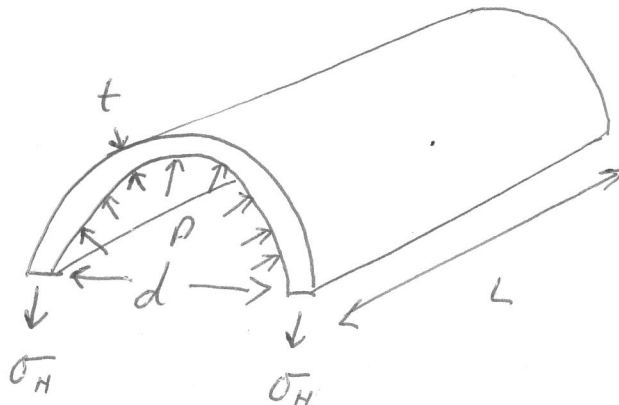


THIN CYLINDERS

TC ①

①

HOOP STRESS



$$\begin{aligned} \text{FORCE ON HALF CYLINDER} &= P \times \text{PROJECTED AREA} \\ &= P \times dL \end{aligned}$$

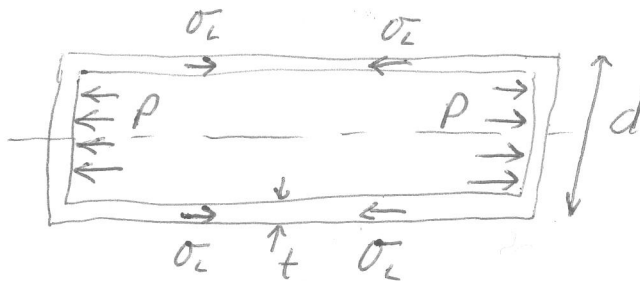
$$\text{TOTAL RESISTIVE FORCE} = 2\sigma_H \times Lt \quad \leftarrow \text{STRESS} \times \text{AREA} \times 2$$

$$2\sigma_H Lt = pdL$$

$$\sigma_H = \frac{pd}{2t}$$

②

LONGITUDINAL STRESS



$$\begin{aligned} \text{TOTAL FORCE ON END OF CYLINDER} &= \text{PRESSURE} \times \text{AREA} = P \times \pi r^2 = P \times \frac{\pi d^2}{4} \quad (\text{APPROX}) \end{aligned}$$

$$\text{AREA OF METAL RESISTING THIS} \approx \text{CIRCUMFERENCE} \times t = 2\pi r t = \pi d t$$

$$\text{STRESS} = \frac{\text{FORCE}}{\text{AREA}} = \frac{P \times \frac{\pi d^2}{4}}{\pi d t}$$

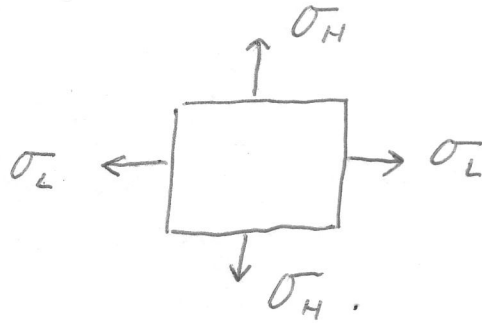
$$\sigma_L = \frac{pd}{4t}$$

CONSIDER A SMALL PART OF THE CYLINDER

(3)

LONGITUDINAL STRAIN

HOOP STRAIN



$$\text{YOUNG'S MODULUS} = \frac{\text{STRESS}}{\text{STRAIN}}$$

$$E = \frac{\sigma}{\epsilon}$$

THE FOLLOWING STRAINS ARE PRODUCED.

* IN L DIRECTION FROM $\sigma_L, \epsilon = \frac{\sigma_L}{E}$

* IN H DIRECTION FROM $\sigma_H, \epsilon = \frac{\sigma_H}{E}$

ALSO POISSON'S RATIO $\nu = \frac{\text{LATERAL STRAIN}}{\text{LONGITUDINAL STRAIN}}$

* IN L DIRECTION FROM $\sigma_H = -\nu \left(\frac{\sigma_H}{E} \right)$ (NEGATIVE AS IT MAKES IT SMALLER)

* IN H DIRECTION FROM $\sigma_L = -\nu \left(\frac{\sigma_L}{E} \right)$

$$\therefore \text{TOTAL LONGITUDINAL STRAIN} = \frac{\sigma_L}{E} - \nu \left(\frac{\sigma_H}{E} \right)$$

$$\boxed{\text{LONGITUDINAL STRAIN} = \frac{1}{E} (\sigma_L - \nu \sigma_H)}$$

ALSO,

$$\text{TOTAL HOOP STRAIN} = \frac{\sigma_H}{E} - \nu \frac{\sigma_L}{E}$$

$$\boxed{\text{HOOP STRAIN} = \frac{1}{E} (\sigma_H - \nu \sigma_L)}$$

LONGITUDINAL STRAIN = $\frac{\delta L}{L}$

(4) CHANGE IN LENGTH

$$\therefore \delta L = \text{LONGITUDINAL STRAIN} \times L$$

$$= \frac{1}{E} (\sigma_L - \nu \sigma_H) \times L$$

$$= \frac{1}{E} \left(\frac{pd}{4t} - \nu \frac{pd}{2t} \right) \times L$$

$$\delta L = \frac{pd}{4tE} (1 - 2\nu) L$$

$$\text{DIAMETRAL STRAIN} = \frac{\delta \text{ DIAMETER}}{\text{ORIG. DIAMETER}}$$

(5) CHANGE IN DIAMETER = DIAMETRAL STRAIN $\times$ ORIGINAL DIAMETER

CONSIDER THE CIRCUMFERENCE

$$\text{CIR. STRAIN} = \frac{\delta \text{ CIR}}{\text{ORIG CIR}}$$

$$\text{CHANGE IN CIRCUMFERENCE} = \text{CIRCUMFERENTIAL STRAIN} \times \text{ORIGINAL CIRCUMFERENCE}$$

$$= \epsilon_H \times \pi d$$

$$= \text{OLD CIR} + \delta \text{ CIR}$$

$$\therefore \text{NEW CIRCUMFERENCE} = \pi d + \pi d \epsilon_H$$

$$= \pi d (1 + \epsilon_H)$$

THIS IS THE CIRCUMFERENCE OF A CIRCLE OF DIAMETER $d(1 + \epsilon_H)$

$$\therefore \text{NEW DIAMETER} = d(1 + \epsilon_H)$$

$$\text{OLD DIAMETER} = d$$

$$\therefore \text{CHANGE IN DIAMETER} = d \epsilon_H$$

$$\text{DIAMETRAL STRAIN } \epsilon_D = \frac{\text{CHANGE IN DIAMETER}}{\text{ORIGINAL DIAMETER}} = \frac{d \epsilon_H}{d} = \epsilon_H$$

$$\therefore \text{DIAMETRAL STRAIN} = \text{HOOP STRAIN}$$

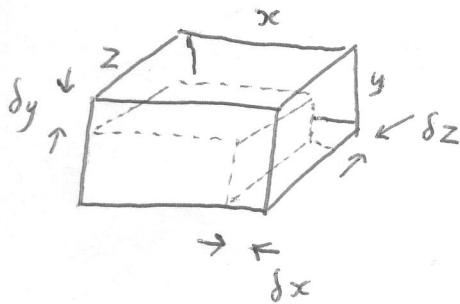
$$\textcircled{6} \quad \text{CHANGE IN DIAMETER} = d \epsilon_H \quad \text{TC (4)}$$

$$= \frac{d}{E} (\sigma_H - \nu \sigma_L) \quad (\text{FROM TC (2)})$$

$$= \frac{d}{E} \left(\frac{pd}{2t} - \nu \frac{pd}{4t} \right)$$

$$\boxed{\text{CHANGE IN DIAMETER} = \frac{pd^2}{4tE} (2 - \nu)}$$

$$\textcircled{7} \quad \text{CHANGE IN INTERNAL VOLUME}$$



$$\text{VOLUMETRIC STRAIN} = \frac{\text{CHANGE IN VOLUME}}{\text{ORIGINAL VOLUME}} = \frac{\delta V}{V} = \frac{\delta V}{xyz}$$

$$\delta V = \text{STRIP AT BACK} + \text{STRIP AT SIDE} + \text{STRIP ON TOP}$$

$$= xy \delta z + y(z - \delta z) \delta x + (x - \delta x)(z - \delta z) \delta y$$

$$= xy \delta z + yz \delta x - y \delta z \delta x + xz \delta y - x \delta x \delta y - \delta x z \delta y + \delta x \delta z \delta y$$

$$= xy \delta z + yz \delta x + xz \delta y$$

$$\text{VOLUMETRIC STRAIN } \epsilon_v = \frac{xy \delta z + yz \delta x + xz \delta y}{xyz}$$

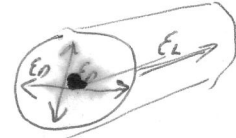
$$= \frac{\delta z}{z} + \frac{\delta x}{x} + \frac{\delta y}{y}$$

$$\epsilon_v = \epsilon_z + \epsilon_x + \epsilon_y$$

T.C (5)

⑧ $\therefore$ VOLUMETRIC STRAIN = SUM OF 3 MUTUALLY PERPENDICULAR DIRECT STRAINS

$$= \epsilon_L + 2\epsilon_D$$



$$= \epsilon_L + 2\epsilon_H \quad (\text{FROM T.C 3})$$

$$= \frac{1}{E} (\sigma_L - \nu \sigma_H) + \frac{2}{E} (\sigma_H - \nu \sigma_L)$$

$$= \frac{1}{E} (\sigma_L + 2\sigma_H - \nu (\sigma_H + 2\sigma_L))$$

$$= \frac{1}{E} \left(\frac{pd}{4t} + \frac{2pd}{2t} - \nu \left(\frac{pd}{2t} + \frac{2pd}{4t} \right) \right)$$

$$= \frac{pd}{4tE} (1 + 4 - \nu (2 + 2))$$

$$= \frac{pd}{4tE} (5 - 4\nu)$$

$\therefore$ CHANGE IN INTERNAL VOLUME = VOLUMETRIC STRAIN $\times$ ORIGINAL VOLUME

$$= \frac{pd}{4tE} (5 - 4\nu) V$$

① Q

T C ⑥

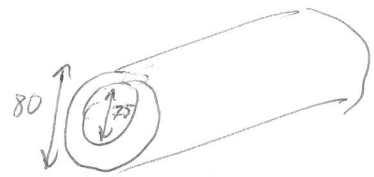
HOOP STRESS = 2 × LONGITUDINAL STRESS ∴ LIMITING FACTOR

$$\sigma_H = \frac{pd}{2t}$$

$$\begin{aligned} \therefore p &= \frac{2t\sigma_H}{d} \\ &= \frac{2 \times 2 \times 10^{-3} \times 150 \times 10^6}{300 \times 10^{-3}} \\ &= \underline{\underline{2 \text{ MPa}}} \end{aligned}$$

Q ② CHANGE IN DIAMETER = $\frac{pd^2}{4tE} (2-\nu)$

$$\begin{aligned} &= \frac{7 \times 10^6 \times 75^2 \times 10^{-6}}{4 \times 2.5 \times 10^{-3} \times 200 \times 10^9} (2-0.3) \\ &= 33.4 \times 10^{-6} \text{ m} \\ &= \underline{\underline{33.4 \mu\text{m}}} \end{aligned}$$



③ CHANGE IN LENGTH = $\frac{pdL}{4tE} (1-2\nu)$

$$\begin{aligned} &= \frac{7 \times 10^6 \times 75 \times 10^{-3} \times 250 \times 10^{-3}}{4 \times 2.5 \times 10^{-3} \times 200 \times 10^9} (1-0.6) \\ &= \underline{\underline{26.2 \mu\text{m}}} \end{aligned}$$

TC (7)

③ HOOP STRESS $\sigma_H = \frac{pd}{2t} = \frac{7 \times 10^6 \times 75 \times 10^{-3}}{2 \times 2.5 \times 10^{-3}} = \underline{\underline{105 \text{ MPa}}}$

LONGITUDINAL STRESS $\sigma_L = \frac{pd}{4t} = \frac{7 \times 10^6 \times 75 \times 10^{-3}}{4 \times 2.5 \times 10^{-3}} = \underline{\underline{52.5 \text{ MPa}}}$

IN ADDITION TO THESE A SHEAR STRESS IS SET UP.

$\frac{\tau}{R} = \frac{T}{J}$: TORSION THEORY

$\tau = \frac{TR}{J}$

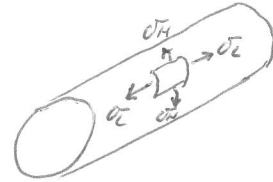
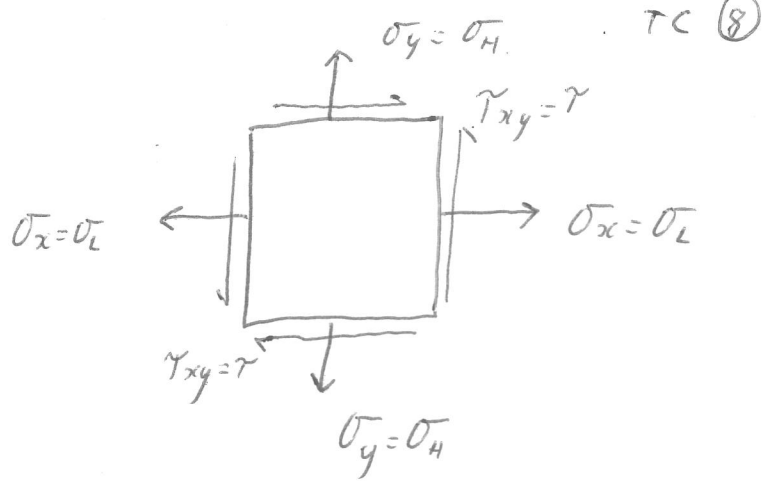
$\leftarrow \frac{\pi d^4}{32}$

now $J = \frac{\pi}{32} (80^4 - 75^4) \times 10^{-12} = 0.92 \times 10^{-6} \text{ m}^4$

$\therefore \tau = \frac{TR}{J}$

$= \frac{200 \times 40 \times 10^{-3}}{0.92 \times 10^{-6}}$

$= \underline{\underline{8.7 \text{ MPa}}}$



PRINCIPAL STRESSES

$$\sigma_1, \sigma_2 = \frac{1}{2} (\sigma_x + \sigma_y) \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2} (52.5 + 105) \pm \frac{1}{2} \sqrt{(52.5 - 105)^2 + 4(8.7)^2}$$

$$= \frac{1}{2} (157.5) \pm \frac{1}{2} \sqrt{(2756 + 302.8)}$$

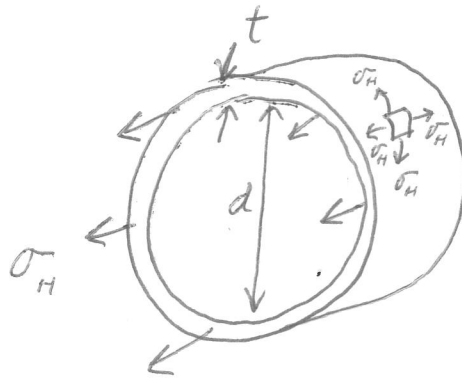
$$= 78.75 \pm 27.7$$

$$= \underline{\underline{106.5 \text{ MPa} \text{ and } 51.1 \text{ MPa}}}$$

BOTH TENSILE

SPHERES

①



FORCE ON HALF SPHERE = PRESSURE $\times$ PROJECTED AREA

$$= p \times \pi r^2$$

$$= p \times \frac{\pi d^2}{4}$$

RESISTING FORCE = $\sigma_H \times \pi d t$ (STRESS $\times$ CIRCUMFERENCE $\times t$)

$$\therefore \frac{p \pi d^2}{4} = \sigma_H \times \pi d t$$

$$\therefore \sigma_H = \frac{p \pi d^2}{4 \pi d t}$$

$$\boxed{\sigma_H = \frac{p d}{4 t}}$$

TC (10)

(10)

CHANGE IN VOLUME = ORIGINAL VOLUME $\times$ VOLUMETRIC STRAIN

(SEE TC (4))

VOLUMETRIC STRAIN = SUM OF 3 MUTUALLY $\perp$ r STRAINS

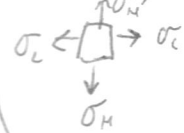
$$= 3 \epsilon_H$$

$$= 3 \times \frac{1}{E} (\sigma_H - \nu \sigma_H)$$

$$= 3 \times \frac{\sigma_H}{E} (1 - \nu)$$

$$= \frac{3 \times pd}{E \cdot 4t} (1 - \nu)$$

(SEE TC (2))



$$\epsilon_H = \frac{1}{E} (\sigma_H - \nu \sigma_L)$$

$$\therefore \epsilon_H = \frac{1}{E} (\sigma_H - \nu \sigma_H)$$

SINCE $\sigma_L = \sigma_H$

$$\text{CHANGE IN VOLUME} = \frac{3pd}{4tE} (1 - \nu) V$$

EFFECTS OF END PLATES & JOINTS:

$$\text{CYLINDER} = \sigma_H = \frac{pd}{2t\eta_L}$$

 η_L = EFFICIENCY OF LONGITUDINAL JOINTS

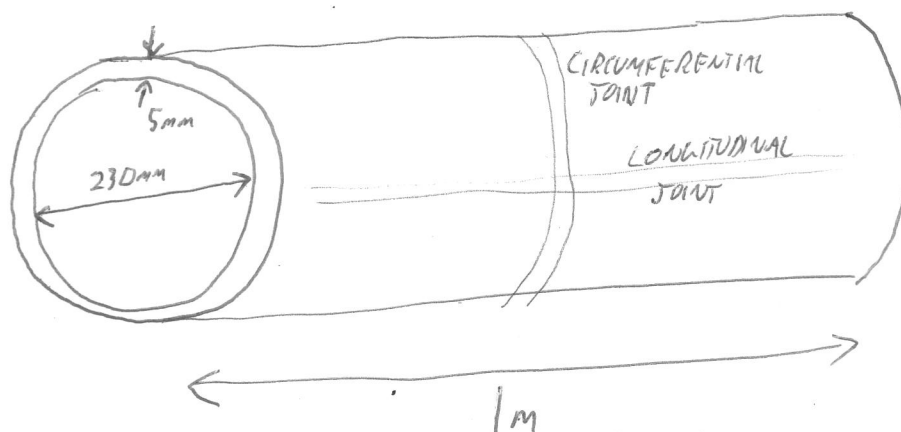
$$\sigma_L = \frac{pd}{4t\eta_c}$$

 η_c = EFFICIENCY OF CIRCUMFERENTIAL JOINTS

$$\text{SPHERES} : \sigma_H = \frac{pd}{4t\eta}$$

Q (3)

TC (11)



$$\delta V = 12.0 \times 10^{-6} \text{ m}^3$$

$$\text{pressure} = p$$

$$E = 200 \text{ GPa}$$

$$\nu = 0.25$$

RIGID END PLATES

$$(a) \quad \text{CHANGE IN INTERNAL VOLUME} = \frac{pd}{4tE} (5-4\nu)V$$

$$\text{ORIGINAL VOLUME } V = \pi \times (115 \times 10^{-3})^2 \times 1 = \underline{\underline{41.6 \times 10^{-3} \text{ m}^3}}$$

$$\therefore 12.0 \times 10^{-6} = \frac{p \times 230 \times 10^{-3}}{4 \times 5 \times 10^{-3} \times 200 \times 10^9} (5 - 4 \times 0.25) \times 41.6 \times 10^{-3}$$

$$12.0 \times 10^{-6} = 9.568 \times 10^{-12} p$$

$$\therefore \underline{\underline{p = 1.25 \text{ MPa}}}$$

$$\begin{aligned}\sigma_H &= \frac{pd}{2t} \\ &= \frac{1.25 \times 10^6 \times 230 \times 10^{-3}}{2 \times 5 \times 10^{-3}} \\ &= \underline{\underline{28.8 \text{ MPa}}}\end{aligned}$$

$$\begin{aligned}\sigma_L &= \frac{pd}{4t} = \frac{1}{2} \sigma_H \\ &= \underline{\underline{14.4 \text{ MPa}}}\end{aligned}$$

NOTE THE HOOP STRESSES
 ACT ON THE LONGITUDINAL JOINT

$$\textcircled{b} \quad \sigma_H = \frac{pd}{2t\eta_L} = \frac{1.25 \times 10^6 \times 230 \times 10^{-3}}{2 \times 5 \times 10^{-3} \times 0.85} = \underline{\underline{33.9 \text{ MPa}}}$$

$$\sigma_L = \frac{pd}{4t\eta_c} = \frac{1.25 \times 10^6 \times 230 \times 10^{-3}}{4 \times 5 \times 10^{-3} \times 0.45} = \underline{\underline{32 \text{ MPa}}}$$

$$\begin{aligned}\textcircled{c} \quad \text{CHANGE IN VOLUME} &= \frac{pd}{4tE} (5-4\nu)V \\ &\propto p\end{aligned}$$

$\therefore$ 15% INCREASE IN PRESSURE = 15% INCREASE IN VOLUME

$$\begin{aligned}\therefore \Delta p &= 0.15 \times 1.25 \times 10^6 \\ &= \underline{\underline{41 \text{ MPa}}}\end{aligned}$$