

1.

MAY 2019 EXAM ANSWERS.

THE 2020 EXAM COVERS ONLY THE PRINCIPLES OF THE CONTENT TAUGHT IN THE LECTURES THIS YEAR.

THE FIRST 3 QUESTIONS OF THE 2019 EXAM SHOW THE TYPE OF QUESTIONS THIS YEAR ALTHOUGH THE DETAILS, NUMBER OF CYLINDERS ETC CAN BE DIFFERENT.

YOU NEED TO UNDERSTAND THE PRINCIPLES FROM THE LECTURE NOTES TO BE ABLE TO TACKLE VARIATIONS IN QUESTION DETAILS.

THE ATTACHED ANSWERS SHOW THE REQUIRED EXPLANATIONS NEEDED.

(I MAY HAVE GIVEN FULLER EXPLANATIONS TO HELP UNDERSTANDING. YOU NEED TO JUDGE THE TIME).



LOCATION SWANSEA

LLEOLIAD ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD

CYFNOD ARHOLIADAU

MAY 2019

MAI 2019

MODULE CODE: M6X00410
Competition Engine Design

TIME: 2 HOURS

ANSWER THREE QUESTIONS OUT OF FOUR

Note that in all your answers, marks will be awarded for detailed explanation of the calculation steps. It is essential to include explanations.

QUESTION ONE

You are required to carry out the balancing of a vertically mounted 500 cc single cylinder Enduro engine at first order of crankshaft rotation.

Assume a racing piston and gudgeon pin with a total mass of 240 gms. Assume a forged steel con-rod of total mass of 300 gms. Assume that the big-end bearing shells have a total mass of 20 gms. Assume that the stroke is 50 mm. Assume that the crankshaft is made of steel and has a mass of 2.5 kg. Assume a con-rod length of 100 mm.

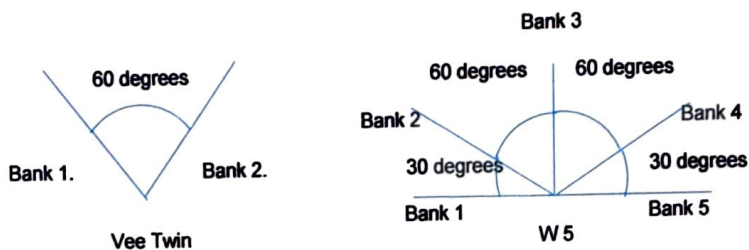
- (a) Using diagrams, and fully explaining the physics, calculate the total mass moment of the resultant first order forward rotating vector acting on the crankshaft as a result of the reciprocating and rotating forces from the piston/connecting rod assembly. Give your answer in kg.mm.
(2 marks)
- (b) Explaining fully the reasons, calculate the required offset in mm of the crankshaft centre of mass from its centre-line of rotation to fully balance the calculated forward rotating vector in part (a). Sketch the crankshaft to illustrate your answer.
(2 marks)
- (c) Calculate the mass moment in kg.mm. of the first order reverse rotating vector.
(2 marks)
- (d) The reverse rotating first order vector is also to be fully balanced for racing. Would you drive the balance shaft from the crankshaft with gears, chain or belt? Sketch a schematic drawing of the drive mechanism of your choice looking at the front of the engine.
(2 marks)
- (e) Assuming the balance shaft has a total mass of 1.5 kg, calculate the required offset in mm of its centre of mass from its centre-line of rotation to fully balance the reverse rotating first order vector.
(2 marks)
- (f) If the crankshaft has rotated clockwise 270° from top-dead-centre, show where the eccentric part of the balance shaft should be pointing. Use a sketch of front view of the engine to illustrate your answer.
(2 marks)
- (g) For applications where a balance shaft will not be used for cost reasons, explain the balancing strategy you will use for the crankshaft to minimise vertical vibrations, using diagrams to support your reasoning. Calculate the required mass moment of the crankshaft in kg.mm. to fulfil your strategy.
(4 marks)
- (h) If the engine were to be installed with the cylinder axis at 45 degrees to the vertical in a sports bike application, explain how you would balance the crankshaft to cause least first order vibration for the rider if it is decided not to use a balance shaft. Explain if it is possible to totally balance the vertical first order in this case and use vector diagrams to explain your best solution.
(4 marks)

4.

QUESTION TWO

You are required to understand the balancing of a 60° Vee Twin Engine and then develop the analysis for a W3 Engine. For the Vee Twin, assume both conrods share a common crankpin. For the W3 Engine, assume all three rods share a common crankpin.

The schematic diagrams below show the layout of the two engines:



For both engines, assume the following specification:

Piston Mass	=	240 gms
Conrod Mass	=	330 gms
Stroke	=	75 mm
Conrod Length	=	130 mm

For all the questions below, use a suitable sketch to show your answer and fully explain the principles.

- For the Vee Twin Engine, put the crankpin along Bank 2 and calculate the magnitude and direction of the resultant first order forward rotating vector. Express the magnitude as a mass moment in kg.mm. (1 mark)
- Again, for the Vee Twin Engine with the crankpin along Bank 2, calculate the magnitude and direction of the resultant first order reverse rotating vector. Express the magnitude as a mass moment in kg.mm. (2 marks)
- For the W5 Engine, put the crankpin along Bank 3 and calculate the magnitude and direction of the resultant first order forward rotating vector. Express the magnitude as a mass moment in kg.mm. (1 mark)
- For the W5 Engine, put the crankpin along Bank 3 and calculate the magnitude and direction of the resultant first order reverse rotating vector. Express the magnitude as a mass moment in kg.mm. (2 marks)
- Explain how you would balance these first order forward and reverse rotating forces in a real engine application. (2 marks)
- If the crankpin rotates clockwise 120° from bank 3 show the position of the resultant first order reverse vector using a suitable schematic diagram. (2 marks)
-

5.

For the W5 Engine, put the crankpin along Bank 3 and calculate the magnitude and direction of the resultant **second order forward** rotating vector. Express the magnitude as a mass moment in kg.mm.

(5 marks)

- (h) For the W5 Engine **with the** crankpin again along Bank 3 calculate the magnitude and direction of the **resultant** second order **reverse** rotating vector. Express the magnitude **as a** mass moment in kg.mm.

(5 marks)

6.

QUESTION THREE

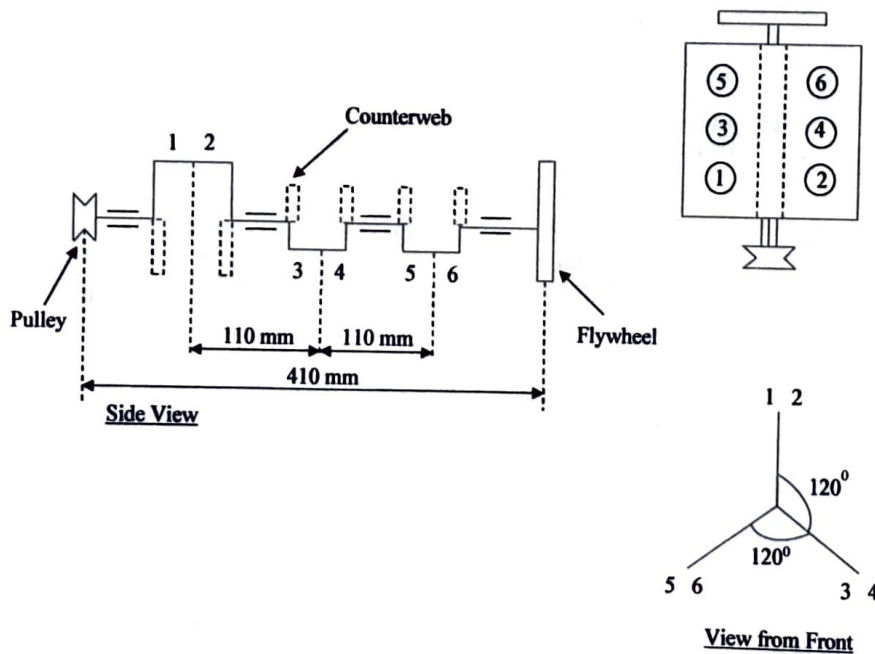
You are required to develop balancing strategies for a current F1 V6 engine with a 90° bank angle. The crankshaft design is one where 2 conrods share one crankpin.

You are required to look at a local balancing strategy and a lighter weight global balancing strategy.

The engine specification is as follows:

Piston Mass	=	240 gms
Conrod Mass	=	240 gms
Stroke	=	65mm
Conrod Length	=	110 mm
Crankpin Phasing	=	120°

The figures below show views of the crankshaft when the crankpin for cylinders 1 and 2 is vertical.



Part 1 - Local Balancing

- Calculate the mass moment in kg.mm. of the effective first order forward rotating vector acting at the centre of each crankspan. (2 marks)
- If each crankspan of the crankshaft has a mass of 3.0 kg, calculate the required centre of mass offset in mm needed to achieve 100% local balance. (2 marks)

7.

- (c) Make a sketch from the front of each crankspan, showing the offset centre of mass obtained in part (b) and explain mathematically why bolt-on tungsten counterweb masses can reduce the overall moment of inertia of the crankshaft whilst maintaining the same balance. Why is this important for racing engines?

(2 marks)

Part 2 - Global Balancing

For a particular racing application, it is now required to minimise all six counterwebs to reduce the crankshaft mass and achieve the required balancing through an out-of-balance front pulley and flywheel.

Commented [MM1]:

- (a) Calculate the magnitude of the resultant first order forward rotating moment of the piston and conrod mass moments along the entire crankshaft. Express your answer in kg.mm.mm.

(4 marks)

- (b) Looking from the front of the crankshaft (ie. view from the front), draw a sketch showing the angle (with calculation proof) of the resultant forward moment of the mass moments relative to crankpin (1 and 2).

(2 marks)

- (c) With the same view from the front, show where the direction of the required centres of mass of the pulley and flywheel need to be to achieve overall first order forward rotating balance.

(2 marks)

- (d) If six equal small counterwebs were created so that the centre of mass of each crankspan of the bare crankshaft was along the axis of crankshaft rotation (ie, no centre of mass offset), calculate the required mass moment of the flywheel and pulley in kg.mm. to achieve overall first order balance along the entire crankshaft.

(2 marks)

- (e) Calculate the magnitude of the resultant first order reverse rotating moment of the mass moments along the crankshaft in kg.mm.mm. for the Vee 6 Engine.

(4 marks)

8. QUESTION 1

RECIPROCATING MASS

$$\begin{aligned}M_{\text{rec}} &= 240 + \frac{1}{3} \times 300 \\&= 340 \text{ gms} \\&= \underline{\underline{0,34 \text{ kg}}}.\end{aligned}$$

ROTATING MASS

$$\begin{aligned}M_{\text{rot}} &= \frac{2}{3} \times 300 + 20 \\&= 220 \text{ gms} \\&= \underline{\underline{0,22 \text{ kg}}}.\end{aligned}$$

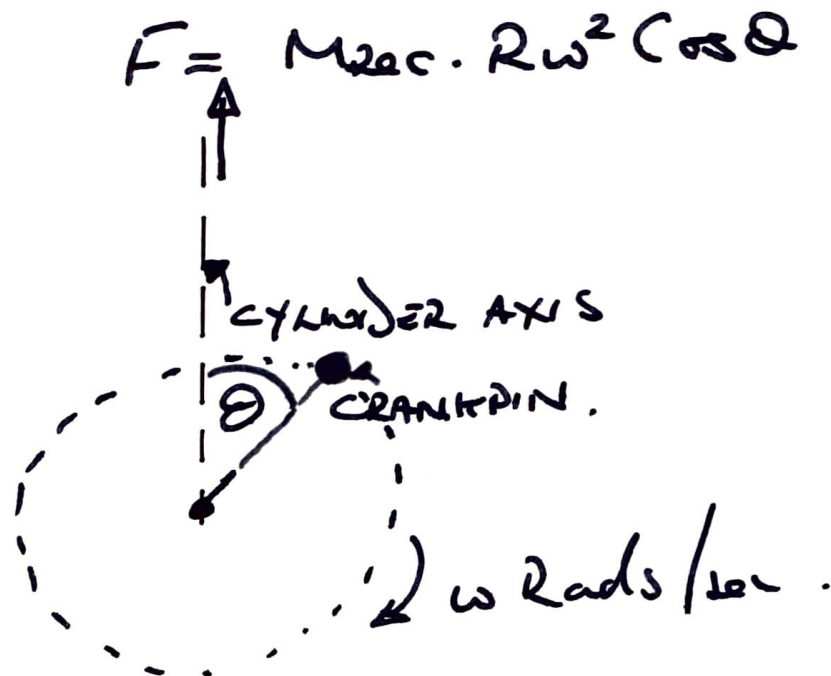
$$\begin{aligned}\text{CRANK RADIUS } R &= \frac{50}{2} \\&= \underline{\underline{25 \text{ mm}}}.\end{aligned}$$

$$\text{CROOKED LENGTH } L = \underline{\underline{100 \text{ mm}}}.$$

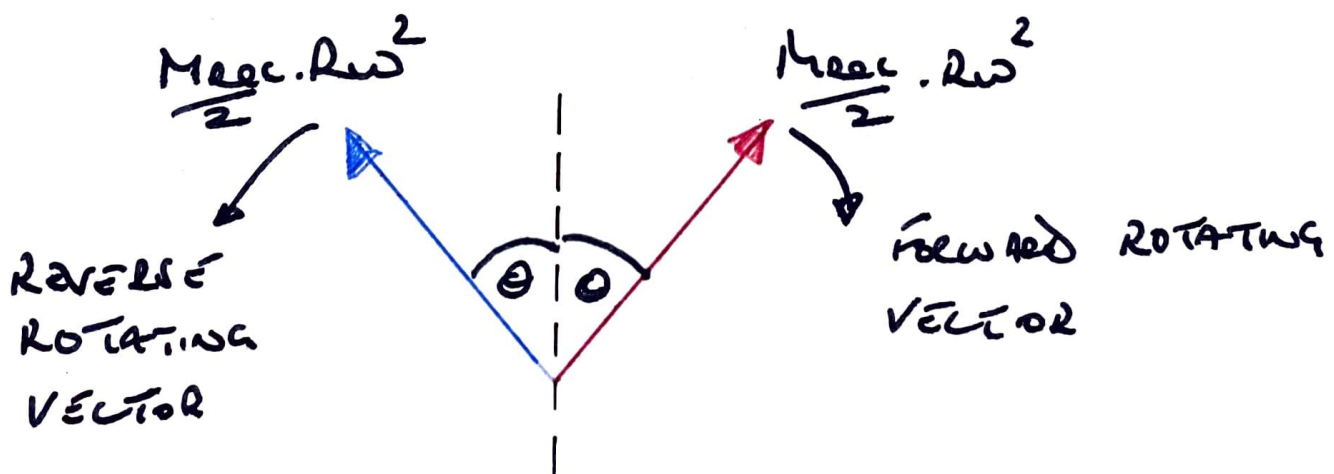
(a) THE RECIPROCATING MASS RESULTS IN FIRST ORDER FORCES ALONG THE VERTICAL CYLINDER AXIS GIVEN BY:

$$F = M_{\text{rec}} \cdot R \cdot \omega^2 \cos \theta$$

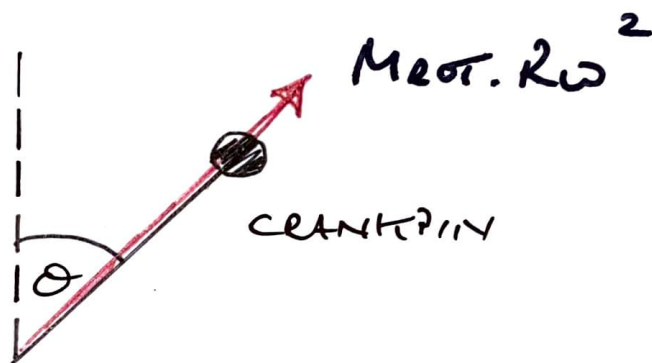
9.



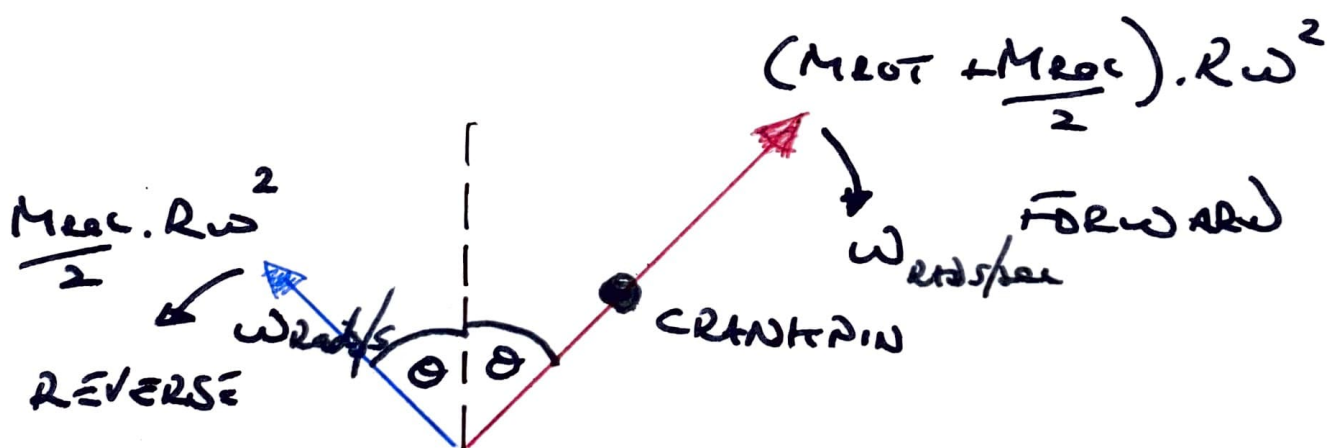
THIS VERTICAL FIRST ORDER FORCE
ALONG CYLINDER AXIS IS MATHEMATICALLY
IDENTICAL TO AND CAN BE REPRESENTED
BY 2 COUNTER-ROTATING CONSTANT
AMPLITUDE VECTORS AS SHOWN IN
DIAGRAM BELOW:



THE ROTATING PART OF THE
 CONROD ALSO CREATES A CENTRIFUGAL
 FORCE OF MAGNITUDE $M_{ROT} \cdot R \omega^2$
 ALONG DIRECTION OF CRANKPIN AS
 SHOWN BELOW:



THE TOTAL FIRST ORDER FORCE CAN
 BE REPRESENTED AS BELOW:



11.

FIRST ORDER FORWARD ROTATING

$$\text{VECTOR} = \left(M_{ROT} + \frac{M_{REC}}{2} \right) \cdot R \omega^2$$

FIRST ORDER FORWARD

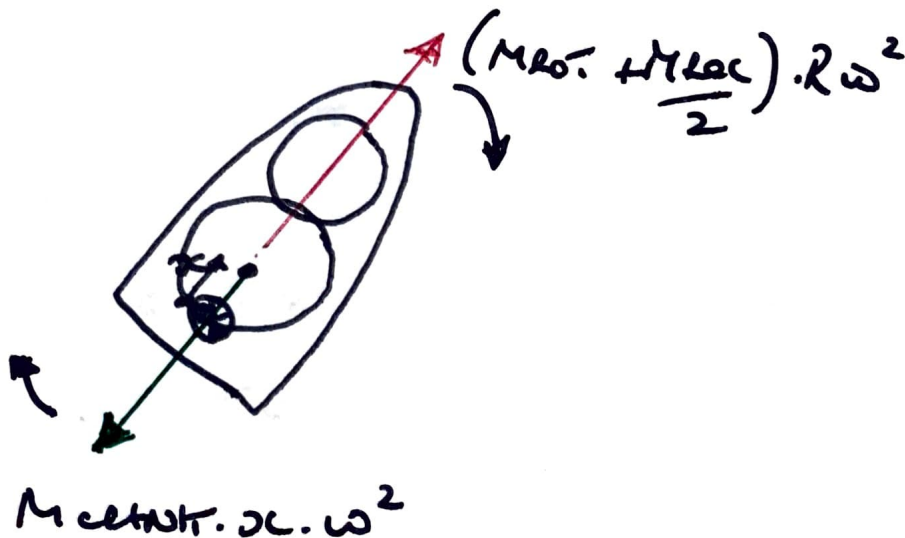
$$\text{MASS MOMENT} = \left(M_{ROT} + \frac{M_{REC}}{2} \right) \cdot R$$

$$= \left(0,22 + 0,34 \frac{2}{2} \right) \cdot 25$$

$$= 0,39 \times 25$$

$$= \underline{\underline{9,75 \text{ kg.m.}}}$$

6) AN OFFSET CENTRE OF GRAVITY /
CENTRIFUGAL CREATS A BALANCING
CENTRIFUGAL FORCE AS SHOWN BELOW:



TO FULLY BALANCE FIRST ORDER FORWARD
VECTOR MAKE

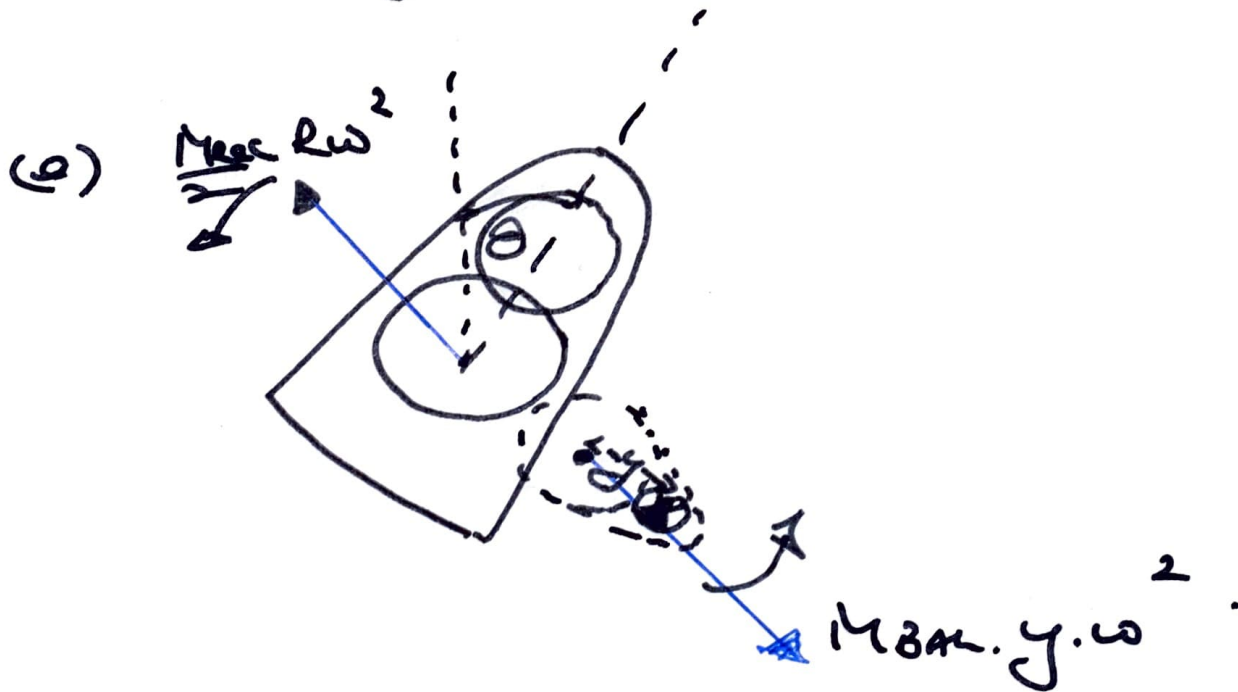
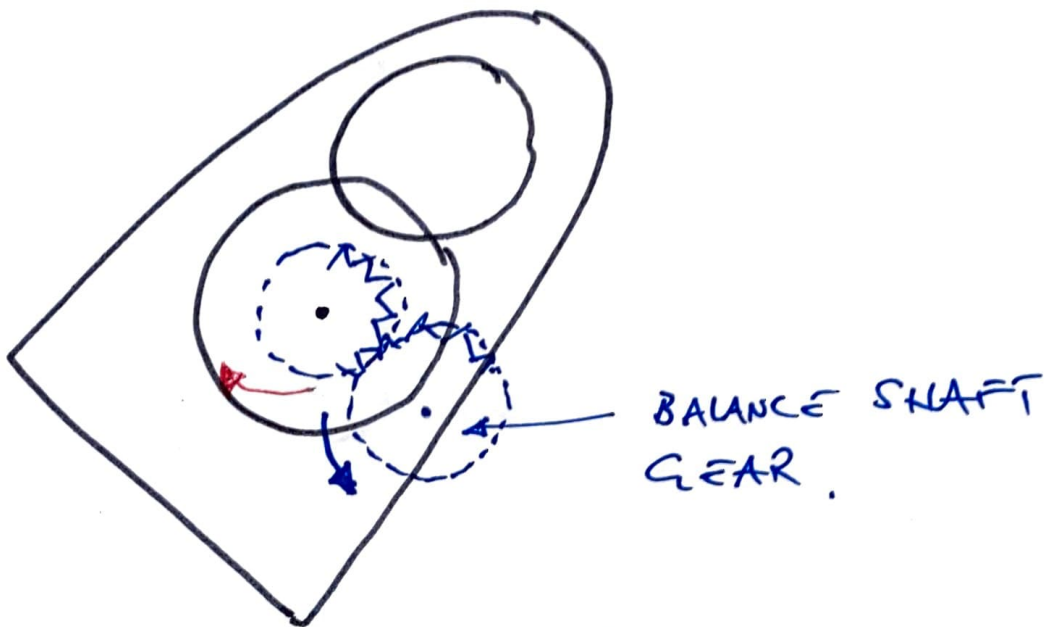
$$M_{CENTRIF.} \cdot R \cdot \omega^2 = \left(M_{ROT} + \frac{M_{REC}}{2} \right) \cdot R \cdot \omega^2$$

$$\begin{aligned}
 12. \quad \therefore M_{\text{CENTR. AC}} &= (M_{\text{ROT}} + \frac{M_{\text{REV}}}{2}) \cdot R \\
 &= 2,5 \text{ kg} \\
 \therefore M_{\text{CENTR}} &= 9,75 \\
 2,5 \cdot x &= 9,75 \\
 x &= \underline{\underline{3,9 \text{ mm}}}
 \end{aligned}$$

(c) MASS MOMENT OF FIRST AND = R
REVERSE

$$\begin{aligned}
 &= \frac{M_{\text{REV}}}{2} \cdot R \\
 &= 0,17 \times 25 \\
 &= \underline{\underline{4,25 \text{ kg} \cdot \text{mm}}}
 \end{aligned}$$

(d) BALANCE SHAFT NEEDS TO ROTATE
COUNTERCLOCKWISE. YOU CAN DRIVE
FROM END OF CRAWLSHAFT USING
A GEAR-PAIR WITH EQUAL DIAMETERS
TO CREATE SAME ROTATION SPEED.



TO FULLY BALANCE, $M_{ROTOR} =$

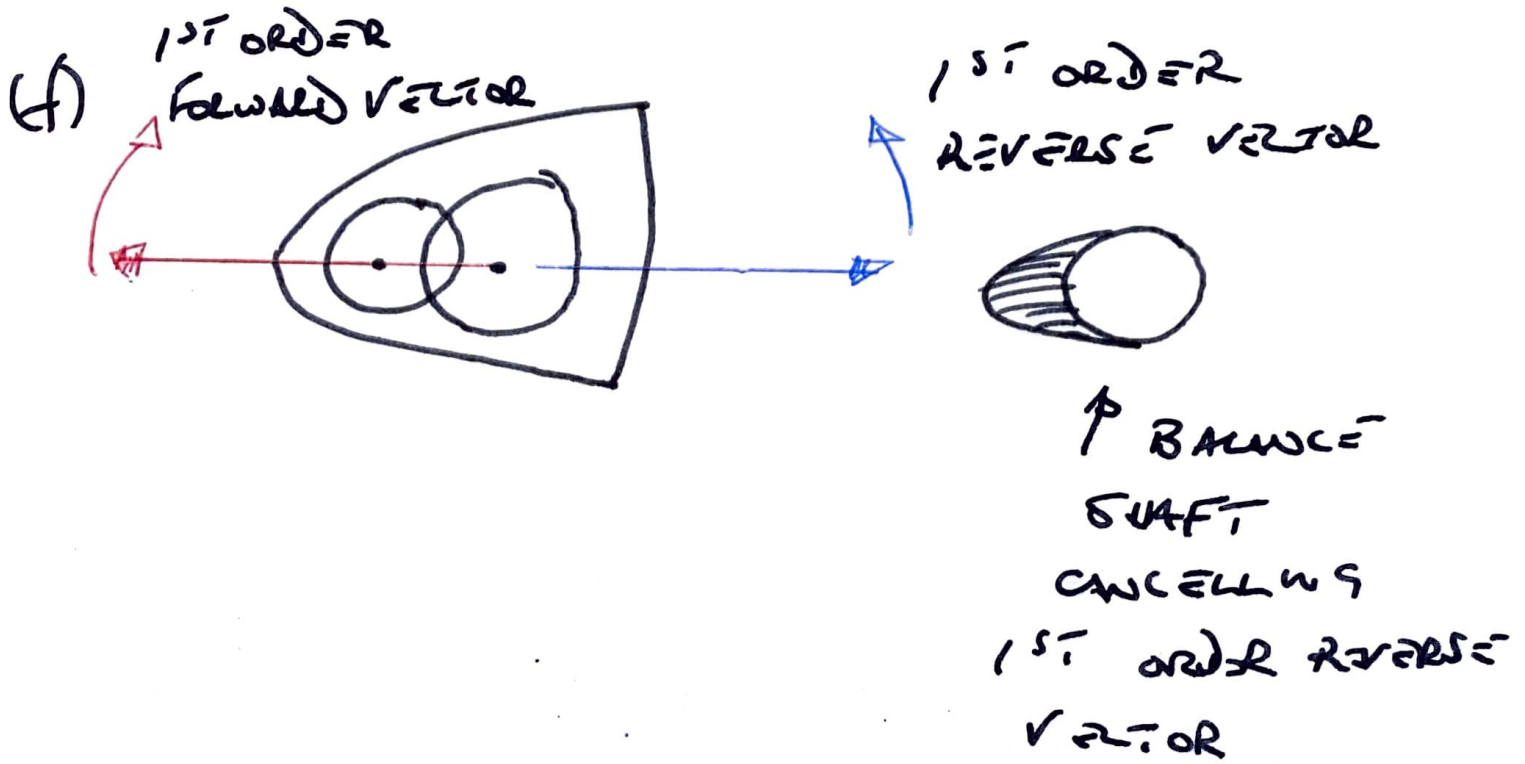
$$\frac{M_{ROTOR}}{2} \cdot R \omega^2 = M_{BAL} \cdot y \cdot \omega^2$$

$$1 \frac{M_{ROTOR}}{2} \cdot R = M_{BAL} \cdot y$$

$$4,25 = 1,5 \cdot y$$

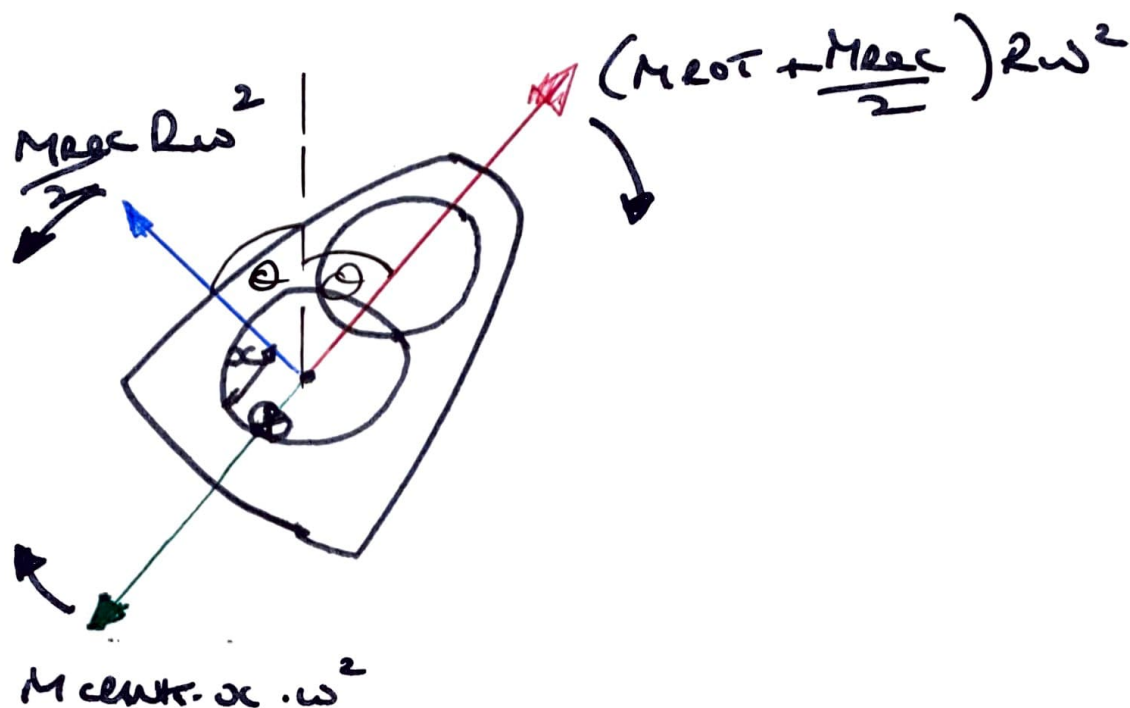
14.

$$y = \underline{\underline{2.83 \text{ mm.}}}$$



(g) THE ENGINE IS VERTICAL. IF A BALANCE SHAFT, CANNOT BE FITTED OVER BALANCE THE COUNTERSHAFT AS BELOW TO CREATE A PURRY HORIZONTAL FORCE WHICH WILL NOT BE FELT SO EASILY BY A RIDER:

15.



TO CREATE HORIZONTAL FORCE MATTE.

$$M_{centric} x \omega^2 - (M_{rot} + \frac{M_{rec}}{2}) R \omega^2 = \frac{M_{rec}}{2} R \omega^2$$

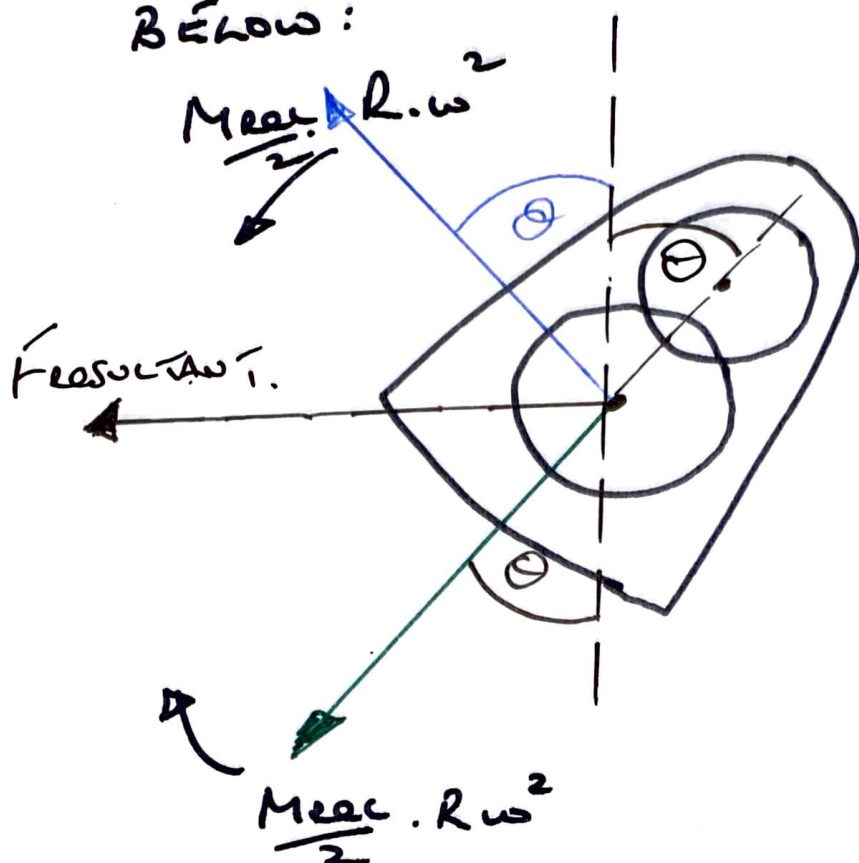
$$\begin{aligned} \text{i.e. } M_{centric} x \omega^2 &= (M_{rot} + \frac{M_{rec}}{2}) R \omega^2 + \frac{M_{rec}}{2} R \omega^2 \\ &= (M_{rot} + M_{rec}) R \omega^2 \end{aligned}$$

$$\begin{aligned} \text{i.e. } M_{centric} x &= (M_{rot} + M_{rec}) R \\ &= (0,22 + 0,34) \cdot 25 \\ &= 0,56 \times 25 \\ &= \underline{14} \text{ kg. mm.} \end{aligned}$$

16.

Resultant Force Diagram is as

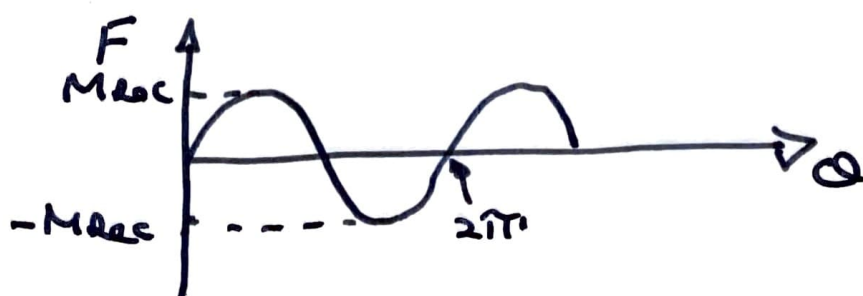
Below:



$F_{resultant}$ is given by

$$\begin{aligned} F_{resultant} &= \frac{M_{acc}}{2} \cdot R \omega^2 \sin \theta + \frac{M_{acc}}{2} R \omega^2 \sin \theta \\ &= \underline{\underline{M_{acc} \cdot R \omega^2 \sin \theta}} \end{aligned}$$

Amplitude of force = $M_{acc} \cdot R \omega^2$.

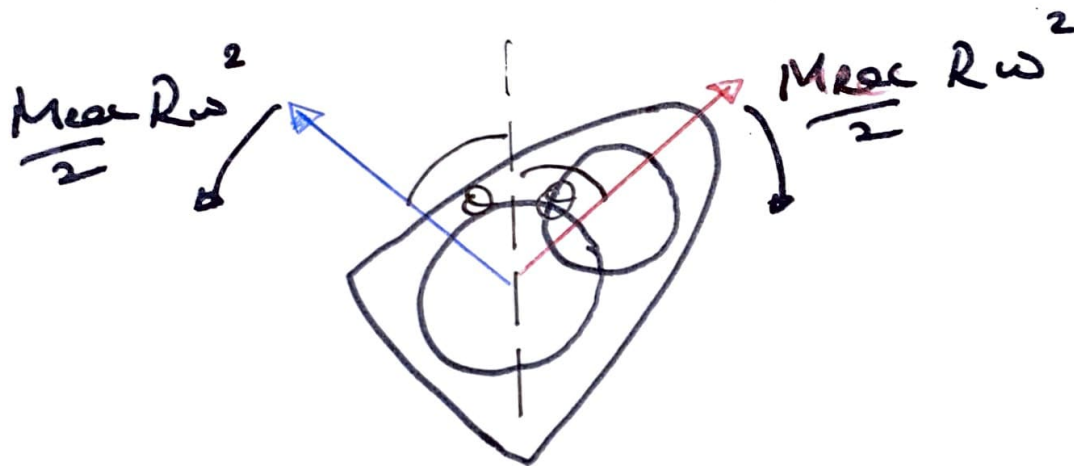


$$F = M_{acc} \cdot R \omega^2 \sin \theta$$

17.

(h) IF YOU UNDERTAKE CRUIT
AND HAVE
 $M_{CENT.OC.} \omega^2 = M_{TOT.} R \omega^2$

THE FOLLOWING FORCE VECTOR IS
CREATED:



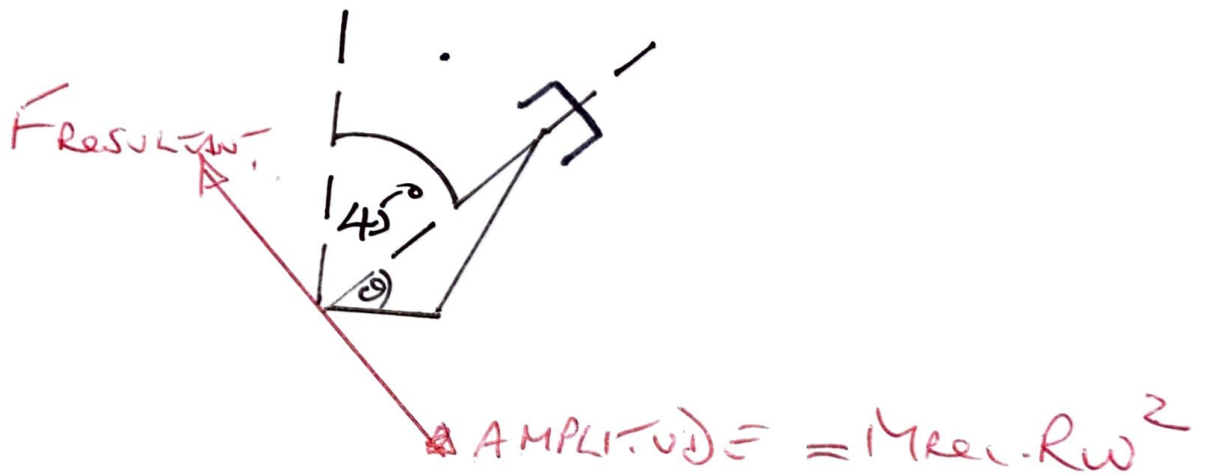
YOU NOW HAVE A RESULTANT
VERTICAL FORCE ALONG CYLINDER
AXIS OF MAGNITUDE

$$= \frac{M_{acc} R \omega^2 \cos \theta}{2} + \frac{M_{acc} R \omega^2 \cos \theta}{2}$$

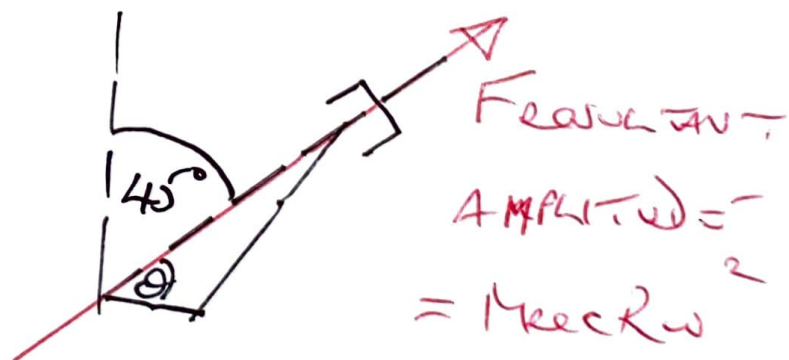
$$= M_{acc} R \omega^2 \cos \theta$$

AMPLITUDE OF FORCE = $M_{acc} R \omega^2$

18. IF ENGINE IS AT 45° TO VERTICAL, OVERBALANCING CREATES RESULTANT FORCE AS SHOWN BELOW.



IF ENGINE IS UNDERBALANCED, RESULTANT FORCE IS AS SHOWN BELOW:



CASES
IN BOTH ~~CASES~~, YOU HAVE AN EQUAL AMPLITUDE VIBRATION AT 45° TO THE VERTICAL.

19.

IT IS NOT READILY POSSIBLE
TO TOTALLY BALANCE THE VERTICAL
FIRST ORDER FORCE.

=

20.

QUESTION 2.

PLEASE NOTE THIS IS A WS ENGINE.
(MISPRINT!).

$$M_{rec} = 240 + \frac{1}{3} \times 330$$

$$= 350 \text{ gms}$$

$$= \underline{\underline{0,35 \text{ kg.}}}$$

$$M_{rot} = \frac{2}{3} \times 330$$

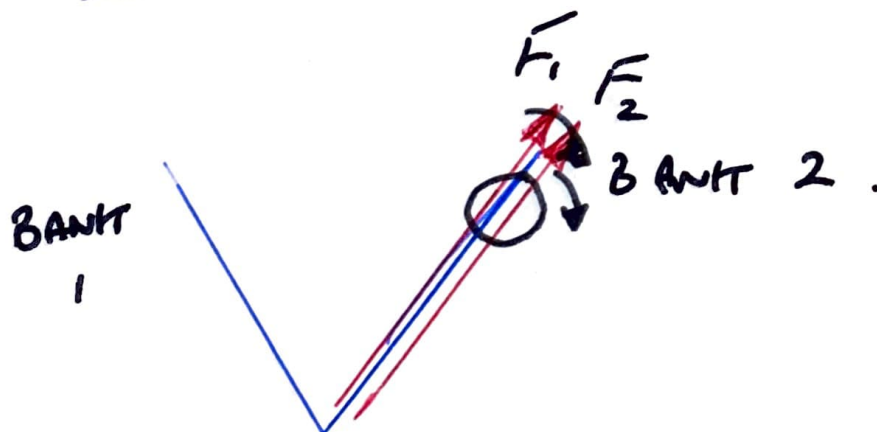
$$= 220 \text{ gms}$$

$$= \underline{\underline{0,22 \text{ kg.}}}$$

$$R = \frac{75}{2} = \underline{\underline{37,5 \text{ mm.}}}$$

$$L = \underline{\underline{130 \text{ mm.}}}$$

(a)



BOTH FIRST ORDER IDENTICAL VALUES

21.

ARE ALONG SAME BANK.

$$F_1 = F_2 = (M_{ROT} + \frac{M_{REC}}{2}) \cdot R \omega^2.$$

RESULTANT FIRST ORDER VECTOR

$$= 2 \cdot (M_{ROT} + \frac{M_{REC}}{2}) R \omega^2$$

RESULTANT MASS MOMENT

$$= 2 \cdot (M_{ROT} + \frac{M_{REC}}{2}) \cdot R.$$

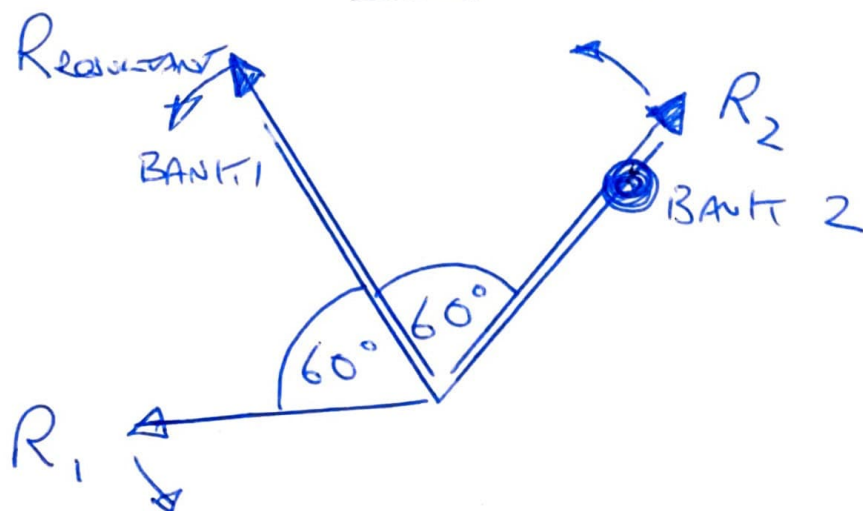
$$= 2 \cdot (0,22 + \frac{0,35}{2}) \times 37,5$$

$$= 2 \cdot (0,22 + 0,175) \times 37,5$$

$$= 2 \cdot 0,395 \times 37,5$$

$$= \underline{\underline{29,6 \text{ Kg.m.}}}$$

(b)



22.

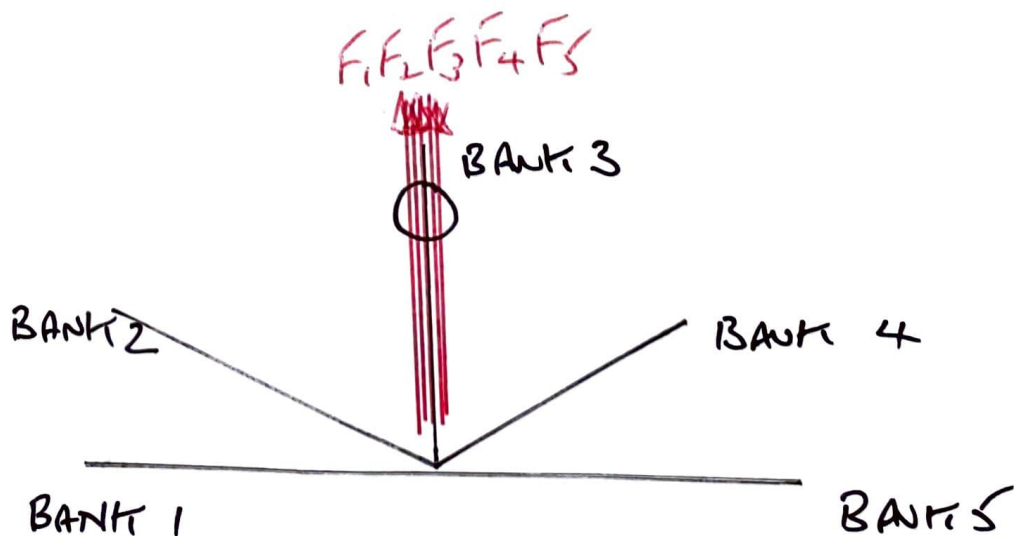
Resultant Reverse Vector is Along BANK 1.

$$R_1 = R_2 = \frac{M_{rec}}{2} \cdot R \omega^2$$

$$\begin{aligned} \text{Resultant} &= \frac{M_{rec}}{2} \cdot R \omega^2 \cos 60^\circ \\ &\quad + \frac{M_{rec}}{2} \cdot R \omega^2 \cos 60^\circ \\ &= M_{rec} \cdot R \omega^2 \cdot \cos 60^\circ \end{aligned}$$

$$\begin{aligned} \text{MRS MOMENT} &= M_{rec} \cdot R \cdot \cos 60^\circ \\ &= 0,35 \times 37,5 \times 0,5 \\ &= \underline{\underline{6,56 \text{ kg.m.}}} \end{aligned}$$

(c)



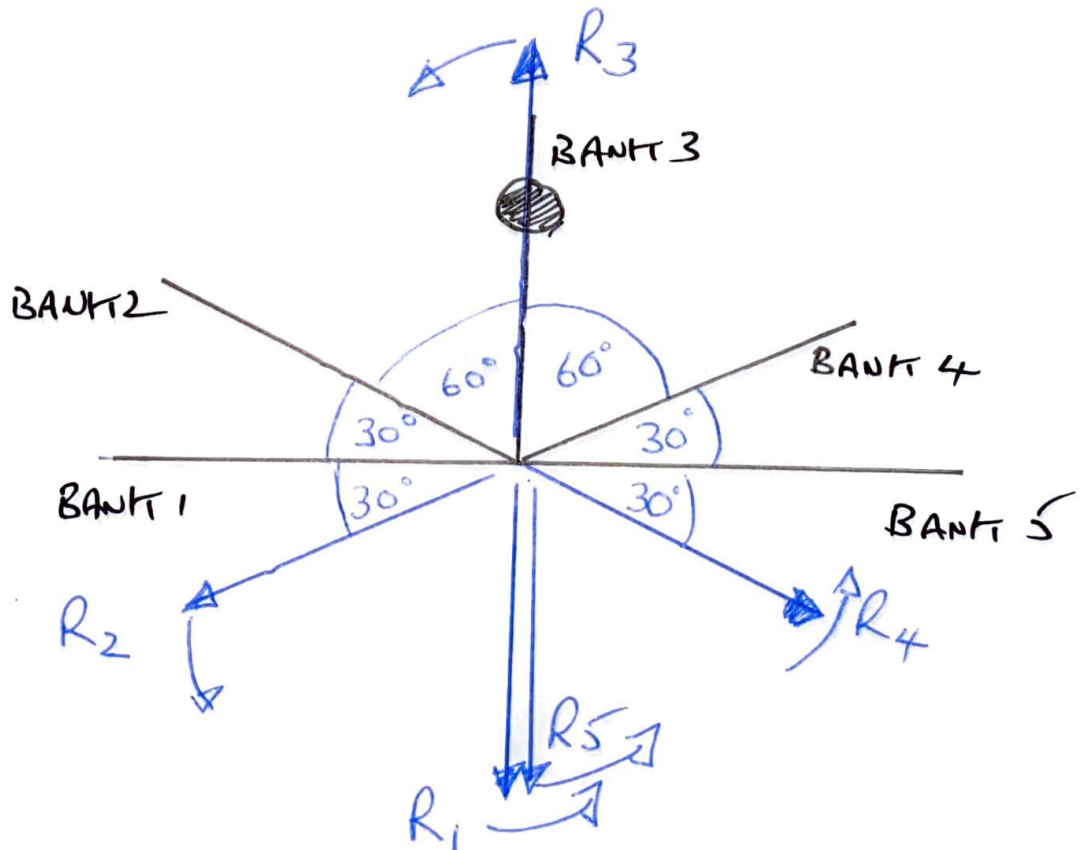
TOTAL FIRST ORDER FORWARD VECTOR

23.

IS ALONG BANK 3.

$$\begin{aligned}
 \text{MASS MOMENT} &= 5 \cdot \left(M_{20T} + \frac{M_{10R}}{2} \right) \cdot R \\
 &= 5 \cdot (0,22 + 0,175) \cdot 37,5 \\
 &= \underline{\underline{74,1 \text{ kg} \cdot \text{mm}}}
 \end{aligned}$$

(d)



LET $R_1 = R_2 = R_3 = R_4 = R_5 = R$.

RESULTANT VERTICAL COMPONENT DOWN

24.

$$\begin{aligned} &= R_1 + R_5 - R_3 + R_2 \sin 30 + R_4 \sin 30 \\ &= R + R - R + 0,5R + 0,5R \\ &= \underline{\underline{2R \text{ Down.}}} \end{aligned}$$

$$\begin{aligned} \text{MASS MOMENT} &= 2 \cdot \frac{M_{\text{rec}}}{2} \cdot R \\ &= M_{\text{rec}} \cdot R \\ &= 0,35 \times 37,5 \\ &= \underline{\underline{13,1 \text{ kg} \cdot \text{mm.}}} \end{aligned}$$

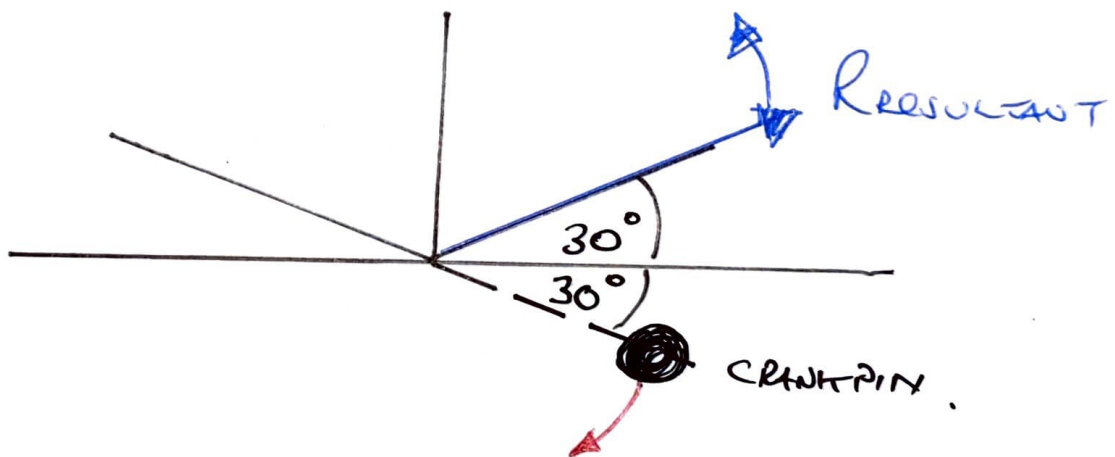
(c) BALANCE THE FIRST ORDER
FORWARD BY USING A COUNTERSHAFT
WITH OFFSET C.O.G.

BALANCE THE RESULTANT REVERSE
VECTOR WITH COUNTER-ROTATING
BALANCE SHAFT.

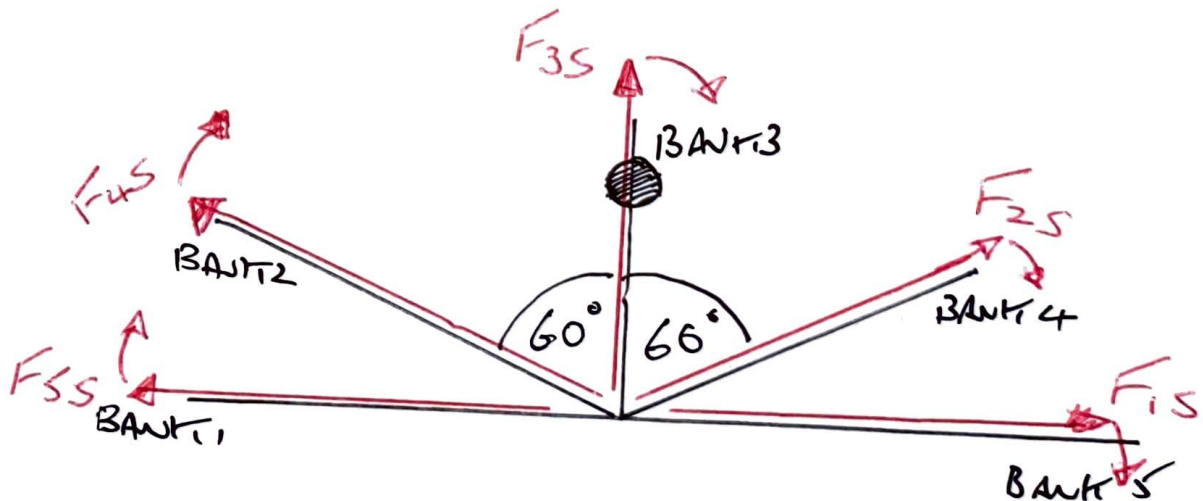
$$\begin{aligned} \text{CRAFT MASS MOMENT} &= 74,1 \text{ kg} \cdot \text{mm} \\ \text{BALANCE SHAFT MASS MOMENT} &= 13,1 \text{ kg} \cdot \text{mm.} \end{aligned}$$

25.

(f). IF CRANK ROTATES 120°
CLOCKWISE FROM BANK 3,
RESULTANT REVERSE FIRST ORDER
ROTATES 120° ANTI-CLOCKWISE
FROM OPPOSITE BANK 3 AS
SHOWN BELOW.



(g)



26.

$$F_{1S} = F_{2S} = F_{3S} = F_{4S} = F_{5S}$$

$$= \frac{1}{2} \cdot \frac{R}{L} M_{rec} \cdot R \omega^2 = F$$

RESULTANT HORIZONTAL FORCE = 0

$$\begin{aligned} \text{RESULTANT VERTICAL FORCE} &= F_{4S} \cos 60 \\ &\quad + F_{3S} \\ &\quad + F_{2S} \cos 60 \\ &= F \times 0,5 + F + F \times 0,5 \\ &= \\ &= \underline{\underline{2F}} \\ &= 2 \cdot \frac{1}{2} \cdot \frac{R}{L} \cdot M_{rec} R \omega^2 \\ &= \frac{R}{L} M_{rec} \cdot R \omega^2. \end{aligned}$$

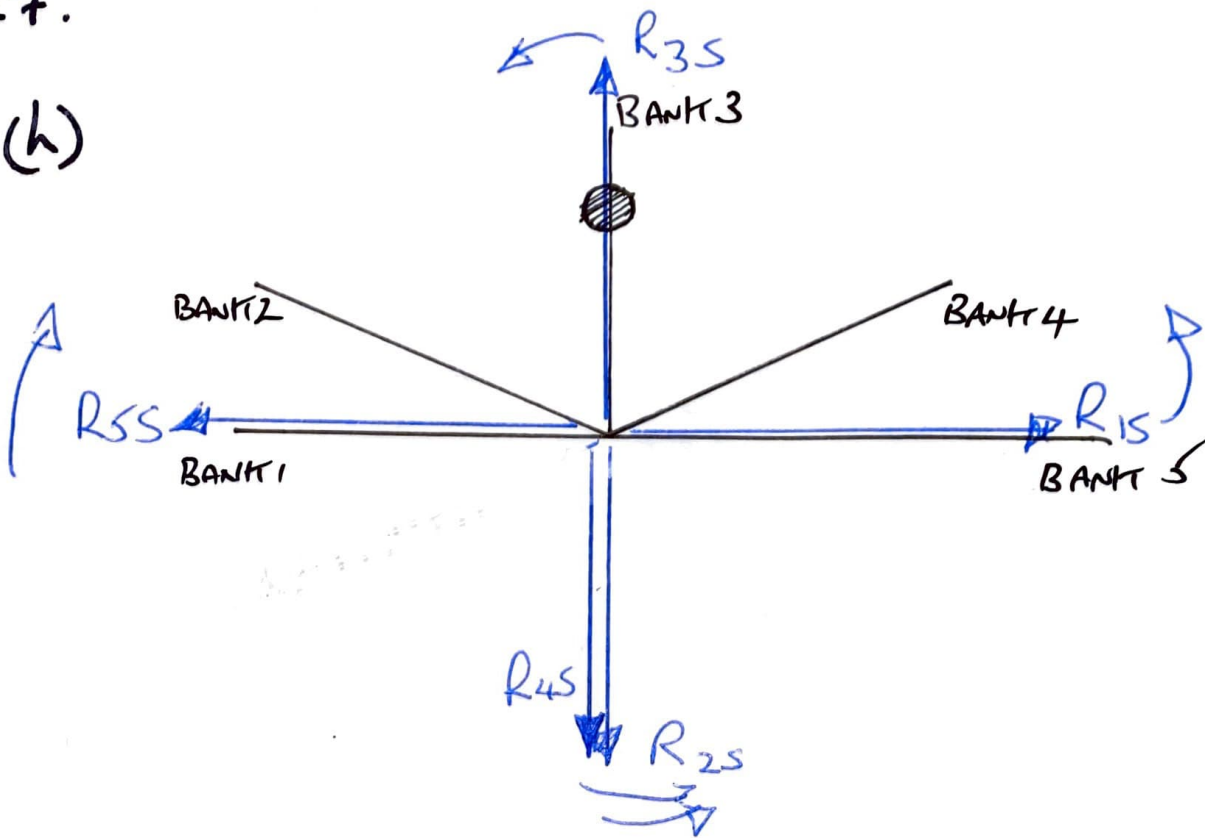
$$\text{MASS MOMENT} = \frac{R}{L} M_{rec} \cdot R$$

$$= \frac{37,5}{130} \times 0,35 \times 37,5$$

$$= \underline{\underline{3,79 \text{ kg} \cdot \text{mm}}}$$

27.

(h)



$$\text{LET } R_{1S} = R_{2S} = R_{3S} = R_{4S} = R_{5S} = R$$

RESULTANT = R DOWN OPPOSITE
BANK 3

$$R = \frac{1}{2} \frac{RM}{L} R_{ec} \cdot R \omega^2.$$

$$\begin{aligned} \text{MASS MOMENT} &= \frac{1}{2} \cdot \frac{R}{L} \cdot M_{ec} \cdot R \\ &= \frac{1}{2} \cdot \frac{37,5}{130} \times 0,35 \times 37,5 \\ &= \underline{\underline{1,89 \text{ kg.m.}}} \end{aligned}$$

28 .

QUESTION 3.

$$M_{\text{REC}} = 240 + \frac{1}{3} \times 240$$

$$= 320 \text{ gms}$$

$$= \underline{\underline{0,32 \text{ kg.}}}$$

$$M_{\text{ROT}} = \frac{2}{3} \times 240$$

$$= 160 \text{ gms}$$

$$= \underline{\underline{0,16 \text{ kg.}}}$$

$$R = \frac{65}{2} = \underline{\underline{32,5 \text{ mm.}}}$$

$$L = \underline{\underline{110 \text{ mm.}}}$$

PART 1

(a) TOTAL FIRST ORDER FORWARD
VECTOR PER SPAN

$$= 2 \left(M_{\text{ROT}} + \frac{M_{\text{REC}}}{2} \right) \cdot R \omega^2$$

$$\text{MARS MOMENT} = 2 \left(M_{\text{ROT}} + \frac{M_{\text{REC}}}{2} \right) \cdot R.$$

29.

$$= 2 \left(0,16 + \frac{0,32}{2} \right) \cdot 32,5$$

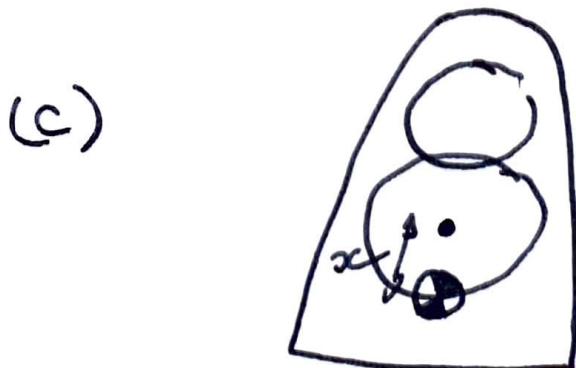
$$= \underline{\underline{20,8 \text{ kg} \cdot \text{mm}}}$$

(b). IF EACH COUNTERSPAN HAS
MASS 3 kg.



$$3. \quad \omega = 20,8$$

$$\omega = \underline{\underline{6,9 \text{ mm}}}$$



TUNGSTEN HAS A
CONSIDERABLY HIGHER
DENSITY THAN STEEL.

30.

$$\rho_{\text{STEEL}} \approx 7800 \text{ kg/m}^3$$

$$\rho_{\text{TUNGSTEN}} \approx 19300 \text{ kg/m}^3.$$

i.e. TUNGSTEN IS OVER TWICE AS DENSE. THIS MEANS MORE MASS CAN BE CONCENTRATED NEAR CENTRE OF WEBS ALLOWING x (DISTANCE OF CG FROM CENTRE) TO BE REDUCED.

HOWEVER, BALANCE CONDITIONS STAY THE SAME.

$$\begin{aligned} \text{THEREFORE } M_{\text{TUNGSTEN}} \cdot x_{\text{TUNGSTEN}} \\ = M_{\text{STEEL}} \cdot x_{\text{STEEL}}. \end{aligned}$$

THE TUNGSTEN CRAWL WILL BE HEAVIER BUT x WILL BE SMALLER.

IMAGINE

$$M_{\text{TUNGSTEN}} = 2 \times M_{\text{STEEL}}. \quad - (1)$$

FOR SAME BALANCE

$$x_{\text{TUNGSTEN}} = 0.5 \times x_{\text{STEEL}}. \quad - (2)$$

31.

$$\text{MOMENT OF INERTIA TUNGSTEN}^2 = \frac{1}{2} M_{\text{TUNGSTEN}} \cdot x_{\text{TUNGSTEN}}^2$$

$$\text{MOMENT OF INERTIA STEEL}^2 = \frac{1}{2} M_{\text{STEEL}} \cdot x_{\text{STEEL}}^2$$

$$\frac{\text{MOMENT OF INERTIA TUNGSTEN}}{\text{MOMENT OF INERTIA STEEL}}$$

$$= \frac{M_{\text{TUNGSTEN}} \cdot x_{\text{TUNGSTEN}}^2}{M_{\text{STEEL}} \cdot x_{\text{STEEL}}^2}$$

AND SUBSTITUTING
FOR ① & ②

$$= \frac{2 M_{\text{STEEL}} \cdot (0,5 x_{\text{STEEL}})^2}{M_{\text{STEEL}} \cdot x_{\text{STEEL}}^2}$$

$$= 2 \times 0,5 \times 0,5$$

$$= \underline{0,5}$$

32.

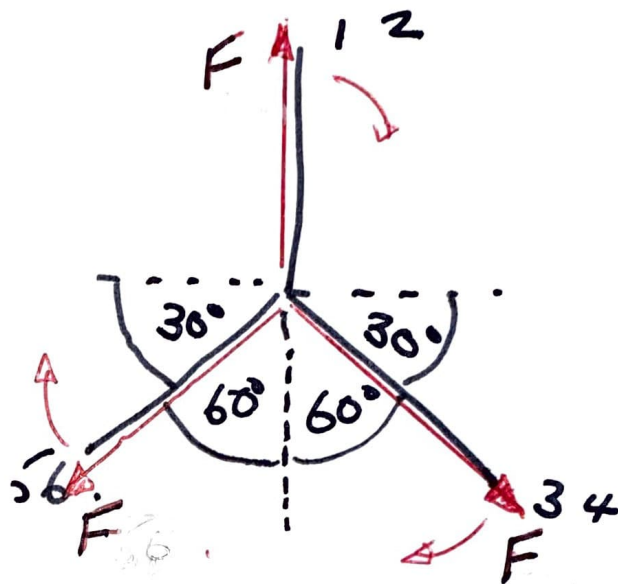
THIS LEANS TUNGSTEN CRANK
IS HEAVIER BUT HAS NEARLY HALF
THE MOMENT OF INERTIA OF STEEL
CRANK.

[QUICK ANSWER IS THAT
TUNGSTEN INCREASES MASS AND
PROPORTIONALLY REDUCES OC .
BECAUSE MOMENT OF INERTIA
IS PROPORTIONAL TO $M \cdot OC^2$,
MOMENT OF INERTIA WILL FALL
IF M INCREASES & OC REDUCES IN
PROPORTION.]

REDUCED MOMENT OF INERTIA ALLOWS
MORE ENGINE TORQUE TO BE AVAILABLE
TO ACCELERATE CAR.

33.

- (a) EACH CRUTSPIN HAS FIRST ORDER MASS MOMENTS ALONG DIRECTION OF EACH CRUTPIN AS SHOWN

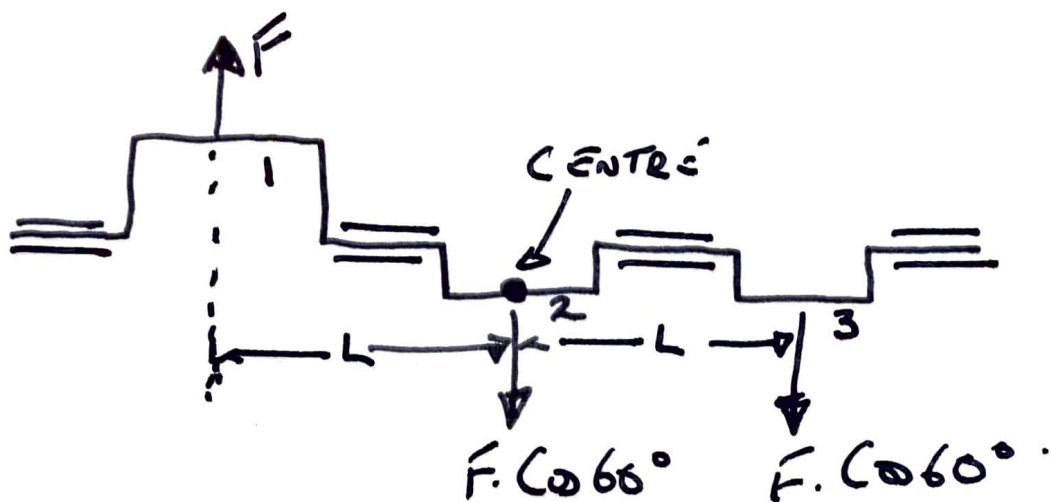


$$\text{WHERE } F = 2(M_{\text{rot}} + M_{\text{cr}}) \cdot \frac{R}{2}$$

MOMENT PARALLEL TO PIN 1 M_{11}

TO TAKE MOMENT OF MASS MOMENTS IN PLANE OF PIN 1, LOOK SIDWAYS AT PIN 1. THE DIAGRAM BELOW SHOWS THE MAGNITUDES OF THE MASS MOMENTS IN PLANE OF PIN 1 (IMAGINE LOOKING SIDWAYS WITH PIN 1 VERTICAL)

34.



MOMENT OF MASS MOMENTS ABOUT CENTRE

$$M_{11} = F \cdot L + F \cdot \cos 60^\circ \cdot L$$

$$(\cos 60^\circ = 0,5)$$

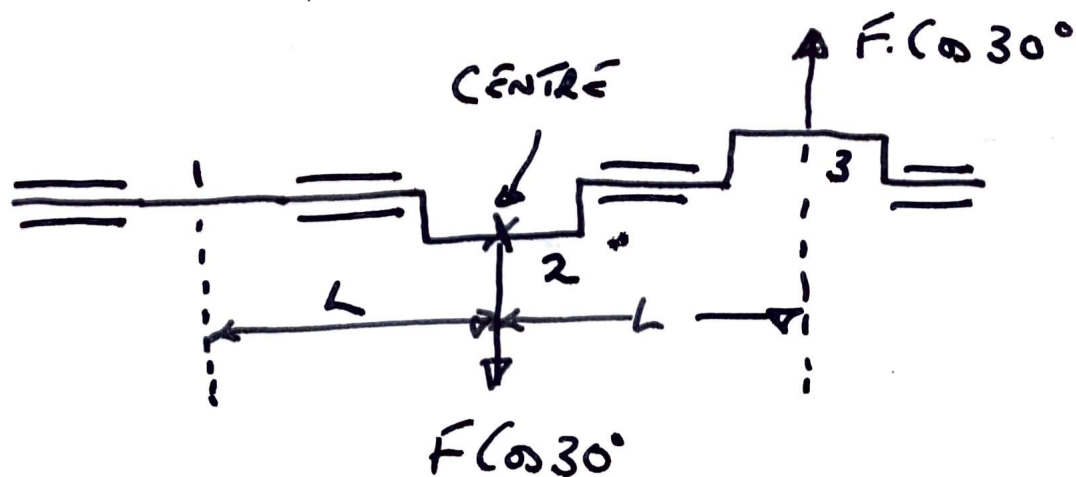
$$M_{11} = FL + 0,5 FL$$

$$= \underline{\underline{1,5 FL}} \quad \text{CLOCKWISE.}$$

MOMENT PERPENDICULAR TO PIN 1 M_1

LOOK ALONG PIN 1 TO SEE
MASS MOMENTS PERPENDICULAR TO IT
AS SHOWN BELOW:

(IMAGINE LOOKING DOWN WITH
PIN 1 VERTICAL)

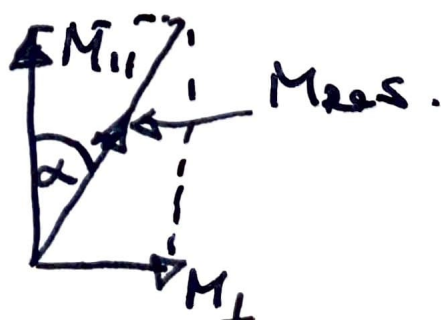


TAKING MOMENTS ABOUT CENTRE

$$M_1 = F \cos 30^\circ \cdot L$$

$$= 0,866 \cdot L \quad \text{ANTI-CLOCKWISE}$$

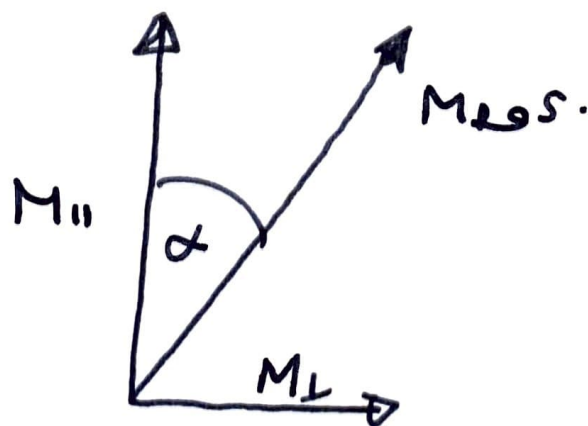
TO GET RESULTANT MOMENT WE
 PYTHAGORAS. THE FURTHER SPAN 5,6
 MOVES REAR OF CRANK TO LEFT AND
 PIVOTING AROUND CENTRE MOVES FRONT
 OF CRANK TO RIGHT.



36.

$$\begin{aligned}
 M_{205} &= \sqrt{M_{11}^2 + M_{1\perp}^2} \\
 &= \sqrt{(1,5 FL)^2 + (0,866 FL)^2} \\
 &= F.L. \cdot \sqrt{1,5^2 + 0,866^2} \\
 &= \underline{\underline{1,732 F.L.}} \\
 &= 1,732 \cdot x 2 \cdot \left(M_{207} + \frac{M_{208}}{2} \right) \cdot R.L. \\
 &= 1,732 \times 20,8 \times 110 \\
 &= \underline{\underline{3962,9 \text{ kg.mm.mm.}}}
 \end{aligned}$$

(b)



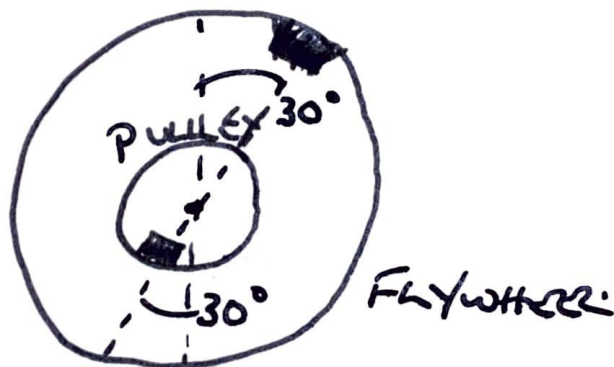
$$\tan \alpha = \frac{M_{1\perp}}{M_{11}} = \frac{0,866 FL}{1,5 FL}$$

37.

$$= \frac{0,866}{1,5} = 0,577.$$

$$\alpha = 30^\circ.$$

(c)



(d) THE CALCULATED MOMENT OF THE FORWARDED MASS MOMENTS IS TO BE FULLY COUNTERACTED BY OUT OF MASS FLYWHEEL AND PULLEY AS THERE IS NO LOCAL BALANCE FROM CRAFTSHAFT IN THIS CASE.

38 .

BECAUSE THERE MUST BE NO
ADDITIONAL NET. FORCE FROM COMBINATION
OF FLYWHEEL AND PULLEY/

MASS MOMENT FLYWHEEL = MASS MOMENT
PULLEY/

$$\text{i.e. } MM_{\text{FLYWHEEL}} = MM_{\text{PULLEY}} = MM.$$

$$\begin{aligned} \text{DISTANCE BETWEEN FLYWHEEL \& PULLEY/} \\ = 410 \text{ mm.} \end{aligned}$$

THEREFORE, COUNTERBALANCING MOMENT

$$= MM \times 410 = 3962,9$$

$$MM = \frac{3962,9}{410}$$

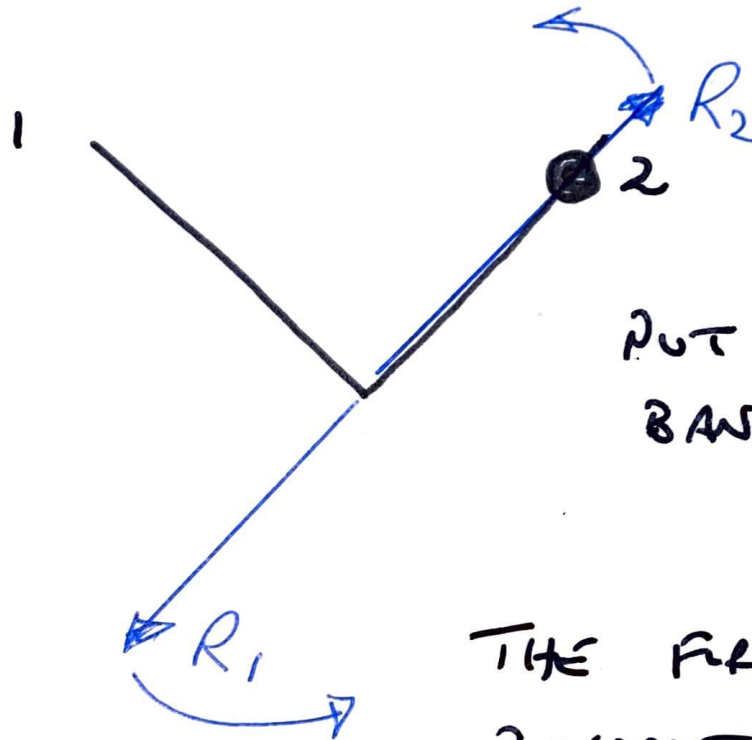
$$= \underline{\underline{9,67 \text{ kg.mm.}}}$$

39

(0) BECAUSE IT IS A $90^\circ \sqrt{6}$,

LOOK AT EACH CRANKSPAN

$\sim 4 \quad 12$



PUT PIN ON
BAR 2.

THE FIRST ORDER
REVERSE VECTORS
CANCEL AT EACH
CRANKSPAN.

THEREFORE, MAGNITUDE OF RESULTANT
MOMENT $= 0$.