

Stress analysis exam

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Stress analysis exam 2004975

A b c d e p g (student number)
2 0 0 4 9 7 5

$$g = 9.81 \text{ m/s}^2$$

$$1 \text{ bar} = 10^5 \text{ N/m}^2 (\text{Pa})$$

Question 1

$$\sigma_x = 25 \times 10^{-6} \text{ N/mm}^2$$

$$\sigma_y = -17 \times 10^{-6} \text{ N/mm}^2$$

$$\nu = 0.28$$

$$E = 170 \text{ GPa} = 170 \times 10^9$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{170 \times 10^9}{1-(0.28)^2} (25 \times 10^{-6} + 0.28 \times -17 \times 10^{-6})$$

$$\sigma_x = 3733506.944$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{170 \times 10^9}{1-(0.28)^2} (-17 \times 10^{-6} + 0.28 \times 25 \times 10^{-6})$$

$$\sigma_y = -1823251.823$$

ii)

$$\sigma_x = 87 \times 10^6$$

$$\sigma_y = 45 \times 10^6$$

$$\nu = 0.28$$

$$E = 170 \times 10^9$$

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$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{170 \times 10^9} (45 \times 10^6 - (0.28 \times 87 \times 10^6))$$

$$\epsilon_y = +2.4360000 - 4.08 \times 10^{-4}$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{170 \times 10^9} (87 \times 10^6 - (0.28 \times 45 \times 10^6))$$

$$\epsilon_x = +2.6000000 - 0.00058$$

Question 2

$$i) \sigma_{1,2} = \frac{(\sigma_x + \sigma_y)}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{(109 \times 10^6 + 207 \times 10^6)}{2} \pm \sqrt{\left(\frac{109 \times 10^6 - 207 \times 10^6}{2}\right)^2 + (4 \times 125 \times 10^6)^2}$$

$$\sigma_1 = 169.8736422 \text{ MPa}$$

$$\sigma_2 = -31.8736422 \text{ MPa}$$

$$\sigma_1 = 347.8736422 \text{ MPa}$$

$$\sigma_2 = -31.8736422 \text{ MPa}$$

$$ii) \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = 347.87 \text{ MPa}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = 31.87 \text{ MPa}$$

$$\tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2}$$

$$\tau_{max} = \frac{\sqrt{(100 \times 10^6 - 207 \times 10^6)^2 + 4 \times (125 \times 10^6)^2}}{2}$$

$$\tau_{max} = 176.638 \text{ MPa}$$

$$\tau_{min} = -\frac{\sqrt{(100 \times 10^6 - 207 \times 10^6)^2 + 4 \times (125 \times 10^6)^2}}{2}$$

$$\tau_{min} = -176.638 \text{ MPa}$$

Question 4

i)

$$d = 280 \text{ mm} \quad 0.28$$

$$L = 600 \text{ mm} \quad 0.60$$

$$n_L = 0.57\%$$

$$n_C = 0.65\%$$

$$\sigma_{\max} = 150 \times 10^6$$

$$P = 14.9 \text{ bar} \quad 14.9 \times 10^5$$

ii)

$$\sigma_c = \frac{Pd}{2Ln_L}$$

$$t = \frac{Pd}{2\sigma_c n_L}$$

$$t = \frac{14.9 \times 10^5 \times 0.28}{2 \times 150 \times 10^6 \times 0.57}$$

$$t_{\min} = 2.439 \text{ mm}$$

ii)

$$\sigma_c = \frac{Pd}{4Ln_C}$$

$$t = \frac{Pd}{4\sigma_c n_C}$$

$$t = \frac{14.9 \times 10^5 \times 0.28}{4 \times 0.65 \times 150 \times 10^6}$$

$$t = 1.069 \text{ mm}$$

iii)

$$t_{\min} = 2.439 \text{ mm}$$

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v) Substitute $\sigma_c = 1.97 \text{ MPa}$ into equation iii

$$E_{ig} = \frac{19694(1.97)^3}{6} - \frac{450(1.97-0.5)^4}{24} + \frac{450(1.97-1.5)^4}{24} - \frac{2300(1.97-1.5)^3}{6} - \frac{800(1.97-1.5)^4}{24} + \frac{800(1.97-2.5)^4}{24} + 7263.28 \times 1.9$$

$$y = 16692.69469$$

Question 5

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$$\begin{aligned} L &= 1.37 & n &= 0.25 \\ d_i &= 0.01905 \text{ m} & I &= 3.93 \times 10^{-9} \\ d_o &= 0.02145 \text{ m} & E &= 2 \times 10^{-8} \end{aligned}$$

$$i) \quad I = \frac{\pi d_o^4 - d_i^4}{64} = \frac{\pi \times 0.02145^4 - 0.01905^4}{64}$$

$$I = 3.93 \times 10^{-9} \text{ m}^4$$

$$ii) \quad n = 0.25 \quad (\text{unrestrained})$$

$$iii) \quad F_{cr} = \frac{n \pi^2 E I}{L^2} = \frac{0.25 \times \pi^2 \times 2 \times 10^8 \times 3.93 \times 10^{-9}}{1.37^2}$$

$$= 103.3287677 \text{ N}$$

$$iv) \quad A = \frac{\pi d_o^2}{4} - \frac{\pi d_i^2}{4} = 7.63 \times 10^{-5}$$

$$I = \frac{\pi \times 0.02145^4}{4} - \frac{\pi \times 0.01905^4}{4}$$

$$k = \frac{\sqrt{I}}{A} = \frac{\sqrt{3.93 \times 10^{-9}}}{7.63 \times 10^{-5}} = 2.18 \times 10^{-3} \text{ m}$$

Question 3

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iv

i)

$$\frac{EI d^2 y}{dx^2} = \frac{1969.4(x)}{2} - \frac{450(x-0.5)^2}{2} + \frac{450(x-1.5)^2}{2} - \frac{2300(x-1.5)}{2} - \frac{800(x-1.5)^2}{2} + \frac{800(x-2.5)^2}{2}$$

ii)

$$\frac{EI dy}{dx} = \frac{1969.4(x)^2}{2} - \frac{450(x-0.5)^3}{6} + \frac{450(x-1.5)^3}{6} - \frac{2300(x-1.5)^2}{2} - \frac{800(x-1.5)^3}{6} + \frac{800(x-2.5)^3}{6} + A$$

iii)

$$EI y = \frac{1969.4(x)^3}{6} - \frac{450(x-0.5)^4}{24} + \frac{450(x-1.5)^4}{24} - \frac{2300(x-1.5)^3}{6} - \frac{800(x-1.5)^4}{24} + \frac{800(x-2.5)^4}{24} + Ax + B$$

iv)

@ $x=0$ $y=0$

Sub into equation iii

$$B=0$$

@ $x=3.1m$ $y=0$

Sub into equation iii

$$EI y = \frac{1969.4(3.1)^3}{6} - \frac{450(3.1-0.5)^4}{24} + \frac{450(3.1-1.5)^4}{24} - \frac{2300(3.1-1.5)^3}{6} - \frac{800(3.1-1.5)^4}{24} + \frac{800(3.1-2.5)^4}{24} + 3.1 + 0$$

$$A = 7263.28$$