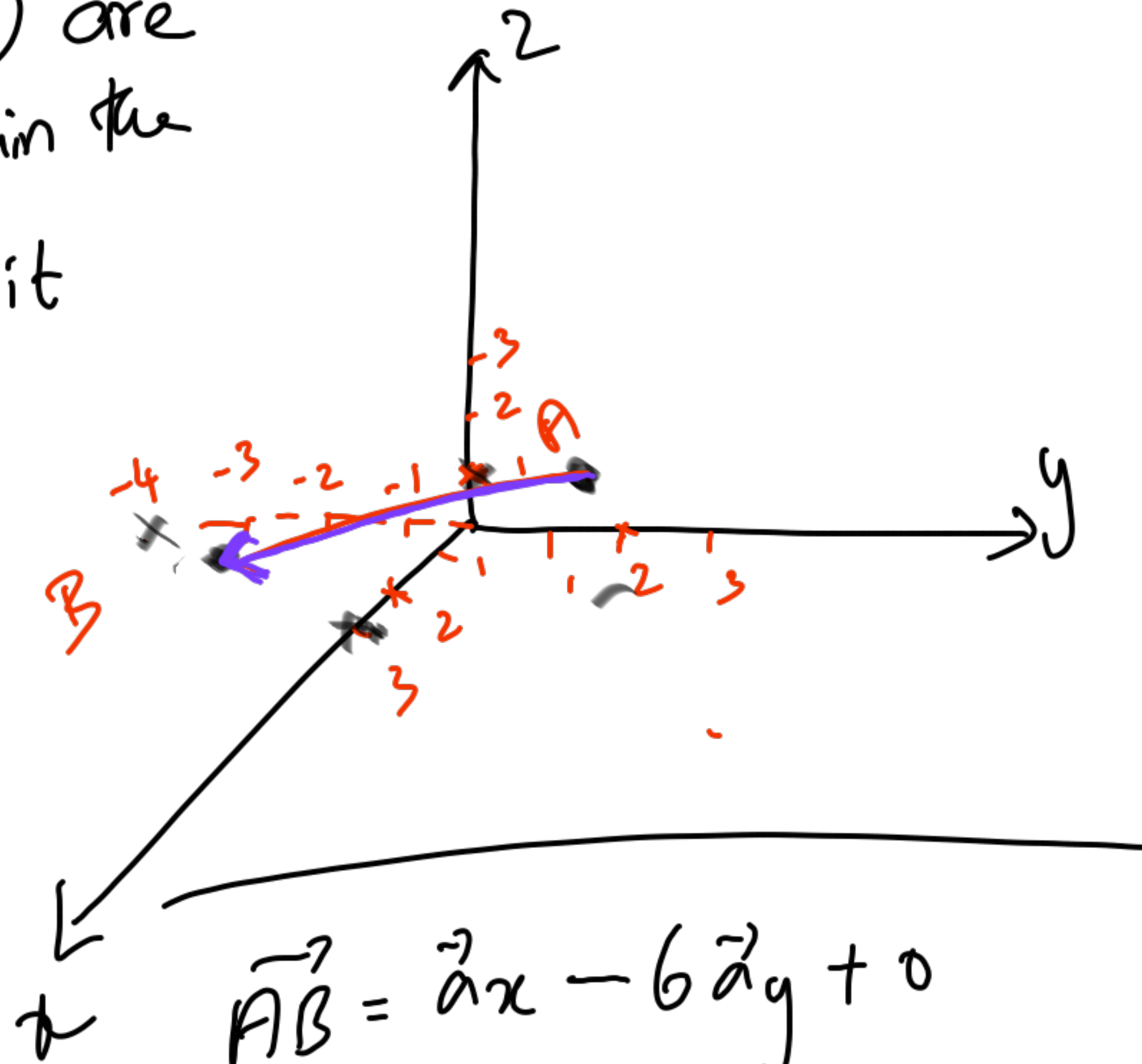
Vector from A to B

Soln.

$$\vec{A} = 2\vec{a}_x + 2\vec{a}_y + \vec{a}_z$$

$$\vec{B} = 3\vec{a}_x - 4\vec{a}_y + \vec{a}_z$$

$$\vec{AB} = \vec{B} - \vec{A} = (3-2)\vec{a}_x + (-4-2)\vec{a}_y + (1-1)\vec{a}_z$$



$$\vec{AB} = \vec{a}_x - 6\vec{a}_y + 0$$

$$\vec{AB} = \vec{a}_x - 6\vec{a}_y$$

$$|\vec{AB}| = \sqrt{(1)^2 + (-6)^2}$$

$$|\vec{AB}| = \sqrt{1 + 36} = \sqrt{37}$$

$$|\vec{AB}| = 6.08$$

$$\vec{a}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{\vec{a}_x - 6\vec{a}_y}{6.08}$$

$$= \frac{1}{6.08} \vec{a}_x - \frac{6}{6.08} \vec{a}_y$$

$$\vec{a}_{AB} = 0.16\vec{a}_x - 0.99\vec{a}_y$$

3) Show that $\vec{A} = 4\vec{a}_x - 2\vec{a}_y - \vec{a}_z$ and $\vec{B} = \vec{a}_x + 4\vec{a}_y - 4\vec{a}_z$ are perpendicular to each other.

Soln:

If $\boxed{\vec{A} \cdot \vec{B} = 0}$ $\vec{A} \perp \vec{B}$ perpendicular.

$$\vec{A} \cdot \vec{B} = (4 \times 1) + ((-2) \times 4) + ((-1) \times (-4))$$

$$\vec{A} \cdot \vec{B} = 0$$

$\vec{A} \perp \vec{B}$ perpendicular.

f) Given two vectors $\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$ and $\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$. Find the vector that is perpendicular to both the vector and verify the resulting vector is perpendicular to $\vec{A}$ and $\vec{B}$.

Soln. $\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$
 $\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$

$$\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$$

$$\vec{B} = B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z$$

$$\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 3 & 2 & -1 \\ -1 & 4 & 2 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \overset{+}{\vec{a}_x} & \overset{-}{\vec{a}_y} & \overset{+}{\vec{a}_z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} - \begin{vmatrix} \overset{+}{\vec{a}_x} & \overset{-}{\vec{a}_y} & \overset{+}{\vec{a}_z} \\ 3 & 2 & -1 \\ -1 & 4 & 2 \end{vmatrix}$$

$$= \vec{a}_x \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \vec{a}_y \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \vec{a}_z \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \vec{a}_x [A_y B_z - A_z B_y] - \vec{a}_y [A_x B_z - A_z B_x] + \vec{a}_z [A_x B_y - A_y B_x]$$

$$\begin{aligned}
 \vec{A} \times \vec{B} &= \vec{a}_x \begin{vmatrix} 2 & -1 \\ 4 & 2 \end{vmatrix} - \vec{a}_y \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix} + \vec{a}_z \begin{vmatrix} 3 & 2 \\ -1 & 4 \end{vmatrix} \\
 &= \vec{a}_x [(2 \times 2) - (4 \times (-1))] - \vec{a}_y [(3 \times 2) - ((-1) \times (-1))] \\
 &\quad + \vec{a}_z [(3 \times 4) - ((-1) \times 2)] \\
 &= \vec{a}_x [4 - (-4)] - \vec{a}_y [6 - 1] + \vec{a}_z [12 - (-2)]
 \end{aligned}$$

$$\vec{A} \times \vec{B} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

$$\vec{A} \times \vec{B} = \vec{R} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

To prove if perpendicular.

$$\vec{A} \cdot \vec{R} = 0$$

$$\vec{B} \cdot \vec{R} = 0$$

To prove $\vec{A} \cdot \vec{R} = 0$.

$$\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$$

$$\vec{R} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

$$\vec{A} \cdot \vec{R} = (3 \times 8) + (2 \times (-5)) + ((-1) \times 14)$$

$$\vec{A} \cdot \vec{R} = 24 - 10 - 14 = 0$$

$$\vec{A} \cdot \vec{R} = 0$$

To prove $\vec{B} \cdot \vec{R} = 0$

$$\vec{B} = -\vec{a}_x + 4\vec{a}_y + 2\vec{a}_z$$

$$\vec{R} = 8\vec{a}_x - 5\vec{a}_y + 14\vec{a}_z$$

$$\vec{B} \cdot \vec{R} = ((-1) \times 8) + (4 \times (-5)) + (2 \times 14)$$

$$= -8 - 20 + 28$$

$$\vec{B} \cdot \vec{R} = \underline{0}$$