

MODULE CODE: M4X01305
TITLE: Engineering Mathematics and Structures
DATED: 25/09/2017

LEVEL: 4
CREDITS: 20

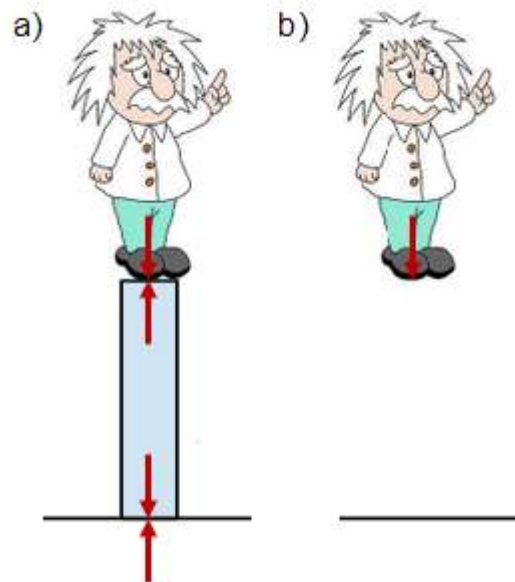
JACS CODE: G120
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Date: 14.11.2017

Forces:

A force in physics is any interaction, when un-opposed, that will result in the change of motion of an object. If the object was stationary, due to the application of the force, the object could start to move. If the object was moving, the force could change the direction of its motion or accelerate or decelerate the motion.

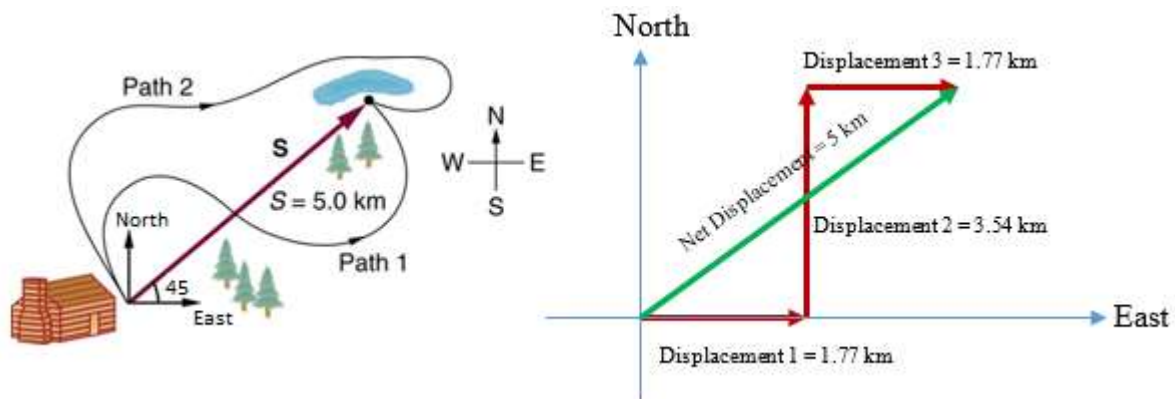
When forces are equal and opposite on an object, they cancel each other out and we call such objects to be in equilibrium.



Vectors:

A vector is a quantity that has a direction as well as a magnitude. A quantity that only has a magnitude, but no direction is called a scalar.

For example, distance is a scalar, whereas displacement is a vector



In the above diagrams:

Path 1 - the distance travelled may have a quantity, but it is a scalar, as there is no clear direction to the travel

Similarly, Path 2 is also a scalar

$S = 5.0 \text{ km}$ is a vector (quantity = 5 km, Direction = North East)

Displacement 1 is a vector (quantity = 1.77 km, Direction = East)

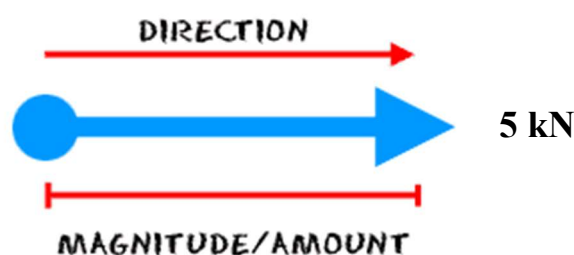
Displacement 2 is a vector (quantity = 3.54 km, Direction = North)

Displacement 3 is a vector (quantity = 1.77 km, Direction = East)

Net Displacement is a vector (quantity = 5 km, Direction = North East)

Forces as Vectors:

Forces will always have a magnitude and a direction. Therefore, forces are vectors. Since they are vectors, we can draw them on paper with an arrow pointing in the direction of the force and the dimension (length, to scale) of the arrow equal to the magnitude of the force.

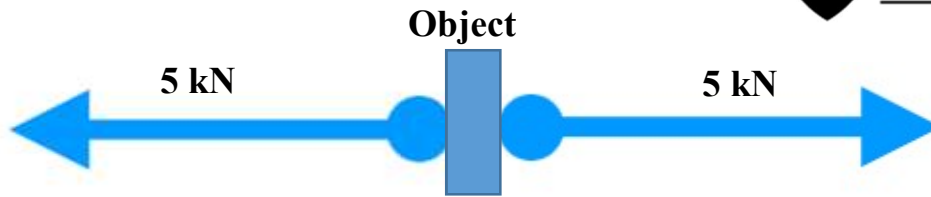


The above could therefore be a force of 5kN in the direction of the east (or to the right).

Vector Addition:

When adding two vectors together the direction has to be taken in to account.

Taking forces as an example of vectors, the effects of a force of 5 kN acting on an object in the direction of east, will be negated by an equal and opposite force (a force of 5kN in the direction of west). This can be shown in a “force” diagram as follows.



The net result of the two forces above is zero, as the effects of the forces cancel each other out.

We could write this mathematically as;

$$5 \text{ kN east} + 5 \text{ kN west} = 0$$

Or, if we consider forces to the east as positive (for our ease of reference), then we could write the above as;

$$+ 5 \text{ kN} - 5 \text{ kN} = 0$$

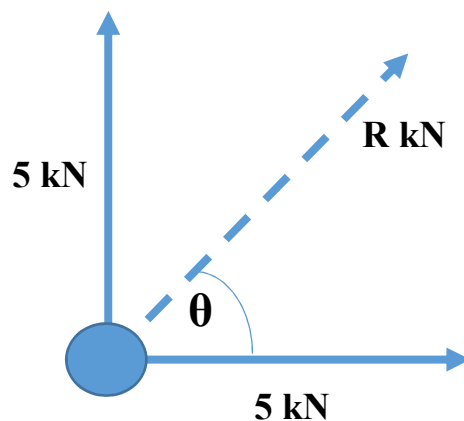
Here we treated the force to the west as negative according to the “sign convention” we selected (that is, forces to the east are positive).

Resultant Forces:

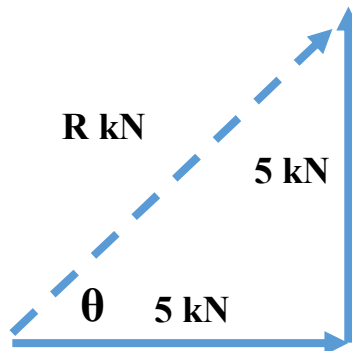
Using vector addition as above, we could find the “resultant” force on an object due to several applied forces as follows.

Taking the above example, the resultant force on the object is 0.

However, take an example where the two forces applied are not equal and opposite.



It is not difficult for us to visualise the object moving in the general direction of R, which we call the resultant direction. If the resultant force is in the direction of R, we have to define the direction (θ) and the magnitude (R) of the force. We could do this following the idea of the displacement diagram shown earlier and redrawing the forces to scale as shown below.



The above diagram was obtained by first drawing the 5 kN force to the east to scale, say 5cm, and then drawing the 5 kN force to the north, again to scale, say 5cm, from where the earlier force ended. The resultant force (both the magnitude and the direction) is now shown by the Hypotenuse of the right angled triangle created.

Using Pythagoras's theorem, we can find;

$$R = \sqrt{5^2 + 5^2} \text{ kN} = 7.07 \text{ kN}$$

Using trigonometry, we can write;

$$\tan \theta = \frac{5}{5} = 1$$

$$\theta = \tan^{-1}(1) = 45^\circ$$

By finding the angle (θ) and the magnitude (R) of the resultant force we have found the effect of the two forces applied on the object.