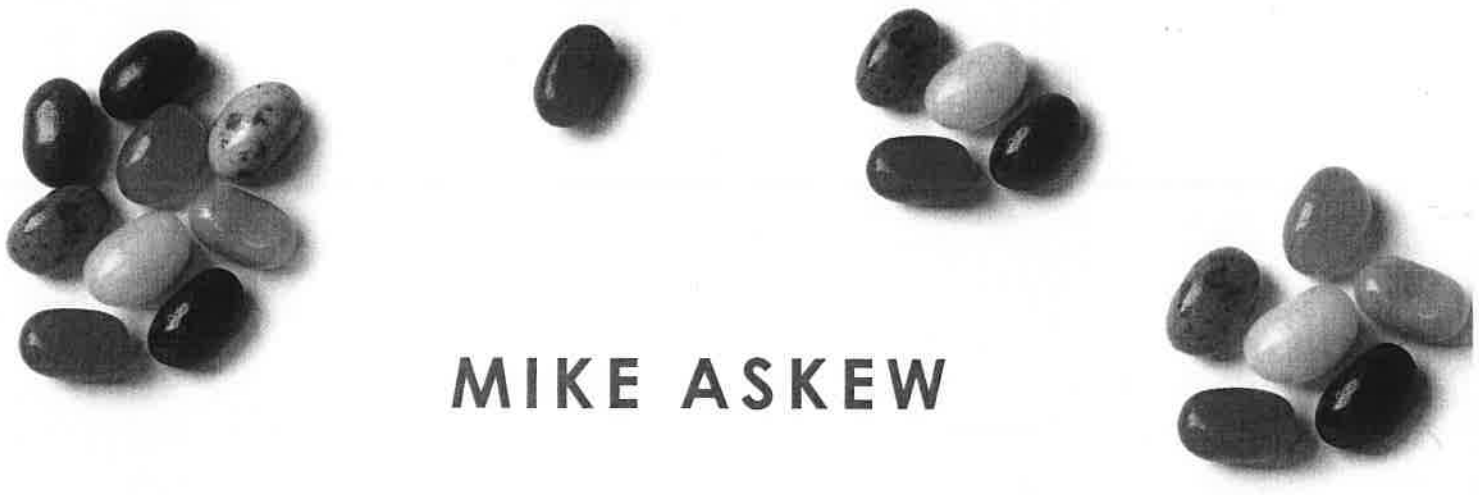


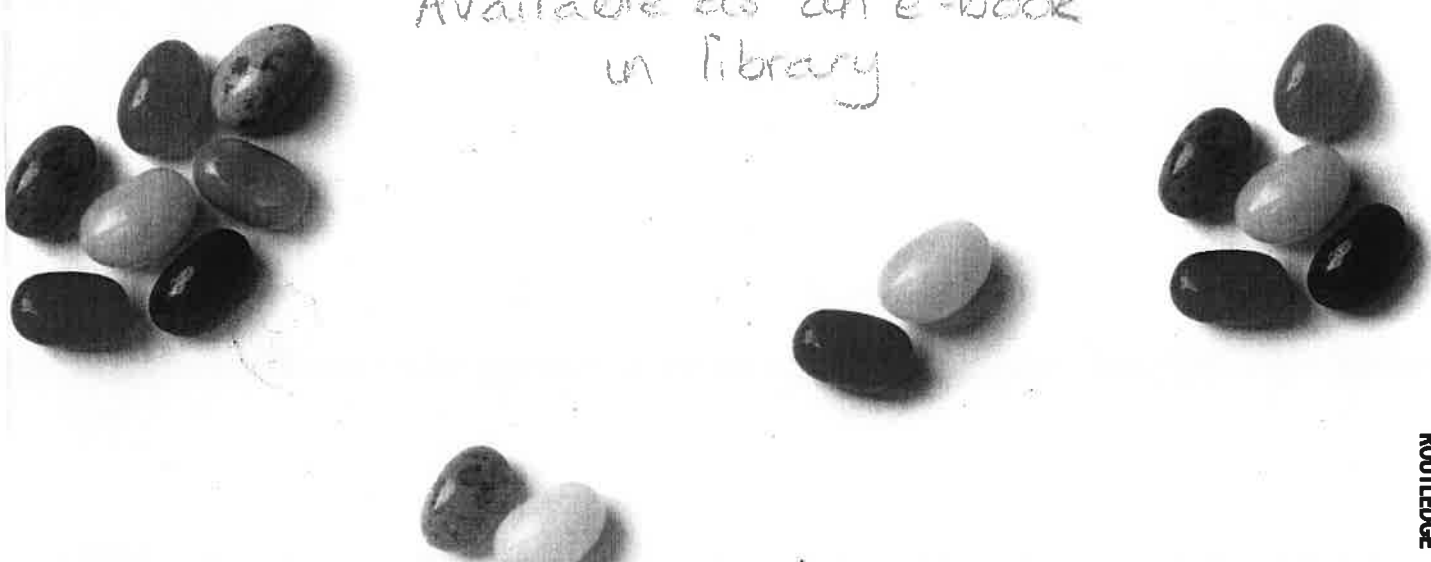


MIKE ASKEW

A PRACTICAL GUIDE TO TRANSFORMING PRIMARY MATHEMATICS

Activities and tasks
that really work

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8 Measures

This chapter explores how to help pupils develop a sound understanding of measuring, with a particular focus on length, mass (weight) and volume and capacity as these measures are all based in the metric system and so also help develop understanding of number and place value. Some ideas on teaching about time are also included at the end of the chapter.

Big ideas

- Measuring is a way of making continuous quantities 'countable'.
- Measuring involves repeatedly using a unit.
- Benchmark measures provide a way of estimating quantities and checking whether or not measures are reasonable.

Making connections

- We can make measurements more accurate by creating smaller units - in our system this leads to decimals.
- Using a unit that is longer than the object being measured connects to understanding fractions.
- Using scales helps to develop understanding of multiplicative reasoning.

Aspects of measurement

Comparison is at the heart of measuring. Knowing that a bowl of flour weighs 150 g, or that a jug contains 50 ml of milk, are not, in themselves, useful pieces of knowledge, but if you are hoping that your muffins will compare favourably to the ones your mother makes, then your quantities have to measure up to the ones she uses. Based on the big idea of comparing then learning about the common measures involves similar stages:

- direct comparing;
- indirect comparing with informal units, such as sticks, shells or egg cups;
- indirect comparing using formal units, such as metres, kilograms or litres.

For example a sequence of teaching about measuring length in Key Stage 1 might include directly comparing the lengths, indirectly comparing two lengths and indirectly comparing using informal units.

Directly comparing the lengths of objects

Directly comparing the lengths of objects - ribbons, books, pencils, children's heights - establishes the language of longer than, shorter than, about the same length and so forth. Although pupils come to school with some informal understanding of such comparisons, there is evidence that they initially find it easier to talk about something being longer rather than shorter, possibly because the comparisons they will have met at home tend to focus more on the greater - who has the most sweets, who ran further, who is the tallest in the family and so forth. Pupils need plenty of experience so that they can talk about the opposite comparisons of less than and shorter.

At the time of writing, the new National Curriculum in England has introduced the use of the symbols $<$ and $>$ as well as the equals sign. Evidence from countries that introduce these symbols early in the mathematics curriculum suggests that young learners do not experience difficulty working with all three symbols. It makes sense to use all three as we meet inequalities far more often than we meet equalities (a statement that itself expresses an inequality!). The idea of equality then becomes a particular case on the continuous scale of something going from being less than something else to being greater than it. For example, with a piece of elastic that is shorter than a length of ribbon pupils can explore when the elastic is shorter than the ribbon, when it is stretched to become longer than the ribbon and the one, unique, moment when the length of the elastic and the ribbon are equal. Measures provide the ideal context for linking these three symbols to the actions of comparing.

Indirectly comparing two lengths

Once learners are confident in the language arising out of direct comparisons they need problems comparing two objects that cannot be placed side-by-side, by asking, for example, is the gap on the display board long enough for a painting on paper to fit in? Can pupils decide without picking up the paper and positioning it on the board? What could they use to make the comparison? Or, if we move the bookcase would it fit between the two tables opposite? Such comparisons are best experienced using a unit that is longer than either object (in the way that many of us will have used a length of string or dressing gown cord to decide if a wardrobe will fit in an alcove or some similar problem) before moving on to using smaller units, either informal ones such as cubes or the formal ones of centimetres and millimetres.

Setting up two towers (made, say, out of cardboard tubes covered in foil) and placing them on classroom surfaces at different heights (on the floor and on a desk, for example) provides the opportunity not only for dialogue about which is the taller tower, but also how this idea relates to being higher or lower - the top of one tower may be higher than the top of the other but that does not necessarily mean it is the taller tower. A metre stick (used simply as a stick) can help pupils find out which is taller. Setting the stick alongside one tower and marking the height by putting a sticker there or wrapping an elastic band is better than trying to keep a finger and thumb in place and then comparing this to when the stick is set against the other tower lays the foundation for reading scales.

Indirectly comparing using informal units

Which is longer - the length of the classroom or the width? Unless a very long piece of string is available, a question like that means repeatedly using a unit. As the question is which is longer, not how much longer, the unit can be informal: pencils, shoes, cubes. Offering the learners a range of possible units like this (rather than simply giving them one to use) provokes a conversation about choosing appropriately - cubes might be useful for comparing the height of the two towers, but may not be the best unit for measuring the length of the classroom.

Recording the number of units provides the ground for an informal conversation about inverse proportion - how was it that the classroom was 24 shoes long but only eight sticks long? Of course, the classic story of the Emperor's new bed has to be told (Google it if you do not know the story).

Restricting the number of units available - providing, say, only three metre sticks - leads into the idea of having to repeat the use of a single unit. How many metres long is the hall? Providing three metre rulers takes away the difficulty of marking the end of each unit when measuring with only one ruler.

As metres are quite long and centimetres rather short, pupils can make their own informal 'unit cube' rulers. Joining ten interlocking cubes (of sides about 2.5 cm) together, they cut a strip of card to this length and use it to measure and compare various things around the classroom.

Using formal units

Understanding how formal units work, and in particular the importance of lining up the zero on a ruler (and ignoring the 'dead-end' if there is one), can be deepened by giving pupils broken rulers to work with. These can be actual rulers with the end cut off or card images like the one in Figure 8.1. Realising that measuring lengths by finding the difference between two numbers on the scales reinforces the ideas that simply looking at the number on the ruler at one end of the thing being measure is not always sufficient.

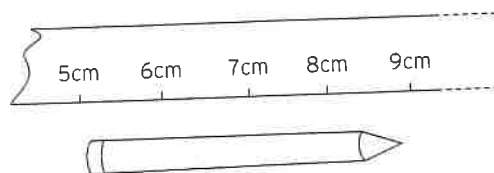


Figure 8.1

Although I have presented these ideas as a neat sequence of progression, the reality is that some pupils will come to class knowing about centimetres and metres, and with some sense of how to use formal tools. Rather than asking these learners to work only with informal units, I suggest making use of their knowledge. There is no harm in having some learners using formal units alongside others using informal ones - they bounce ideas off each other and everyone gains.

Progression in length, area and perimeter

Some ideas that we take for granted as adults are not intuitively obvious to young learners. One such idea is that a measure of height does not necessarily have to be done with a person standing up - measuring the length of your body lying down will produce the same (or very close to) the same result. *Standing up and lying down* addresses this idea.

In moving away from direct and indirect comparison towards the use of units, it is common practice to introduce children to units that are shorter than the object being measured: cubes for the length of a ribbon or hand spans for the length of the mat. As noted earlier, learners' understanding of the need to measure using tools is developed when they begin by using units that are longer than the thing being measured. *Secret ribbons* provides such a challenge.

Most arithmetical relationships that primary pupils encounter obey simple direct or indirect rules: for example, the larger the number you add, the larger the total (a direct relationship), or the smaller the unit of measure, the greater the number of units needed (an indirect relationship). The relationship between perimeter and area is probably the first connection that pupils meet where the 'rules' are not simple. While it is often the case that, as the areas of shapes increase, so too do the perimeters, this is not universally true. Similarly increasing the perimeter does not necessarily lead to a greater area. Pupils' 'common sense' will lead them to expect that the rule 'make one bigger and the other will always be bigger' will hold with respect to area and perimeter. Finding the perimeters of shapes in isolation can reinforce the erroneous idea that shapes with larger areas always have larger perimeters, so rather than introduce finding perimeters in isolation I suggest using tasks that link area and perimeter from the beginning - *Ideal gnome homes* provides such a task. Pupils need time, experience and lots of talk about measuring shapes to work through to understanding that there is no simple relationship between area and perimeter. *Down on the farm* challenges pupils to investigate the relationship between area and perimeter further, and the task also provides consolidation in calculating areas and perimeters, as well as working systematically.

When learning about area, pupils usually start with counting: initially covering surfaces with unit squares and then creating areas on squared paper. However, the move to finding areas of rectangles using the formula has a problematic connection to counting squares.

Consider the rectangle shown in Figure 8.2.

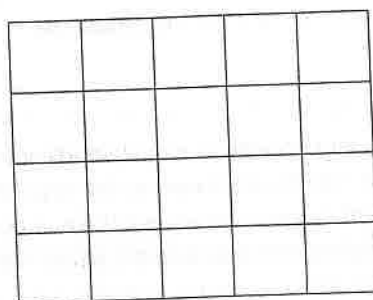


Figure 8.2

Counting the squares is straightforward and talking about there being four rows of five or five columns of four can make some links to multiplication. But compare this with the version of the rectangle shown in Figure 8.3.

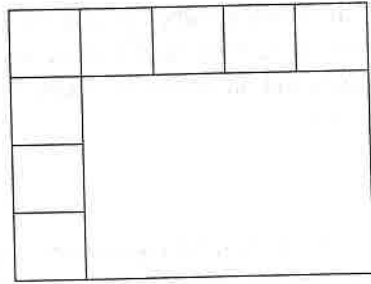


Figure 8.3

In moving from counting all the squares to using the limited information provided in the diagram, a shift has to occur in relation to attending to the square in the top left hand corner. When counting all the squares to find the area, this corner square must be counted once and once only, but in moving to use multiplication to find the area, this square must be counted twice: once to determine the number of squares in length of the rectangle and once again to find the number in the width. *Ripped and torn* is an activity that can help children move away from counting every square, while still supporting the underlying idea of area being about covering a surface with squares and supporting the shift in understanding from thinking of squares to considering lengths of sides.

Given that calculating the area of rectangles is, once understood, relatively easy to do, it is sensible to find ways of using rectangles to calculate the areas of non-rectilinear shapes. *Into rectangles* explores ways of doing this.

STANDING UP AND LYING DOWN

Foundational inquiry.

Stages

Key Stage 1.

Direct object of learning

Compare, describe and solve practical problems involving lengths and heights.

Indirect object of learning

Reasoning: understand that quantity is conserved when switching being between measuring height and length.

Tools

Long strips of paper around 5 cm wide (till-rolls are ideal). Chalk or post-its to mark positions on the wall or floor. Blu-tak.

Task

Public talk

Show how, working in pairs, pupils can measure each other's heights: one learner stands against the wall and their partner marks their height with a book or ruler. The first learner steps away and their height is recorded with a chalk line or a post-it with an arrow on it. They swap roles and record their partner's height.

Private conversation

As the pupils are finding and marking each other's heights, engage them in conversation using appropriate language such as taller than, shorter than. Help them cut paper strips that match their heights - they label these in whatever way they wish.

Get pairs then to find out how 'long' they each are when lying down: feet against the wall, they mark where their heads end on the floor. They check, using their height strips, how their lying down 'lengths' compare with their heights. If there is any difference, talk about why this might be so (and so in a simple way introduce the idea that measuring can involve errors).

Assessment

- Can pupils explain the relationship between how tall they are and the length of the paper strips they cut?
- Can they use the paper strips to put three or more pupils in height order and then check if they were correct by pupils lining up?

Public talk

Have a dialogue about what the pupils noticed. Why is their height the same as their length when lying down?

Get around six children to bring up their height strips - can the class help them to sort their strips into order, shortest to longest? Attach the strips to the wall with the bottom end of each on the ground. Get the children to line up in the same order as the strips - are they in height order? Why is that so?

Variation

Pupils cut strips the same length as, say, their foot or the distance from their middle finger tip to their elbow. They find objects around the room that are longer than or shorter than this strip, recording these appropriately.

Source

Adapted from an activity in Biggs and Shaw (1995) *Mathematics Alive!*

**SECRET RIBBONS**

Foundational inquiry.

Stages

Key Stage 1.

Direct object of learning

Choose and use appropriate standard units to estimate, measure, compare and order lengths.

Indirect object of learning

Reasoning: appreciate the need for standard units of measurement.

Tools

Metre rulers, elastic bands.

Task

As a lead into the main activity, set up some simple activities involving direct comparison, for example comparing two shoelaces.

Experience of comparisons that cannot be done directly also help, for example, pupils can draw around their foot and their closed hand, and are asked:

'If an ant crawled around the edge of your foot and another ant crawled around the outline of your hand, which ant would crawl further?'

Pupils can cut lengths of thick cotton (or fine string), dampened to help it stay in place, matching the distances around each outline and so find which is the longer by comparing the two lengths of cotton.

Public talk

Set up two 'secret' ribbons in boxes on opposite sides of the classroom. Tell the pupils that there are two ribbons, one in each box and, without moving the ribbon away from where it is in the room, they have to find out which hidden ribbon is the longer.

Private conversation

If the ribbons are close enough in length for the difference not to be discerned simply by looking, pupils can be introduced to the use of a tool that is longer than each ribbon: a metre ruler (pupils who have not been formally introduced to the standard metre ruler can ignore the marks). They also need elastic bands. Lining up one of the ribbons along the stick they mark the length of it by rolling an elastic band onto the ruler and matching it up with the end of the ribbon, repeating this with the other ribbon. The two elastic bands provide the visual evidence for the longer ribbon but are the basis of beginning to work with and understand scales.

Assessment

- Do the pupils measure from the same end of the ruler each time?
- Does anyone know anything about the numbers on the ruler?

Public talk

Look at the metre rulers and discuss how the pupils decided which ribbon was the longer.

Draw their attention to the centimetre markings on the ruler. Have a dialogue about whether they could use these markings to compare the two ribbons if they did not have any elastic bands to mark the lengths.

Talk about whether it makes any difference which end of the ruler they measure from.

Set the pupils off to use the rulers to see which was the longer - the length of cotton that was the distance around their hand or the one around their foot and to record what they find in some way.

Variation

Similar challenges could involve finding the taller of two towers of bricks, one on the floor and one on a table.

IDEAL GNOME HOMES

Foundational inquiry.

Stages

Lower Key Stage 2.

Direct object of learning

Measure the perimeter of simple 2D shapes.

Indirect object of learning

Reasoning: explore how the perimeter of a 2D shape can change while the area stays constant.



Tools

Plastic or strong cardboard squares around 3 cm square.

Task*Public talk*

Set up the problem - it is a fantasy context where pupils are asked to imagine that gnomes all like their houses to be the same size (in terms of the area they cover) but also like them to look different.

Introduce the problem:

A community of gnomes are building houses to live in. Each house has a 'footprint' made up by fitting four small square tiles together and then building walls around the space enclosed.

The challenge is to investigate the different houses that can be made from putting together four tiles and whether they all need the same amount of material for building walls. All buildings are one storey high.

Model how the tiles can be arranged in different configurations (the tiles need to be arranged so that their sides line up, as shown in Figure 8.4) and how the side of one tile is the 'unit' to be used for measuring the distance around the outside. If anyone introduces the term perimeter then pick up and use it; otherwise, the term can be introduced at the end of the lesson when having a dialogue about what the learners have found.

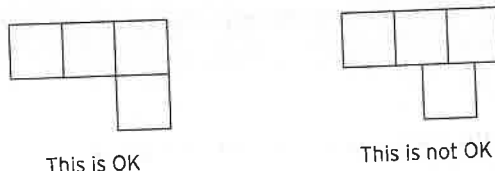


Figure 8.4

Private conversation

Set the pupils off in pairs to work on the challenge - they can draw around the different arrangements of the four tiles and record the amount of walling needed in whatever fashion they choose.

As the pupils are working, look to see who is working systematically to make sure they have found all the different arrangements of the four tiles - prepare these pupils for sharing what they did with the rest of the class. Excluding rotations and reflections there are essentially four different possible arrangements.

Assessment

- Do pupils think that the different arrangements of four tiles all provide the same amount of space to live in? Can they explain why?
- Can they suggest any ways to check that they have found all possible arrangements of the four tiles?

Public talk

Invite pupils to share what they have found. Is anyone confident that they found all the different ways to arrange the four squares? Introduce into the conversation, if not already present, the language of perimeter.

What can the pupils say about the perimeters of the different shapes?

Variation

Extend to five squares.

Provide centimetre squared paper and get pupils to create their initials (or a short name) out of exactly ten squares for each letter. What is the perimeter of each letter?

**RIPPED AND TORN**

Foundational inquiry.

Stages

Lower Key Stage 2.

Direct object of learning

Find the area of rectilinear shapes by counting squares.

Indirect object of learning

Reasoning: find areas when not all squares are visible.

Tools

Squared paper.

Task*Public talk*

Have prepared some demonstration rectangles made up of, say, 4 cm $\times$ 4 cm squares. Hold these up (or display on the IWB) long enough for children to be able to count the number of squares in each row and column, but not long enough for them to be able to count every individual square.

Encourage pupils to use the language of multiplication in sharing their answers with the class:

'I could see that it was 6 squares across and 5 down, so there must have been 6 times 5 squares. That's 30 squares altogether.'

Have some rectangles prepared with a corner torn off. Hold them up or display them one at a time and invite pupils to talk in pairs and decide how many squares were in the rectangle before it got torn (see Figure 8.5).

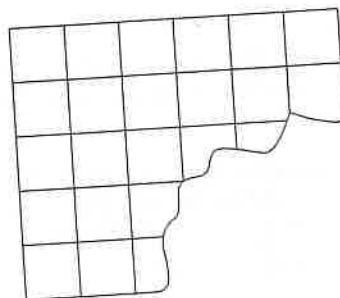


Figure 8.5

Have prepared some rectangles with more than one piece torn off where it is not immediately obvious how many squares there are in each row and column (see Figure 8.6).

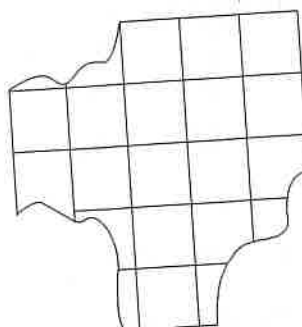


Figure 8.6

How do the pupils now figure out how many squares there were originally?

Also have some rectangles where so much has been torn off that finding a unique answer is not possible (see Figure 8.7).

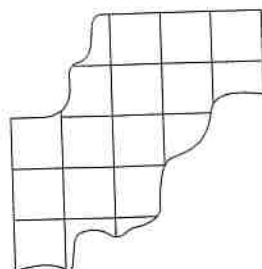


Figure 8.7

What are some possible answers?

Private conversation

Working in pairs or small groups have the learners prepare *Ripped and torn* puzzles for each other to solve. They cut out rectangles from squared paper, note down the dimensions of the rectangle (say, 4×7) and the area (28 square units). They rip pieces from the rectangle and swap with a partner. Can their partner write down the dimensions and find the area of the original rectangle?

Assessment

- Can anyone see a relationship between the lengths and breadths recorded and the area of each rectangle?
- Were any of the puzzles created impossible to solve? Why?

Public talk

Bringing the class back together, put up a selection of the data in a table and discuss the relationship between the numbers

Side 1 (squares)	Side 2 (squares)	Area (square units)
2	6	12
5	5	25
3	8	24

Work on articulating the rule for finding the area given the lengths of two sides.

Variation

Provide learners with simple shapes on squared paper made up of two rectilinear shapes joined along a common edge (see Figure 8.8). They find the area of the overall shape.

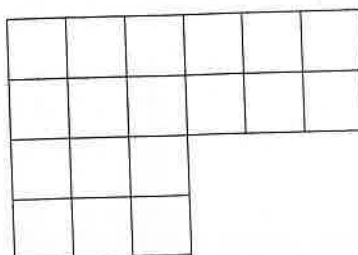


Figure 8.8



DOWN ON THE FARM

Foundational inquiry.

Stages

Upper Key Stage 2.

Direct object of learning

Measure and calculate the perimeter of composite rectilinear shapes and calculating the area of rectangles (including squares).

Indirect object of learning

Reasoning: explore the relationship between perimeter and area.

Tools

Alongside pencils and rulers (including some metre rules), centimetre squared paper is the only thing needed, although some pupils may benefit from having square tiles to move around.

Task

Public talk

Set up the context by talking about farms and how some have an enclosure (pen) where children can go and see animals close up. Talk about a farmer, Jenny say, who was setting up such pens and she wanted each pen to have a total area of 16 square metres. Use metre rules to mark out a square metre on the floor and talk about what a total of 16 of these might look like.

(As my work is usually in urban schools I set the context of the lesson within a city farm. Obviously children in rural schools will be more familiar with farms proper.)

Jenny thinks it would be interesting to have the animals' pens different shapes. She wants each one to be a rectangle but was wondering if 16 square metres can be arranged to create different rectangles. Turn to your partner. How could 16 squares be arranged to create a rectangle?

Take suggestions from the pupils and note these on the board. Invite pupils up to make quick sketch of each suggestion to check that it does have an area of 16 squares. Discuss whether, say, a rectangle 8 squares long and 2 squares wide is different from one that is 2 long by 8 wide. Establish that these are essentially the same rectangle. Work on listing all the solutions (there are only three) in an organised fashion.

$$1 \times 16$$

$$2 \times 8$$

$$4 \times 4$$

Explain that as well as wanting to build some pens with an area of 16 square metres, Jenny also wanted to have some with areas of 24 square metres. Working in pairs, can the pupils find all the different arrangements of 24 squares to make rectangular shaped pens?

Private conversation

As the pupils begin to generate examples, direct their attention back to the board. Explain that Jenny is going to put a wire fence around the outside of the pens and wants to know how many metres of fencing will be needed for each pen.

Look again at these three pens that all have an area of 16 square metres. Talk to your partner. Will Jenny use the same length of chicken wire for putting a fence up around each one?

Discuss whether the pupils think the 16 square metre pens will all need the same length of fencing or whether different pens require different lengths. Work on finding the perimeter of the 1×16 and 4×4 , recording the lengths of all four sides to make the overall calculation clear.

Pupils continue to look at the possible rectangles with an area of 24 square metres and also the perimeters of these.

Assessment

- Are pupils being systematic in listing all the possible rectangles?
- Can pupils calculate the areas of the pens they create or are they still counting squares?

Public talk

At the end of the lesson gather all the information and order it into a table:

Rectangle	Area in square metres	Perimeter in metres
1×24	24	50
2×12	24	28
3×8	24	22
4×6	24	20

What do you notice about the areas of all the rectangles? What about the perimeters? Why does the 4×6 rectangle need so much less fencing than the 1×24 ? Talk to your partner.

Discuss the fact that it is possible for shapes to have the same area but to have different perimeters.

Variation

Change the problem to areas with a fixed perimeter:

Jenny has been given some rolls of chicken wire. Each roll is 12 m long so she has decided to make some rectangular pens that need exactly 12 m of chicken wire to fence them in. Talk to your partner and see if you can suggest a rectangle that Jenny can make.

Into rectangles

Extension challenge.

Stages

Upper Key Stage 2.

Direct object of learning

Recognise that shapes with the same areas can have different perimeters and calculate the area of parallelograms.

Indirect object of learning

Reasoning: relate areas of simple non-rectilinear shapes to areas of rectangles.

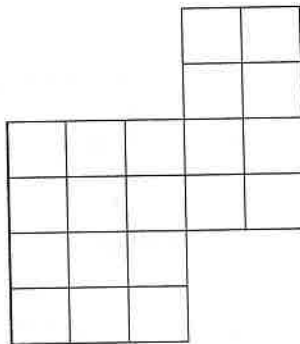
Tools

Centimetre dot squared paper, rulers.

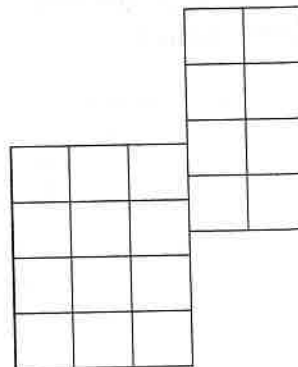
Task

Public talk

Model how to put two rectangles together to create a new shape that is not a rectangle. The new shape has to have at least part of a side of each rectangle fitted together for a whole number of squares: Figure 8.9 shows examples of what is acceptable and what is not.



This is OK



This is not OK

Figure 8.9

Private conversation

Working in pairs, each pupil cuts out a rectangle from squared paper. They can choose the dimensions, but to make things manageable you might want to specify that each side of the rectangle should be no longer than ten units.

They put their two rectangles together to create a non-rectangular shape like the ones they have just seen modelled.

Pairs record the shape on centimetre squared or dot squared paper, only drawing the outline and not indicating where the two rectangles are joined. They swap this with another pair. They find the area of the shape they have been given by dividing it up into rectangles. Can they do this in at least two different ways? (Even though the shape they were given was originally made from two rectangles, they can divide it up into three or more.)

Public talk

Share some examples with the class.

Private conversation

Working in pairs, learners draw a parallelogram on the centimetre dot paper – the corners of the parallelogram have to be at dots on the paper. They find, using whatever method they like, the area of the parallelogram.

They cut out the parallelogram and decide how and where to cut it, with a single cut, so that the two pieces can be put back together to create a rectangle. They record the dimensions of the parallelogram (lengths of sides, area) and the related rectangle (lengths of sides, area). They do this for two or three different parallelograms.

Assessment

- Can pupils explain why the area of a parallelogram can be made equal to the area of a rectangle?
- Can they say what they need to know about the parallelogram in order to be able to find the lengths of the sides of the rectangle with the same area?

Public talk

Talk about what they did and noticed.

Draw up a table of results of the inquiry into parallelograms and turning these into rectangles. Are there any patterns that the learners notice? Once you have established that the areas of the parallelograms and the rectangles are the same, look at the dimensions of the rectangle and how these relate to that of the parallelogram. Explore how making a vertical cut across the parallelogram means you can always make a rectangle with one side the same length as the original parallelogram and the other side the height of the parallelogram (see Figure 8.10). Can learners use this construction to express a rule for finding the area of a parallelogram without actually doing the cutting?

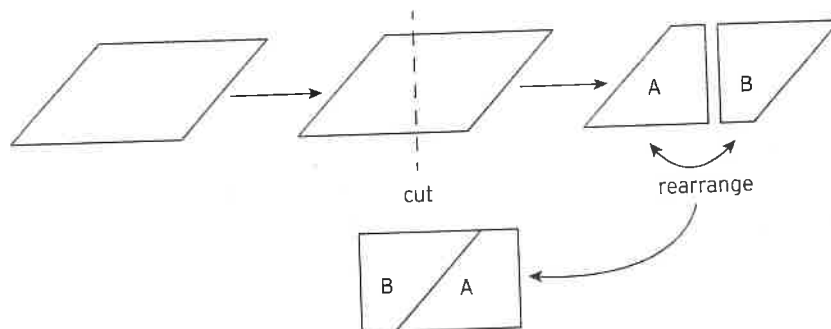


Figure 8.10

Variations

Work on finding areas of triangles in a similar way. In this case, rather than cutting up triangles and trying to rearrange them to make rectangles, work on enclosing the triangle in a 'snug' rectangle. What is the relationship between the area of the rectangle and the area of the triangle? What do they need to know about the triangle in order to be able to find the lengths of the sides of the rectangle?

**PROGRESSION IN MEASURING MASS AND WEIGHT****A note on terminology: mass or weight?**

Mass is the measure of the amount of 'matter' in an object, whereas weight is a measure of the effect of gravity on an object. A 1 kg weight, on the moon, will no longer 'weigh' 1 kg although its mass will be unaltered. Some purists argue that we are really teaching about mass. I don't think the more everyday language of weight lays down any problems for later understanding of this distinction and would use the terms interchangeably, as I do in the following.

Directly comparing weight

Lengths can be compared directly simply by putting things side-by-side: pupils have little difficulty knowing who is the taller of a pair of children or which is the longer ribbon, as they can see the difference. Comparing masses is more challenging, in that bodily sensations arising from the actions of informal comparing can be misleading. Judging which is the heavier by looking at the size of objects, or being a human 'balance' and comparing the 'feel' of an object in each hand does not always lead to a correct comparison. The large parcel might look as though it should be heavier than the small one, but place one on each of your upturned hands, and the pressure in the palm of your hand from the small parcel may make it feel heavier than the large one. Only balance scales can really tell us which perception is correct, so these need to be around in the classroom to supplement bodily experiences. It is helpful to have a collection of sealed, wrapped, identically sized containers with different amounts of clay or dried beans in them, so that pupils can act as human balance scales to decide which of a pair of these containers they think is heavier before testing out their choice on balance scales.

Balance scales can only compare the weight of objects pair wise: ordering five or six objects by 'feel' of weight and then checking the order using the scales is a challenging problem to solve. *Heavier or lighter* provides a practical context for exploring this.

Indirectly comparing weight

Using non-standard units to measure the weight of objects requires multiple units to be used, which is subtly different from what happens when measuring length. I can find the length of, say, a rug by repeatedly using only one pencil. I cannot, however, weigh a box using only one pebble or a single 20 g weight - I must have several. Although ultimately having an extensive collection of weights is replaced by using scales, weighing tools with vertical or circular scales provide no visible sense of the 'weights' making up the measure. Understanding here is helped by getting pupils to make an elastic band (spring) weighing 'machine'. Cutting about 12 cm off the bottom of a juice carton provides a suitable 'pan': punch a hole midway along each side, thread four knotted strings through and join them together about 20 cm above the pan. This can then be attached to a large rubber band or length of elastic and suspended from a hook, in front of a sheet of paper. Learners mark the position of the empty pan - either its top or bottom edge and its position with objects placed in the pan (making sure that they are consistent in where they are measuring from) to create a spring balance scale.

Using formal units

Given plenty of experience of non-formal units and creating scales to measure weight, pupils will come to understand that measurement is approximate, even when using formal units. As they seek out smaller units to make their measures more accurate it is helpful to discuss that although the measuring becomes more precise even at the level of grams it can never be completely accurate: however small our unit, there is always a possible error of half a unit either under or over.

Being able to read scales with increasing accuracy is an important skill but it needs to be supplemented with good estimation skills. Pupils often come to believe, largely through their experiences in school, that estimating is of lesser importance than actually measuring, yet in our daily lives we estimate quantities far more often than we measure them. *Weight scavenger hunt* introduces the idea of using benchmark masses to make reasonable estimates. Share with the pupils typical benchmarks such as a 250 g pack of butter, 1 kg bag of sugar. Encourage the children to look out for other everyday objects that can be added to a poster of favourite benchmark quantities.

Make weights provides a foundational inquiry for exploring how sets of weights for balance pans are constructed. Converting between the different metric weights is put in the context of problem solving in *Victoria's sponge*, which is a classic problem where the unknown (the weight of one egg) is at the start of the problem, rather than at the end. So you have to flip things and work backwards from the known end result (the weight of one pan of cake mixture) reversing the steps (doubling that to get the weight of the total mix, and so on). The power of working on such problems is not so much in finding the solution but drawing pupils' attention to the strategy of working backwards.

Exploring the connection between metric measures and imperial measures is dealt with in *Vegetable market*. Some learners will find pleasure in investigating the links between the different metric measures - how in the metric system a cubic centimetre of water has a mass of 1 g (at a particular temperature and pressure). They could research the 'neatness' of the metric system compared to the imperial system of pints and fluid ounces and pounds and ounces.

Finally *Strong as an ant* brings multiplicative reasoning into focus through an intriguing context that extends understanding of mass.

HEAVIER OR LIGHTER?

Foundational inquiry.

Stage

Key Stage 1.



Direct object of learning

Compare, describe and solve practical problems for mass/weight and use the language of heavy/light, heavier than, lighter than.

Indirect object of learning

Problem solving: solve practical problems.

Tools

A collection of sealed, wrapped containers or boxes of different dimensions containing different amounts of clay or dried beans. Balance scales.

Task

Public talk

Take a pair of the wrapped containers, show them to the class and invite the pupils to say which they think is going to be the heavier. Pass the containers round the class, Which one of the pair do pupils now think is the heavier? Invite learners to share with the class why they think one is heavier than the other. Did anyone change her or his mind after holding the packages? Why?

Use a pair of balance scales to check which is the heavier object.

Private conversation

From the collection of containers label one as the 'target' weight: groups sort the collection into two sets, those containers they think are heavier than the target and those they think are lighter. They do this by using only their bodies to decide - they do not use balance scales to check. They record this sorting in whatever way they choose.

They then re-sort the objects by using scales and comparing each object to the 'target' mass. How does this fit with their first sorting?

They collect objects from around the room that they think are either lighter or heavier than the target object. Do they get better with practice?

Assessment

- When comparing the mass of objects only using their bodies, do pupils take into consideration how an object 'feels' as well as how it looks?
- If pupils can use the balance scales to compare two objects, can they use the scales to put three objects in order from heaviest to lightest?

Public talk

Talk about what the pupils did. Were there any objects that were tricky to decide if they were heavier or lighter? What made them tricky?

Variation

Provide learners with a 1 kg or 500 g weight as the object to make the comparison against.



WEIGHT SCAVENGER HUNT

Practice and consolidation.

Stages

Key Stage 1.

Direct object of learning

Use appropriate standard units to estimate and measure mass (kg/g) and compare and order mass.

Indirect object of learning

Reasoning: develop a sense of 'benchmark' weights.

Tools

A selection of everyday items that can act as 'benchmark' weights: a 500 g bag of pasta, a 250 g tub of spread (use a recycled tub filled with rice or plasticene to make the weight up to 250 g), a 1 kg bag of sugar. Standard 250 g, 500 g and 1 kg weights. Balance and kitchen scales.

Task*Public talk*

Introduce the pupils to the idea of having 'benchmark' items: items that help you make an estimate about the weight of things when you cannot actually weigh them. Pass around the 1 kg bag of sugar and the 1 kg weight and discuss whether these feel the same weight. It is quite likely that some pupils think they do not - the smaller, more compact 1 kg can feel heavier because it applies more pressure to the hand. Use the balance scales to check that they do weigh the same. Repeat this with the 500 g object and weight and the 250 g pair.

Private conversation

Set the pupils off to find, in pairs, collections of sets of three objects for each of the 1 kg, 500 g and 250 g weights: something they think is a little bit lighter than the weight, something they think weighs almost exactly the same and something they think is a bit heavier.

Once they have collected their nine objects, they use balance scales to check if they were correct. They devise a way to record their findings.

Assessment

- Do pupils use a range of information - both size and 'feel' - to find their objects?
- Are they prepared to risk being wrong by choosing objects that feel close to the benchmark weights or do they play safe and choose objects that are much heavier or lighter?

Public talk

Have some pupils share their findings. How good were they at estimating whether something weighed almost 500 g or 1 kg? Get pupils to show how they used the balance scales to check.

Use the kitchen scales to weigh and record the masses of some of the items the pupils had collected. Over the remainder of the day send pupils off to use the scales to find the weights of their items and to add this to their records.

Variations

Learners can use rice or small pasta to weigh out 250 g and 500 g and other amounts that you specify.

Extension challenge.

Stages

Lower Key Stage 2.

Direct object of learning

Measure, compare, add and subtract mass.

Indirect object of learning

Problem solving: solve practical problems.

Tools

Plasticene or similar dough/clay material. Balance scales, weights totalling 400 g. Rice or small pasta.

Task

Public talk

Set up the challenge:

Explain that pairs are going to be given weights that total 400 g. They are to use this and the balance scale to measure out a 400 g ball of plasticene. They will then return the 400 g weight(s) as they will not use them again.

The challenge is to use their 400 g ball of plasticene to create balls of other weights: 200 g, 100 g, 50 g and 25 g.

Once they have made these different weights you will put a list up on the board of various masses. The learners have to use the weights that they created to measure out those masses of rice (or whatever material you have made available).

Private conversation

Watch as the learners set about creating a set of weights - they need to use the scales to halve the various masses. Encourage them to be as accurate as possible when halving the quantities.

Put the following list of quantities on the board:

75 g
175 g
225 g
350 g

Get the pupils to use their plasticene weights to measure out these quantities. They choose how they are going to record what they did. Can they find a different way to measure out the same quantity?

Assessment

- Do pupils realise that they might put plasticene weights on each side of the balance pan?
- Can they list three other masses that they could measure out using their weights?

Public talk

Invite pupils to share their recordings of how they measured out the various amounts. What different methods were used?

Variation

Learners could investigate the system of measures used in pounds and ounces: 1 oz, 2 oz, 4 oz, 8 oz, 16 oz. How did these weights allow every amount from 1 oz to 31 oz to be measured out? They could make a similar

set of weights by sealing marbles into small plastic bags, one-marble bag, two-marble bag, four-marble bag and so forth.



VICTORIA'S SPONGE

Extension challenge.

Stages

Lower Key Stage 2.

Direct object of learning

Convert between different units of measure, between grams and kilograms.

Indirect object of learning

Problem solving: work backwards.

Tools

Some children may find balance scales, weights and plasticene helpful. Others may be confident enough to simply work with the numbers.

Task

Public talk

Set up the challenge. Explain that the following recipe was passed down by Grandmother Victoria and it is how she was able to bake a sponge cake without using any formal weights. You (or someone you know) tried it and it worked.

Here is how she did it:

- Place two large eggs in one of the pans on a balance scale.
- Measure out into the other pan an amount of sugar that exactly balances the weight of the eggs.
- Also weigh out the same amounts (that is, two eggs' weight) of (soft) butter and the same of flour.
- Beat the sugar and butter together until light and fluffy. Gradually beat in the eggs. Finally stir in the flour.
- Split the mixture between two cake tins and bake.

Explain how you tried this recipe and just before putting the tins in the oven you weighed the contents of one of the tins and it was a quarter of a kilogram.

Can the pupils use this information to figure out the weight of one egg?

Set the pupils off in pairs to investigate.

Private conversation

There is a lot of information here for the learners to make sense of, and it is likely that they may initially feel the problem is overwhelming. In that case, encourage them to represent the problem using a picture or diagram - they might use a bar to represent the weight of one egg. From there it should then

be relatively easy to build up a diagrammatic representation of the problem and go on to find a solution (see Figure 8.11).

egg	egg
sugar	
butter	
flour	

Figure 8.11

You may also need to remind the children that there are 1,000 grams in a kilogram.

Assessment

- If stuck, can pupils start with a trial and improvement approach by guessing the weight of one egg?
- Can they create models of the situation using physical materials or pictures or diagrams?

Public talk

Go over the various solutions with the class. Which solution method do the learners like best? Why?

Variation

A similar problem that involves working backwards is:

Tulela started to read a story book on Saturday and read 15 pages. She read twice that number of pages on Sunday. She finished the book on Monday by reading the last 28 pages. How many pages were there in the whole book?

Source

Adapted from the Nrich problem 'Cherry Buns', available online at <http://nrich.maths.org/2015>.



VEGETABLE MARKET

Extension challenge.

Stages

Upper Key Stage 2.

Direct object of learning

Understand and use approximate equivalences between the metric units and common imperial units of kilograms and pounds.

Indirect object of learning

Reasoning: understand the relationship between different measuring scales.

Tools

Some learners may be helped by having some imperial and metric weights to handle and work with.

Task

Public talk

Talk about shopping for vegetables and the common amounts that people buy. For example, you might buy a 5 kg bag of potatoes, but would you buy that mass of, say, green beans? Elicit a list of common vegetables from the learners and record the typical weight of each that you might buy.

Talk about the imperial system of measures - what do the learners know about this? Introduce this approximate pair of conversion facts:

$$0.45 \text{ kg} = 1 \text{ lb}$$

$$2.20 \text{ lb} = 1 \text{ kg}$$

Talk about how, although they now have to sell things in grams and kilograms some people who run market stalls still think in terms of selling things in pounds (lbs). They can do this because they can calculate mentally what a mass in pound is in kilograms and vice versa.

Put up, or talk through this problem:

You get about 8 onions in a kilo.

So, approximately, what would 32 onions weigh in pounds (lbs)?

Give the pupils a little time to talk with a partner about what they think the answer is and why. Go over the solution on the board.

Private conversation

Put up these problems on the board for pupils to work on in pairs.

You get about three potatoes to the pound.

Roughly, how many potatoes might you get in a 5 kg bag?

About 8 beetroots weigh 1 kg.

About how many pounds would 28 beetroots weigh?

Medium carrots and parsnips weight about the same: you get around six carrots or six parsnips to 1 lb.

What would a bag of nine carrots and nine parsnips roughly weigh in kilograms?

As the pupils are working, note the different strategies they are using to solve the problem and select two or three pairs of learners to share what they have done with the rest of the class.

Learners who finish early can make up similar problems for others to solve.

Assessment

- Do the learners want to work with 'unit' weights (for example, finding the weight of one beetroot) or can they work with simple ratios?
- Are they using models like a t-table or double number line to help them find answers?

Public talk

Invite the pupils you identified to share their solution methods with the class.

Variation

Get children to investigate on the Internet the average weights of fruits and to create similar problems about fruits.

Source

Adapted from the Nrich problem 'Weighing Fruit', available online at <http://nrich.mathematics.org/6055>.



STRONG AS AN ANT?

Extension challenge.

Stages

Upper Key Stage 2.

Direct object of learning

Use and convert between standard units of mass from a smaller unit of measure to a larger unit and use decimal notation to up to three decimal places.

Indirect object of learning

Problem solving: connect different aspects of mathematics.

Tools

Calculators or spreadsheets.

Task

Public talk

Introduce the challenge by discussing the fact that ants are strong for their size. What would the pupils be able to lift if they were as strong as an ant? Give them a moment to chat with a partner and to come up with a list of things they think they would be able to lift if they had, relatively, the strength of an ant.

Then provide the following information:

An average ant weighs about 0.004 g.

An average ant can lift about 0.2 g.

Based on this information, do they want to change the list of things that they think they might be able to lift?

Write up on the board a list of some of the things the pupils think they would be able to lift if they were as strong as an ant.

Private conversation

Set the learners off to work in pairs and figure out what they would be able to lift if they were as strong as an average ant.

If some find this is too challenging, provide them with the information that some ants can carry 10 times their own body weight. It is even claimed that some ants can carry 50 times their own body weight.

Once the pupils have worked out the approximate weight that they could lift, they can use the Internet to investigate what sort of objects would weigh around that amount.

Assessment

- Can pupils work out the scale of the relationship between 0.004 g and 0.2 g?
- Are pupils clear that the relationship here is a multiplicative one - how many times more than bodyweight - rather than additive - how much more than bodyweight?

Public talk

Return to the list of objects that the pupils originally speculated that they might be able to lift if strong as an ant. Did they overestimate or underestimate what their strength would be?

Variation

What about jumping? If the average grasshopper is 2.5 cm long and can jump 50 cm, how far could the average child jump if they could jump as well as a grasshopper?

What other amazing feats can the children find out about minibeasts and convert to superhuman skills? Is what Spiderman purported to be able to do possible?

PROGRESSION IN MEASURING VOLUME OR CAPACITY

Capacity is the amount a container can hold, which is sometimes (confusingly) also referred to as the volume of the container. If a full bottle holds 250 ml of water, then the capacity of the bottle is 250 ml, and the volume of the water in the bottle is also 250 ml. The confusion comes from all solid objects having volume but not necessarily any capacity, so, strictly speaking the volume of a bottle is the amount of space occupied by the solid matter that makes up the bottle, rather than the amount that the bottle holds. And talk of the volume on the TV only makes things worse.

Direct comparison

A wide collection of empty plastic containers - drink and shampoo bottles, yogurt and cream cartons - provides most of the kit needed for direct comparison of capacity. As with mass, sight alone can be deceptive: which has the greater capacity, a tall thin bottle or a short, wide one may not be immediately obvious. Some 'flowing' material is needed to make the comparison - rice, dried beans or small pasta are less messy than sand or water. Cutting the top off a 2 l plastic drinks bottle and inverting it makes a funnel that can reduce spillages. Pupils can select a pair of containers, decide which they think has the greater capacity and check this. Do they fill what they think is the smaller bottle first, pour the contents into the larger and look for unfilled space, or vice versa and look for material left in the larger bottle?

Indirect comparison

In a sense, pupils are indirectly comparing capacities even when just ordering two containers as some sort of intermediate 'measure' that can be poured from container to container has to be introduced. Ordering three or more containers raises the level of challenge. Containers can only be compared in pairs so keeping track of the relationship between, say, four visually different bottles is far from trivial. It is interesting to simply pose this ordering as a challenge without any guidance as to how to do it and see what strategies pupils come up with.

Before introducing a smaller unit - egg-cups say - that needs to be counted to find the capacity, children benefit from pouring the contents of several different containers into larger, similar containers. A collection of 2-l plastic bottles, tops sliced off at the 'neck' provide a set of uniform containers that can be compared side-by-side. Pupils order a collection of different containers first by estimating what order they think they will go in and then filling each with dried goods (rice, sand or small pasta). Pouring the contents of each into several matching larger containers, pupils can then compare the level to which each is filled, thus setting the stage for reading scales on measuring jugs later.

When introducing the use of a repeated smaller unit, it is helpful to start off by filling up multiple copies of the unit, rather than having only one to fill and counting the number of times it is filled. Plastic egg cartons are perfect for this: cut one up to provide a collection of single 'units', and also have complete cartons to hand. Bottles can then be compared by filling them with rice, and then emptying the rice out by repeated filling a single 'egg-cup' unit and transferring each filled unit to a different section in a complete egg-tray. Pupils will then be able to see that, for example, one bottle held seven egg-cups, while the other held nine. Five empty bottles develops this task further.

Standard units

Make a measuring cylinder is a practical problem that helps learners to understand where the scale on a measuring cylinder comes from. Litre box takes this further and helps learners understand the relationship between cubic centimetres and millilitres.

Converting and using the relationship between millilitres and litres is explored through the problems presented in *Sports day*.

Imagining what very large volumes look like is not easy: *Olympic pool* provides a context for examining this. Finally, at the upper end of primary school, exploring the relationship between surface area and volume provides an opportunity to use spreadsheets to plot a graph and think about why it takes the shape that it does. There are also cross curricular links to be explored in terms of how animals' need to eat is related to their size: *Invasion of the giant ants* looks at this.



FIVE EMPTY BOTTLES

Foundational inquiry.

Stages

Key Stage 1.

Direct object of learning

Compare, describe and solve practical problems for capacity and volume.

Indirect object of learning

Reasoning: use a set amount to reason about the capacity of containers.

Tools

A collection of different empty bottles - soft drinks, washing-up liquid, shampoo and so forth. A small container to use as a non-standard unit, for example an egg-cup or small yogurt carton. Water (or rice or sand).

Task

Public talk

Display five empty bottles. Invite pupils to say which bottle they think would hold the most water. Which do they think would hold the least? What order between these two do they think the other bottles should be placed in? Label the bottles and record the order for later reference.

Private conversation

Working in pairs or small groups the pupils take turns to order the collection of five bottles. They do this in three ways. First, they agree on what order they think the bottles need to be placed in from the one holding the most to the one holding the least - this may be different from the order the class had decided upon. They record their chosen order.

They then fill each bottle with five units - say egg-cups - of rice. When each bottle has five units of rice in it, they check again to see if they still think the bottles are in order of capacity.

Finally, they empty the bottles and take it in turns to fill the bottles - one pupil uses the unit and repeatedly fills and pours rice in a bottle. Another pupil tallies the number of times the unit is filled. When the bottle is full, they count up and record the number of tallies. Everyone has a turn at filling and tallying.

Assessment

- Can pupils use appropriate language associated with capacity, for example, more than, less than, most, least?
- Are pupils thinking in three dimensions when deciding on the order of the bottles, not just looking at the height of them?

Public talk

Bring the class back together when every group has ordered the bottles. Was the original order that the class had agreed on correct? If it was, what had helped them to get the order correct? If it was not, why do they think they were originally wrong?

Variation

Have a selection of open topped cardboard boxes - make these 'sand-proof' by putting sticky tape along all the edges. Using sand or rice to fill the measure, repeat the activity.

MAKE A MEASURING CYLINDER

Practice and consolidation.

Stages

Key Stage 1.

Direct object of learning

Choose and use appropriate standard units to estimate and measure capacity.

Indirect object of learning

Reasoning: create and use scales.

Tools

Empty 1 1/2 or 2 l drinks bottles: cut the top off just at the neck where the bottle narrows - keep the cut-off part to invert and create a funnel. Masking tape, 100 ml measures, water. A kitchen measuring jug or measuring cylinder. A collection of clean empty bottles.

Task*Public talk*

With the class, look at and talk about the measuring jug. Has anyone seen one of these being used? What was it used for?

Explain that the pupils are going to make their own measuring jugs. Model sticking a strip of masking tape up the side of a top-cut-off drink bottle. Use the 100 ml measure to pour in 100 ml of water, marking and labelling the water level on the tape. Repeat, until nine units of 100 ml have been poured into the bottle. The last label was 900 ml - does anyone know what the next label will be?

Private conversation

The pupils should not need much guidance in creating their measuring cylinders. When they have completed them, ask them to use their measuring cylinders to find the capacity of some bottles.

Assessment

- Can the pupils use their measuring cylinders to decide which of two bottles holds the most?
- When the level of water in their measuring cylinder comes between two markers, can they make a reasonable estimate of the volume of water?



Public talk

Talk about how to read the scales on their measuring cylinders when the level of water does not exactly match a marker. Put a large version of the scale on the board and indicate various readings. What are sensible estimates?

Variation

Provide kitchen measuring jugs to measure various containers.

**LITRE BOX**

Practice and consolidation.

Stages

Lower Key Stage 2.

Direct object of learning

Measure, compare, add and subtract volume/capacity (l/ml).

Indirect object of learning

Problem solving: solve a practical problem and reason about relationships between measures.

Tools

Squared paper, rulers, scissors, tape, rice, sand or small pasta. Measuring cylinders or jugs. Centimetre cubes.

Task*Public talk*

Set up the practical problem. Working in pairs the pupils are to make, from squared paper, an open-topped box that is 10 cm × 10 cm × 10 cm. Explain that the box needs to be strong enough to be filled with rice (or whatever dry pour material is to hand), so they will also have to decide how to make the box strong enough for that purpose.

Private conversation

As the pupils are working on constructing their boxes, ask them to think about how many centimetre cubes they think it will hold.

Public talk

Have to hand a measuring cylinder containing 1 l of rice.

Take one of the boxes made (a sturdy one!) and show this alongside the litre of rice. In their pairs, pupils decide whether they think the rice, when poured into the box, will be less than a full box, more or about the same. Record the numbers selecting each option. Pour the rice into the box - it should just about fill it.

Having thus established that the box has a capacity of 1 l, work on calculating the volume of the box in cubic centimetres (or centimetres cubed). You may need to model how there will be 100 cubic centimetres in one layer, so given that there are ten layers, the box must hold 1,000 cubic centimetres. What do the pupils know about the number of millilitres in a litre? What can they conclude about the relationship between millilitres and cubic centimetres?

Private talk

Put up a number of quantities on the board such as:

- 500 cubic centimetres
- 250 millilitres
- $1\frac{1}{4}$ litres.

Back in their pairs, pupils use their boxes to measure out these amounts of rice into other containers.

Assessment

- Can pupils confidently convert between cubic centimetres and millilitres?
- Can they use the relationship between litres and millilitres to convert fractions of litres to centimetres cubed?

Variations

Challenge pupils to make boxes that would hold 100 or 500 cubic centimetres.

SPORTS DAY

Practice and consolidation.

Stages

Lower Key Stage 2.

Direct object of learning

Convert between different units of measure, millilitres to litres and vice versa.

Indirect object of learning

Problem solving: relate volume and capacity to ideas of scale.

Tools

Measuring cylinders and empty containers.

Task**Public talk**

Set up the context: the problems are about providing refreshments for sports day. Have these problems available to show on the IWB or on paper (if on paper provide these on separate slips, so that pupils can choose the order in which they want to do the problems).

To make up drinks for the school sports day, you mix 20 ml of squash for 180 ml of water. If a beaker holds 250 ml of drink, how many beakers can be filled from 1 l of squash? If you buy 5 l of squash, how many litres of drink will that make?

A 1 l jug of water will serve eight cups of coffee. How many millilitres are in each cup of coffee? If people add 5 ml of milk to a cup of coffee, how many cups of coffee can be given milk from half a litre of milk?

Private conversation

If pupils are having difficulty, encourage them to use practical equipment to try out some solution methods.

After they have had some time to work on the problems, get pairs to join up with other pairs and share what they have done so far.

Assessment

- Can learners scale up from finding the answer for, say 1 l, to finding it for 5 l?
- Can they record their working clearly enough for someone else to follow it?

Public talk

Invite selected pairs to share their solution methods with the class. Who can re-explain, in their own words, the method of solution that they have just heard?

Variation

Challenge the pupils to make up similar problems for others to work on.

**OLYMPIC POOL**

Extension challenge.

Stages

Upper Key Stage 2.

Direct object of learning

Estimate and calculate volume.

Indirect object of learning

Reasoning: make sense of a large volume and how measures given in metres can be converted to cubic metres and then into litres.

Tools

Internet access.

Task*Public talk*

Talk about swimming pools. Does anyone know how long an Olympic pool is? How wide do they think it is? How deep is a swimming pool at the shallow end? At the deep end? Record the various estimates on the board and draw up a sketch of a pool, labelled with appropriate estimates.

How many litres of water do pupils estimate it would take to fill an Olympic swimming pool?

Note down the various suggestions and agree on the upper and lower bounds that the pupils think the answer is going to lie between.

Tell the pupils they are going to use the Internet to check the dimensions of an Olympic-size swimming pool and the relationship between cubic metres and litres. Once they have found this information they need to figure out the answer to the problem.

Private conversation

Part of the challenge here is that a swimming pool does not have a flat base. If some pupils are struggling to get started, suggest that they imagine that the swimming pool is, say, 2 m deep for its entire length.

There are at least two ways that pupils might deal with working with a sloped base. One is recognising that a pool is a very large prism, calculating the area of the vertical cross section (a trapezium) and then using that information to find the volume of the prism. Another approach is to look at a diagram of the cross section and think about how that could be theoretically cut up and rearranged to make the cross section a rectangle, and so making the pool equivalent to a cuboid (see Figure 8.12).

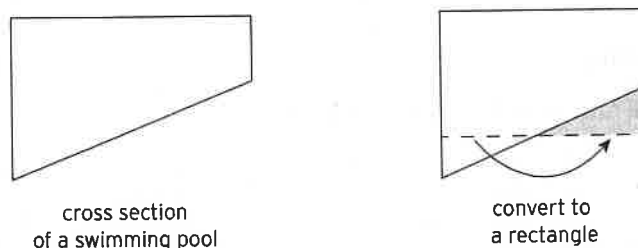


Figure 8.12

The answer that pupils are going to come up with is going to be in cubic metres, so the second challenge is how to convert that into litres. They might figure out how many litres are in a cubic metre, or they might convert cubic metres to cubic centimetres and use the fact that a cubic centimetre is equivalent to a millilitre.

The main teaching challenge here is to encourage the learners to persevere with the challenge and not provide too much guidance so that the thinking is taken away from the learners.

Assessment

- Can pupils set out their workings clearly so that someone else can follow them?
- Can they talk about the relationship between volume of water and the capacity of the pool?

As the pupils are working, select two or three pairs or groups who you will ask to share their workings with the class. These may not be the most sophisticated solutions, but ones that you think will provoke a rich mathematical conversation. Alert the pupils to the fact that you will be calling on them.

Public talk

Invite the pupils you had identified to share their working with the class.

Talk about the use of volume (of water) and capacity (of the pool). In what ways are these similar? In what ways are they different?

Variation

Pose some Fermi-type problems:

- If you were going to fill the pool one litre-jug of water at a time, walking to and from a tap, how long would it take to fill the pool?
- If you used the water in the pool to fill a household bath, and you bathe seven days a week, how long would the water in the pool provide baths?

Source

Adapted from an activity in Bobis, Mulligan and Lowrie (2013).



INVASION OF THE GIANT ANTS

Extension challenge.

Stages

Upper Key Stage 2.

Direct object of learning

Explore the relationship between volume and surface area.

Indirect object of learning

Reasoning: investigate the ratio of surface area to volume.

Tools

Interlocking cubes, spreadsheets.

Task

Public talk

Set the context of the giant bugs beloved of science fiction writers. Who has read books involving such creatures or seen movies with them in? Explain that by exploring the relationship between surface area and volume pupils are going to investigate whether such giant bugs could ever exist for real.

Private talk

Pupils need a number of cubes, preferably interlocking. Starting with a single cube, taking the length of its side as one unit they calculate the surface area and volume of the cube. They then construct a $2 \times 2 \times 2$ cube and find the same measures. They record the results in a table:

Length of side of cube	Surface area	Volume
1	6	1
2	24	8
3	54	27
4		

They can plot two graphs: one each the length of the side of the cube is along the x-axis and two different scales up the y-axis - one for the surface area and one for the volume. Doing this on a spreadsheet allows for playing with the data and including fractional lengths of sides. They then superimpose one graph on top of the other. Although these need two different scales on the y-axis - square units and cubed units - the same size divisions on the scale can be used for each. That is, a step of one unit squared can be the same as a step for one unit cubed.

Public talk

Talk about the graphs - for small cubes the number of units of surface area is greater than the number of units of volume but there comes a point where this crosses over and the number of units of volume is greater than the number of units of area.

What does this have to do with giant bugs? Small creatures can take in oxygen through their skin but for larger creatures the ratio of skin surface to volume is not great enough to let in sufficient oxygen. Hence the need for lungs. As insects have no lungs, those large sci-fi spiders would not take in enough oxygen to survive.

Variation

Pupils could investigate what this also has to do with small children needing to be wrapped up warmly in cold weather, and why mice need to eat almost constantly whereas elephants do not even need to eat every day.

Teaching time

There are two distinct aspects to learning about time: telling the time and elapsed time. Time is a measure but it is markedly different from those discussed earlier and although it is included in the mathematics curriculum, I think a cross-curricular approach to teaching time is best. History, science, PE and other parts of the curriculum present plenty of opportunities for exploring different aspects of time. Telling the time, in particular, I think, is a skill that we need to help pupils develop over time (sorry about the pun). The class clock is the best resource – referring to this across the days and weeks puts telling the time firmly into context. Worksheets on telling the time are really only exercises in reading dials and the fact that a clock showing ten past three can instantly turn into one showing five to six does not help pupils develop a sense of elapsed time. But for completion's sake, here are some tasks for teaching about time.

KEY STAGE 1 TASKS

Silly stories

Talking about and putting in order pictures of everyday activities provides pupils with the opportunity to develop the language associated with time.

Prepare some simple blank 'comic strips' of four or five boxes.

Talk with the children about everyday events such as what they do each evening when they get home from school or what they do on a Saturday morning. Pupils individually choose something that they regularly do that they record in a comic strip form. Help them to describe their event in four or five clear steps, for example, getting ready in the morning: getting out of bed, getting washed, getting dressed, eating breakfast. They draw the comic strip of their event.

When they have finished, the pupils cut up their comic strip into the separate boxes. Taking it in turns, they mix up the illustrations and, turning them over one at a time, tell the 'silly' version of their event: this morning I ate my breakfast, got washed, got dressed and then got out of bed. Everyone in the group helps put the pictures back in a suitable order, using the appropriate language of time: before, after, first, second, next and so forth.

My time line

Pupils can think about elapsed time over a longer period by making a time line showing their personal history.

In preparation for making the time lines, encourage pupils to bring in photographs of themselves and family events. Talk about what has happened in their lives since they were born and get them to draw pictures of those events that they don't have photos of.

Prepare strips of paper about 1 cm × 15 cm with a track of eight boxes (each about 12 cm × 12 cm) on it.

Work with the pupils on labeling the box at the far right with the current year and then working back to label the previous years. Which year were they born in? They put their picture of themselves in that box. What other events have happened? When did they start school? Who had a brother or sister born? How old were they when they learned to read? Help the children flesh out their time lines with details and to write these in or make illustrations.

LOWER KEY STAGE 2 TASKS

One hand clock

The two hands on the clock serve different purposes: the hour hand broadly indicates the approximate time and the minute hand adds precision. In introducing pupils to telling the time it helps to start with a clock with only the hour hand.

Remove the minute hand from a class demonstration clock and discuss telling the time with only the hour hand. As well as positioning the hand to point exactly to an hour, position it so that the pupils can talk about 'just before' and 'just after' the hour. What does it mean when the hour hand is about half way between the three and the four?

Get a cheap analogue clock with exposed hands and take (or break!) off the minute hand. Use this throughout the day to talk about the approximate time.

When the class are familiar with this, provide individual practice by preparing a page of clock faces with only the hour hand drawn in. They record the approximate times using appropriate terms.

Body clock

Use simple activities in PE to help pupils develop a sense of the duration of 30 seconds or a minute. For example, throwing in the air and catching a bean bag for a minute, running on the spot for 30 seconds.

You can also use this time to help pupils explore the beginning of inverse proportion. For example, stride swiftly across the hall and ask a learner to do it at the same time and to keep up with you so that you both start and finish together. Because the pupil will need to take more steps than you they may think that they were 'quicker' at crossing the hall than you were. Discuss how, since you started and ended together, you both took the same amount of time, but the shorter the size of a person's stride, the more steps they need to make to cover the same distance.



UPPER KEY STAGE 2 ACTIVITIES

Cooking problems

Prepare a series of problems involving time elapsed in cooking. Vary these between:

- problems where the elapsed time has to be found:
Mike put a cake in the oven at 15.55 and took it out at 16.35. How long was the cake in the oven for?
- problems where the finished time has to be found:
Helen puts a stew in the oven at 11.15. If it has to cook for 90 minutes, what time will she take it out?
- problems where the start time has to be found:
Tim wants to have supper at 7.55. If he needs to cook a chicken for an hour and ten minutes, what time does Tim need to put it in the oven?

Rather than showing the pupils one way of calculating the answers to elapsed time problems, encourage them to sketch and use a 'empty time line' to figure out the answers.

Pair of timers

Provide pairs or groups of pupils with a three-minute and a five-minute sand timer. There are some times that can obviously be measured using these two, for example, six minutes, or eight minutes.

How can the pair of timers be used to measure four minutes? Or seven minutes?

Is there any number of minutes that cannot be measured out using this pair of timers?