

Q1) $A = \begin{pmatrix} 2 & -5 & 3 \\ -5 & 0 & 5 \\ 3 & 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} -5 & 8 & 0 \\ 2 & 6 & 2 \\ 0 & -2 & 3 \end{pmatrix}$ $C = \begin{pmatrix} 12 & 5 \\ -3 & 2 \\ 2 & 5 \end{pmatrix}$ ACMA 4006

$$D = \begin{pmatrix} 1 & 11 \\ 0.5 & 23 \end{pmatrix}$$

i) $AB = \begin{pmatrix} -10-10+0 & 16-30-6 & 0+10+9 \\ 25+0+0 & -40+0-10 & 0+0+15 \\ -15+0+0 & 24+0-4 & 0+0+6 \end{pmatrix}$

$$AB = \begin{pmatrix} -20 & -20 & -1 \\ 25 & -50 & 15 \\ 15 & 20 & 6 \end{pmatrix}$$

ii) $BA = \begin{pmatrix} -10-40+0 & 25+0+0 & -15+40+0 \\ 4-30+6 & -10+0+0 & 6+30+4 \\ 0+0+9 & 0+0+0 & 0-10+6 \end{pmatrix}$

$$BA = \begin{pmatrix} -50 & 25 & 25 \\ -20 & -10 & 40 \\ 19 & 0 & -4 \end{pmatrix}$$

iii) $AC = \begin{pmatrix} 24+15+6 & 10-10+15 \\ -60+0+10 & -25+0+25 \\ 36+0+4 & 15+0+10 \end{pmatrix}$

$$AC = \begin{pmatrix} 45 & 15 \\ -50 & 0 \\ 40 & 25 \end{pmatrix}$$

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iv) Not possible as C has 2 columns and A has 3 rows.

$$V) |A| = 2(0-0) - 5(-10-15) + 3(0-0)$$

$$|A| = 125$$

$$vi) |B| = -5(18+4) - 8(6-0) + 0(-4-0)$$

$$|B| = -158$$

$$vii) A+B = \begin{pmatrix} -3 & 3 & 3 \\ -3 & 6 & 7 \\ 3 & -2 & 5 \end{pmatrix}$$

viii) $AB+D$ is not possible due to matrix dimensions.

$$b) y = \begin{pmatrix} 2 & 2x & -2 \\ 5 & x & 1 \\ 2 & (x+2) & 1 \end{pmatrix}$$

$$|y| = 2(x-x-2) - 2x(5-2) - 2((5x+10)-2x)$$

$$|y| = -4 - 10x + 4x - 6x - 20$$

$$|y| = -12x - 24$$

$$\text{When } |y| = 5$$

$$5 = -12x - 24$$

$$\frac{29}{-12} = x$$

ACMA 2006

$$\begin{aligned} \text{c)} \quad 3x - y + 4z &= 21 \\ 5x + 3y - z &= 15 \\ 2x + 4y + 3z &= 10 \end{aligned}$$

$$\begin{pmatrix} 3 & -1 & 4 \\ 5 & 3 & -1 \\ 2 & 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 21 \\ 15 \\ 10 \end{pmatrix}$$

← 'A'

$$\star |A| = 3(9 + 4) + 1(15 + 2) + 4(20 - 6)$$

$$|A| = 33 + 17 + 56 + 39$$

$$|A| = 112$$

$$A_1 = \begin{pmatrix} 21 & -1 & 4 \\ 15 & 3 & -1 \\ 10 & 4 & 3 \end{pmatrix}$$

$$|A_1| = 21(9 + 4) + 1(45 + 10) + 4(60 - 30)$$

$$|A_1| = 273 + 55 + 120$$

$$|A_1| = 448$$

$$A_2 = \begin{pmatrix} 3 & 21 & 4 \\ 5 & 15 & -1 \\ 2 & 10 & 3 \end{pmatrix}$$

$$|A_2| = 3(45 + 10) - 21(15 + 2) + 4(50 - 30)$$

$$|A_2| = 165 - 357 + 80$$

$$|A_2| = -112$$

$$A_3 = \begin{pmatrix} 3 & -1 & 21 \\ 5 & 3 & 15 \\ 2 & 4 & 10 \end{pmatrix}$$

$$|A_3| = 3(30 - 60) + 1(50 - 30) + 21(20 - 6)$$

$$|A_3| = -90 + 20 + 294$$

$$|A_3| = 224$$

ACMA 4006

$$x = \frac{|A_1|}{|A|} = \frac{448}{112} = 4$$

$$y = \frac{|A_2|}{|A|} = \frac{-112}{112} = -1$$

$$z = \frac{|A_3|}{|A|} = \frac{224}{112} = 2$$

Q2)

$$z_1 = 2 + 5j \quad z_2 = 1 - 3j \quad z_3 = -5 - 2j$$

i) $z_1 + z_3$

$$-3 + 3j$$

ii) $z_1 - z_3$

$$7 + 7j$$

iii) $\frac{z_1}{z_2} = \frac{2+5j}{1-3j} \times \frac{1+3j}{1+3j}$

$$= \frac{(2+5j)(1+3j)}{(1-3j)(1+3j)}$$

$$= \frac{2 + 6j + 5j + 15j^2}{1 + 3j - 3j - 9j^2}$$

$$j^2 = -1$$

$$= \frac{-13 + 11j}{10}$$

$$= \frac{-13}{10} + \frac{11}{10}j$$

$$iv) (z_3)^2 - \overline{z_1}$$

$$(-5-2j)(-5-2j) - (2-5j)$$

$$= 25 + 10j + 10j + 4j^2 - (2-5j) \quad j^2 = -1$$

$$= 21 + 20j - (2-5j)$$

$$= 19 + 25j$$

$$b) 5x^2 = -3x - 12$$

$$a = 5 \quad b = 3 \quad c = 12$$

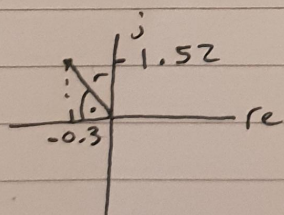
$$5x^2 + 3x + 12 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(5)(12)}}{10}$$

$$x = \frac{-3 \pm \sqrt{-231}}{10}$$

$$x = \frac{-3}{10} + \frac{15.20j}{10}$$

$$x = \frac{-3}{10} - \frac{15.20j}{10}$$



$$r^2 = 0.3^2 + 1.52^2$$

$$r^2 = 2.40$$

$$r = 1.55$$

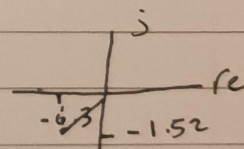
$$\tan \theta = \frac{1.52}{0.3}$$

$$\theta = 78.84^\circ$$

$$\theta = 180 - 78.84$$

$$\theta = 101.16^\circ$$

$$1.55 (\cos 101.16 + j \sin 101.16)$$



$$r^2 = 0.3^2 + 1.52^2$$

$$r = 1.55$$

$$\tan \theta = \frac{1.52}{0.3}$$

$$\theta = 78.84^\circ$$

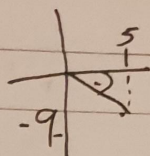
$$\theta = 180 + 78.84$$

$$\theta = 258.84^\circ$$

$$1.55 (\cos 258.84^\circ + j \sin 258.84^\circ)$$

c)

i) $(5-9j)^5$



$$r^2 = 5^2 + 9^2$$

$$r = 10.30$$

$$\tan \theta = \frac{9}{5} \quad \theta = 60.95^\circ$$

$$\theta = 360 - 60.95$$

$$\theta = 299.05^\circ$$

$$10.3^5 (\cos(5)(299.05) + j \sin(5)(299.05))$$

$$115927.41 (0.57 + j 0.82j)$$

$$66078.39 + 95060.14j$$

d) $j^7 = -j$

e) The conjugate of g

Q3)

i) $\int (x^8 + 2x^2)(3x^2 + x) dx$

$$\int 3x^{10} + x^9 + 6x^4 + 2x^3 dx$$

$$= \frac{3x^{11}}{11} + \frac{x^{10}}{10} + \frac{6x^5}{5} + \frac{2x^4}{4} + C$$

ii) $\int (e^x + 1)(e^x - 1) dx$

~~$\int (e^x)^2 - e^x + e^x - 1 dx$~~
 ~~$\int e^{2x} dx$~~
 ~~$\frac{1}{2} e^{2x}$~~

$\int e^{2x} - 1 dx$

$= \frac{1}{2} e^{2x} - x + C$

iii) $\int_2^5 (x+2)^3 + x - 1 dx$

~~$(x+2)^3$~~

$\int_2^5 (x+2)(x+2)(x+2) + x - 1 dx$

$\int_2^5 (x^2 + 4x + 4)(x+2) + x - 1 dx$

$\int_2^5 (x^3 + 2x^2 + 4x^2 + 8x + 4x + 8) + x - 1 dx$

$\int_2^5 (x^3 + 6x^2 + 12x + 8) + x - 1 dx$

$\int_2^5 x^3 + 6x^2 + 13x + 7 dx$

$\left[\frac{x^4}{4} + \frac{6x^3}{3} + \frac{13x^2}{2} + 7x \right]_2^5$

$\left[\frac{(5)^4}{4} + 2(5)^3 + \frac{13(5)^2}{2} \right] - \left[\frac{(2)^4}{4} + 2(2)^3 + \frac{13(2)^2}{2} \right]$

$\left[156.25 + 250 + 162.5 \right] - \left[4 + 16 + 26 \right]$

$= 522.75$

b) $\int x \sin 4x \, dx$

$$u = x \quad v = \frac{-1}{4} \cos 4x$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \sin 4x$$

$$\int u v - \int v \frac{du}{dx} \, dx$$

$$(x) \left(\frac{-1}{4} \cos 4x \right) - \int \frac{-1}{4} \cos 4x \, dx$$

$$\int -0.25 \cos 4x$$

$$(x) (-0.25 \cos 4x) + 0.25 \int \cos 4x \, dx$$

$$\int \cos 4x \, dx$$

$$= \frac{1}{4} \sin 4x$$

$$(x) (-0.25 \cos 4x) + 0.0625 \sin 4x + C$$