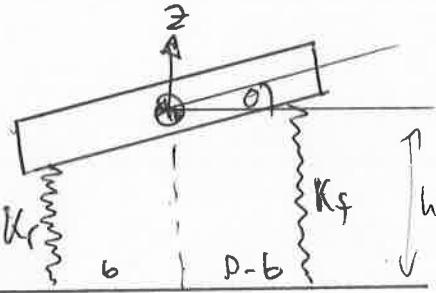




EXAMINATION SOLUTIONS

(to accompany the examination paper sent to the external examiner
but NOT to be seen by students)

Programme:	BEng Motorcycle Engineering	Level:	5
Module:	Motorcycle Dynamics		
Author:	Owen Williams	Page:	

Q	Question	Marks
1a	 The situation is coupled Bounce and Pitch are linked. $M\ddot{z} + (K_f + K_r)z + K_f(P-b)\theta - K_rb\theta = 0$ $I\ddot{\theta} + K_f(P-b)z - K_rbz + K_f(P-b)^2\theta + K_rb^2\theta = 0$	(5)
b	$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C_f + C_r & C_f(P-b) - C_rb \\ C_f(P-b) - C_rb & C_f(P-b)^2 + C_rb^2 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} K_f + K_r & K_f(P-b) - K_rb \\ K_f(P-b) - K_rb & K_f(P-b)^2 - K_rb^2 \end{bmatrix} \begin{bmatrix} z \\ \theta \end{bmatrix} = 0$	(5)
c	 $M\ddot{x} + C\dot{x} + Kx = 0$ $100 \frac{\partial^2 x}{\partial t^2} + 1200 \frac{\partial x}{\partial t} + 18,000x = 0$	



1c
Cont

$$\Rightarrow r^2 + 12r + 180 = 0 \quad (e^{rt} \text{ substitution})$$

$$r = \frac{-12 \pm \sqrt{12^2 - (4 \times 180)}}{2}$$

$$= \frac{-12 \pm \sqrt{144 - 720}}{2}$$

$$= -6 \pm i \frac{24}{2}$$

$$= -6 \pm 12i$$

$$\text{so } x = e^{-6t} [A \cos(12t) + B \sin(12t)] \quad (5)$$

1d

$$\text{when } t=0, x=0 \quad 0 = A + 0$$

$$A = 0$$

$$x = e^{-6t} [B \sin(12t)]$$

$$\dot{x} = -6e^{-6t} [B \sin(12t)] + 12B \cos(12t)$$

$$2 = 12B$$

$$B = \frac{1}{6}$$

So

$$x = e^{-6t} \left[\frac{1}{6} \sin(12t) \right] \quad (5)$$

Total 20



2a

Total Mass 213 kg

Front sprung mass = $(0.57 \times 213) - 11.6 = 109.81$ kg

Rear sprung mass = $(0.43 \times 213) - 68 = 23.59$ kg

Springs:

Suitable ride frequency ~ 2.1 Hz $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$$k_f' = (2\pi f)^2 \times m_f = 19.118 \text{ N/mm}$$

$$k_r' = (2\pi f)^2 \times m_r = 4.107 \text{ N/mm}$$

$$k_f = k_f' \cos^2 \theta = 19.118 \times \cos^2 30 = 14.34 \text{ N/mm}$$

$$k_r = k_r' \times LR^2 = 4.107 \times 3.7^2 = 56.225 \text{ N/mm}$$

Dampers: Assume $\zeta \sim 0.5$

$$\text{So } C_f' = C_{\text{crit}} \times \zeta = 0.5 \times 2\sqrt{19.118 \times 109.81}$$

$$C_f' = 1448.9 \text{ Ns/m}$$

$$C_r' = C_{\text{crit}} \times \zeta = 0.5 \times 2\sqrt{4.107 \times 23.59}$$

$$C_r' = 311.26 \text{ Ns/m}$$

$$\text{So } C_f = C_f' \times \cos^2 30 = 1086.7 \text{ Ns/m}$$

$$C_r = C_r' \times LR^2 = 4261.15 \text{ Ns/m}$$

b

A description detailing applying torque to a gyroscope to change its roll torque can be used to correct for changes in roll angle allowing a motorcycle to self-balance. At higher speeds the motorised would be gyroscopically stable as a normal motorcycle

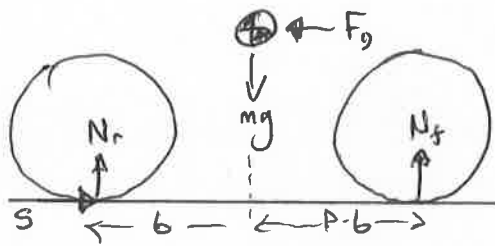
(15)

(5)

20



3a



$$F_D = S \quad (1)$$

$$mg = N_f + N_r \quad (2)$$

Taking moments (CotG)

$$N_f(P-b) + Sh = N_r b$$

sub with (2) for N_r

$$N_f(P-b) + Sh = mg - N_f b$$

$$N_f P - N_f b + Sh = mg - N_f b$$

$$N_f P + Sh = mg b$$

$$N_f = \frac{mg b}{P} - \frac{Sh}{P}$$

Now sub with (2) for N_r

$$(mg - N_r)(P-b) + Sh = N_r b$$

$$mgP - N_r P - mgb + N_r b + Sh = N_r b$$

$$N_r P = mgP - mgb + Sh$$

$$N_r = \frac{mg(P-b)}{P} + \frac{Sh}{P}$$

(3)

b

Using velocity to get acceleration we get driving or braking force (s) which we can use to calculate N_f or N_r . From this $F = kx$ will give suspension position (vertically) which can be resolved to the rake angle as $k_r = k_f \cos^2 \epsilon$ or Rear shock as $k_r = k_r' \times LR^2$.

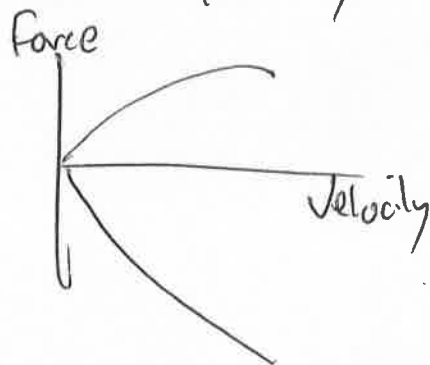


3c The main difference would be the loss of the detail from the small changes in wheel speed due to dynamic conditions, wheelies, gear changes etc. The results from the 'Lap Sim' based Suspension trace would be very smooth without the fine detail from the real wheel speed data.

~~24~~
3

d The differential of the suspension position trace gives us suspension velocity. This is key since $F = CV$ for damping.

The damper 'dyw' charts look like.



and show the force at different velocities

from these charts we can get the damping constant being used

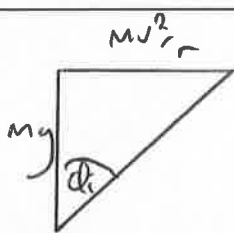
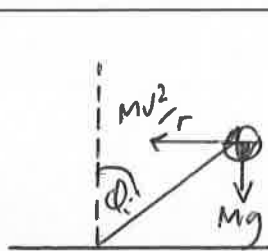
at any velocity (gradient of line) which can be related to the damper velocity traces from the suspension data to see what speeds and hence damping valves are being used.

5

Total 20



4a



so $\tan \phi_i = \frac{mv^2}{r \cdot mg}$

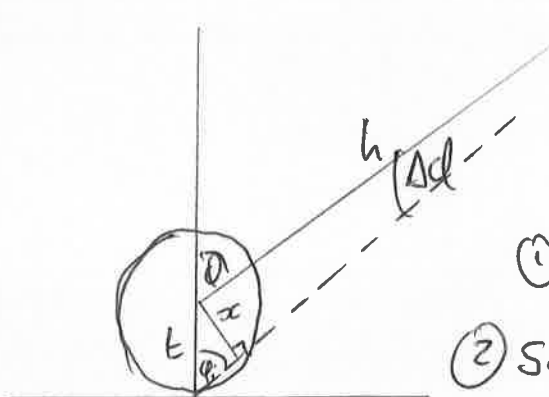
$$\phi_i = \tan^{-1} \left(\frac{v^2}{r \cdot g} \right)$$

(4)

6 The common misconception is that the RS will have a much greater lean angle. This is due to its (usual) much higher corner speed. When travelling at the same speeds, angles are equal (some changes due to tyres and geometry)

(3)

c



$$\phi = \phi_i + \Delta \phi$$

(1) $\sin \phi_i = \frac{x}{t} \therefore x = t \sin \phi_i$

(2) $\sin \Delta \phi = \frac{x}{h-t} \therefore x = (h-t) \sin \Delta \phi$

equating (1) and (2)

$$t \sin \phi_i = (h-t) \sin \Delta \phi$$

$$\frac{\sin \Delta \phi}{t} = \frac{\sin \phi_i}{h-t}$$

(5)



4d The discussion should include 'active' aerodynamics, such that any potential downforce can be 'switched off' mid corner so not to effect tyre loads in a bad way. The effect of vehicle yaw should be considered, either in terms of the riders leg/knee in the airstream or some form of active aerodynamic aid.

(4)

4e The geometry of a modern GP bike has become physically longer, longer wheelbase and more importantly the trail value. This coupled with the added straight line stability from the current generation of aero aids has created a bike which is very straight line stable and difficult to turn. This has resulted in a change of 'racing line' as the fastest solution is now a 'corner to corner' sprint, more than traditional high speed cornering.

(4)

Total 20