

UWTSD MBA (Nov 2018 intake) SBMA7133 Managerial Economics

(2-1) Quantitative Demand Analysis and Elasticity

Che-Cheong POON
20 January 2019 10:00 am to 1:30 pm

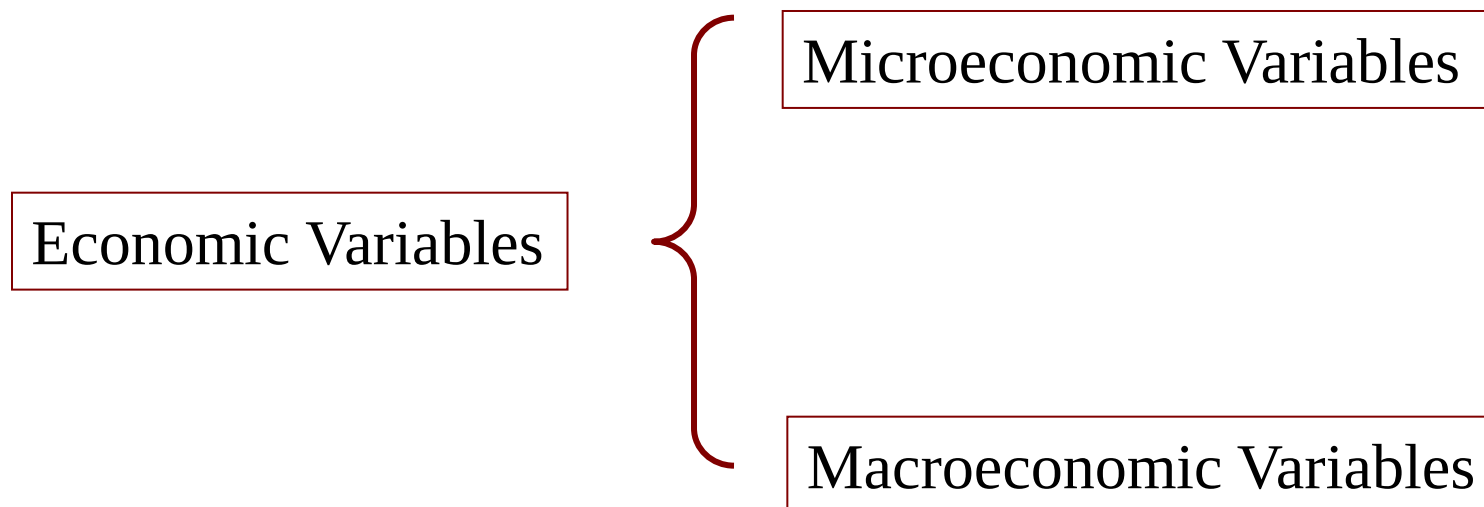
Lecture Agenda :

- Functional Expression of the Relationship between Two Economic Variables
- The Elasticity Concept
- Own Price Elasticity of demand
- Other Coefficients of Elasticity and their applications

Functional Expression of the Relationship between Two Economic Variables

Definition of economic variables

Economic variables are variables used by economists to measure changes in economic phenomena.





Please list ten important economic variables that will be used in your project assignment of Managerial Economics.

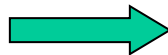
Qualitative analysis and quantitative analysis

(Qualitative analysis)



Considers the direction of movement in the dependent variable (quantity demanded or demand price) resulting from a change in any one of the independent (endogenous or exogenous) variables

(Quantitative analysis)



Considers the magnitude of movement in the dependent variable (quantity demanded or demand price) resulting from a change in any one of the independent (endogenous or exogenous) variables.

Economic Functions

A mathematical function consists of three parts:

1. One dependent variable
2. One or more independent variable(s)
3. The parameters including coefficients of independent variables and the constant term (if any)

An example for a 2-variable linear function

Theoretical function: $Y = f(X)$

Linear model: $Y_i = a + bX_i + e_i$

The estimated equation: $\hat{Y}_i = a + bX_i$

The simple supply and demand model: Determination of the equilibrium quantity

$$P_d = \alpha_1 - \beta_1 Q \dots\dots\dots (1)$$

$$P_s = \alpha_2 + \beta_2 Q \dots\dots\dots (2)$$

$$P_d = P_s \dots\dots\dots (3)$$

By substituting (1) and (2) into (3), we get

$$\alpha_1 - \beta_1 Q = \alpha_2 + \beta_2 Q$$

$$Q_e = \frac{\alpha_2 - \alpha_1}{-\beta_1 - \beta_2} = \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2}$$

The simple supply and demand model: The determination of the equilibrium price

$$Q_d = \frac{\alpha_1}{\beta_1} - \frac{1}{\beta_1} P \quad \dots\dots\dots (1)$$

$$Q_s = -\frac{\alpha_2}{\beta_2} + \frac{1}{\beta_2} P \quad \dots\dots\dots (2)$$

$$Q_d = Q_s \quad \dots\dots\dots (3)$$

By substituting (1) and (2) into (3), we get

$$\frac{\alpha_1}{\beta_1} - \frac{P}{\beta_1} = -\frac{\alpha_2}{\beta_2} + \frac{P}{\beta_2}$$

$$P \left(\frac{1}{\beta_1} + \frac{1}{\beta_2} \right) = \frac{\alpha_1}{\beta_1} + \frac{\alpha_2}{\beta_2}$$

$$P_e = \frac{\alpha_1 \beta_2 + \alpha_2 \beta_1}{\beta_1 + \beta_2}$$



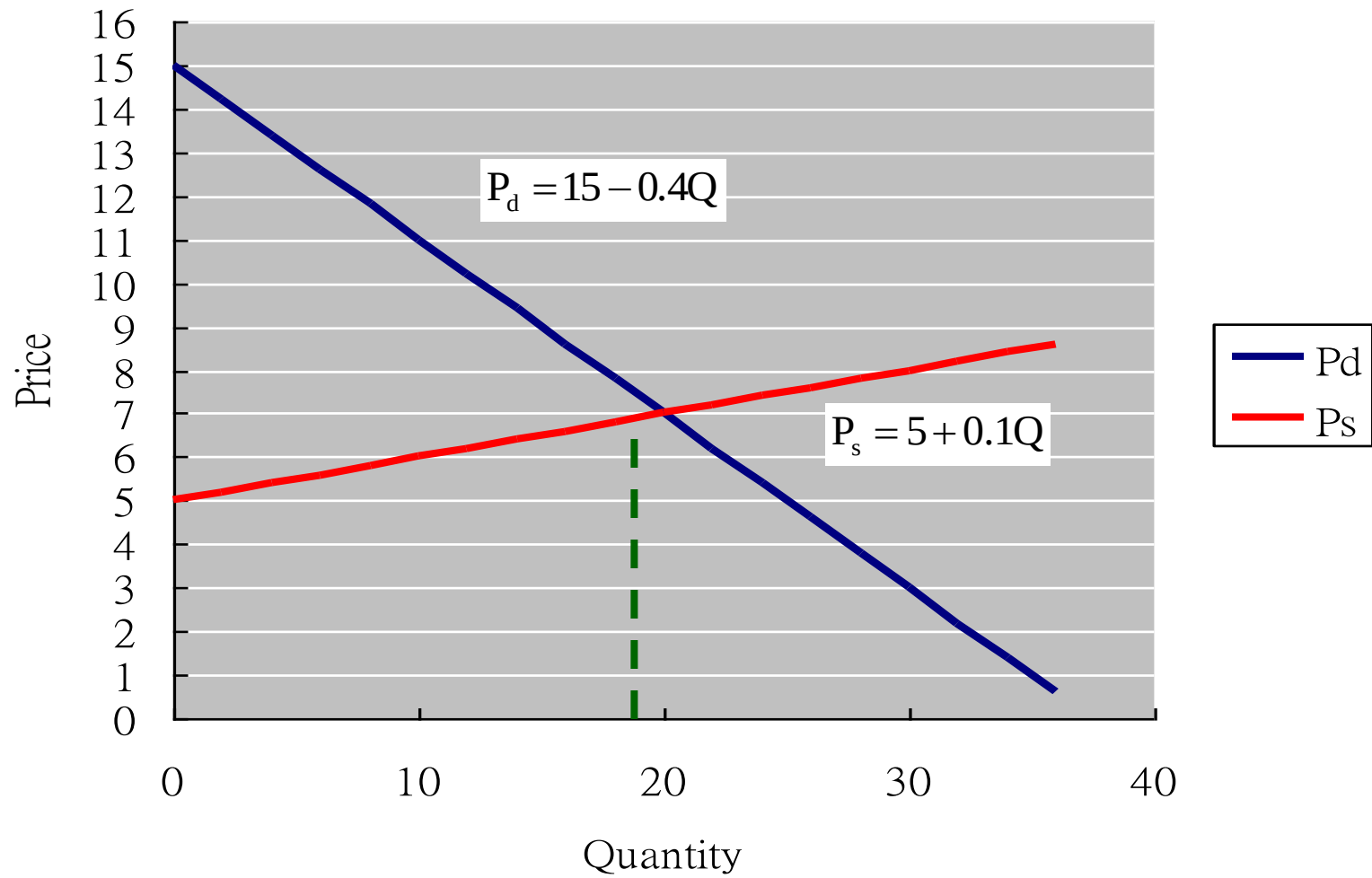
You are given the following supply and demand functions:

$$P_d = 15 - 0.4Q$$

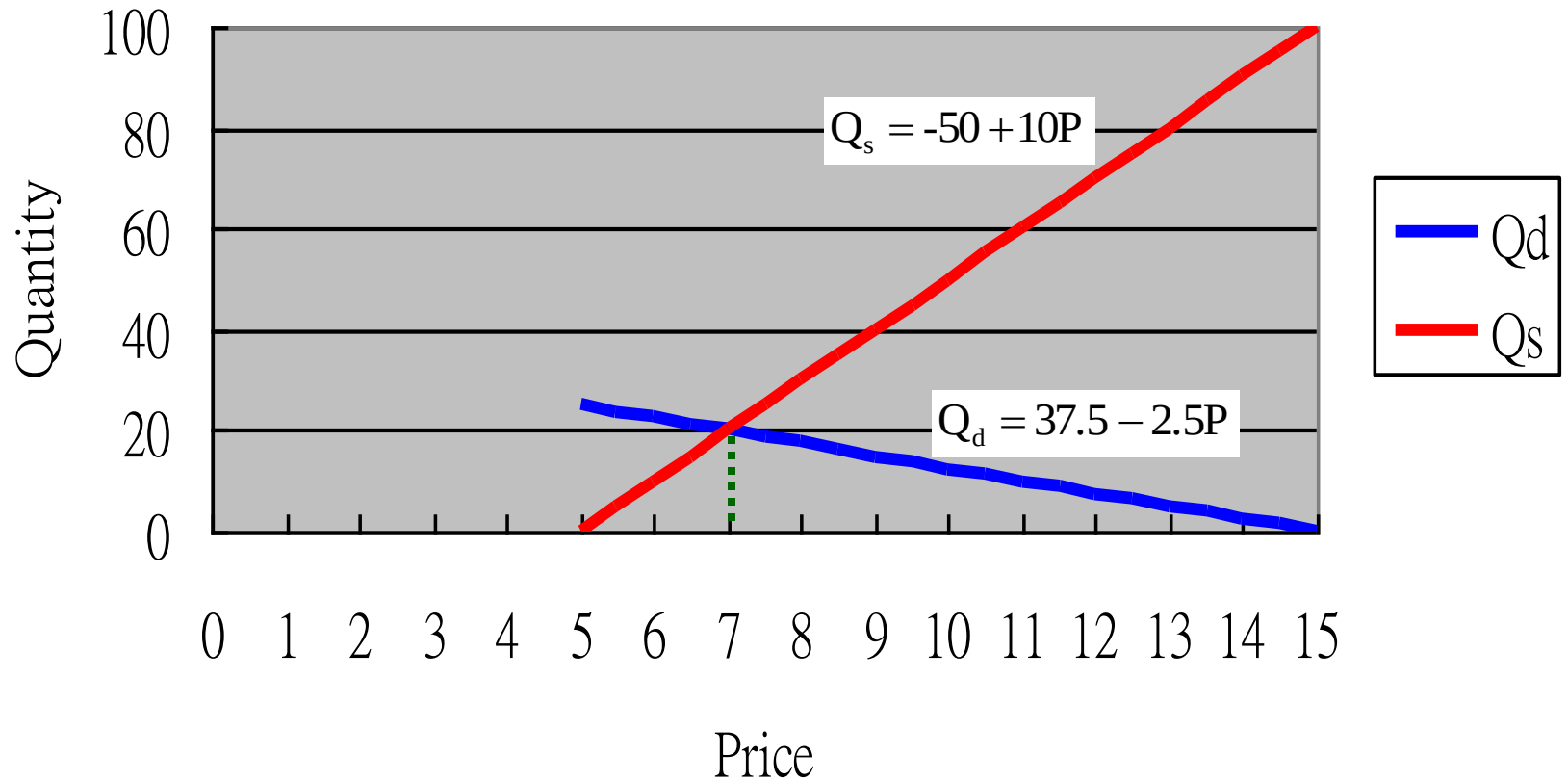
$$P_s = 5 + 0.1Q$$

1. Calculate the equilibrium quantity (Q_e)
2. Calculate the equilibrium price (P_e)

Market equilibrium



Market equilibrium



$$Q_e = \frac{\alpha_1 - \alpha_2}{\beta_1 + \beta_2} = \frac{15 - 5}{0.4 + 0.1} = \frac{10}{0.5} = 20$$

$$\begin{aligned} P_e &= \frac{\alpha_1 \beta_2 + \alpha_2 \beta_1}{\beta_1 + \beta_2} = \frac{15 \times 0.1 + 5 \times 0.4}{0.4 + 0.1} \\ &= \frac{1.5 + 2}{0.5} = 7 \end{aligned}$$

where :

α_1 and α_2 is the vertical intercept of the demand and supply curve respectively.

β_1 and β_2 is the slope of the demand and supply curve respectively.

When there is an autonomous change (a change in the exogenous variable) in demand/supply, the demand/supply curve will shift to a new position parallel to its original position.

An example for a 3-variable linear function

$$P_e = f(Q_s, Q_d)$$

A theoretical function

$$P_e = a - bQ_s + cQ_d + e$$

An observed empirical function

$$\hat{P}_e = a - bQ_s + cQ_d$$

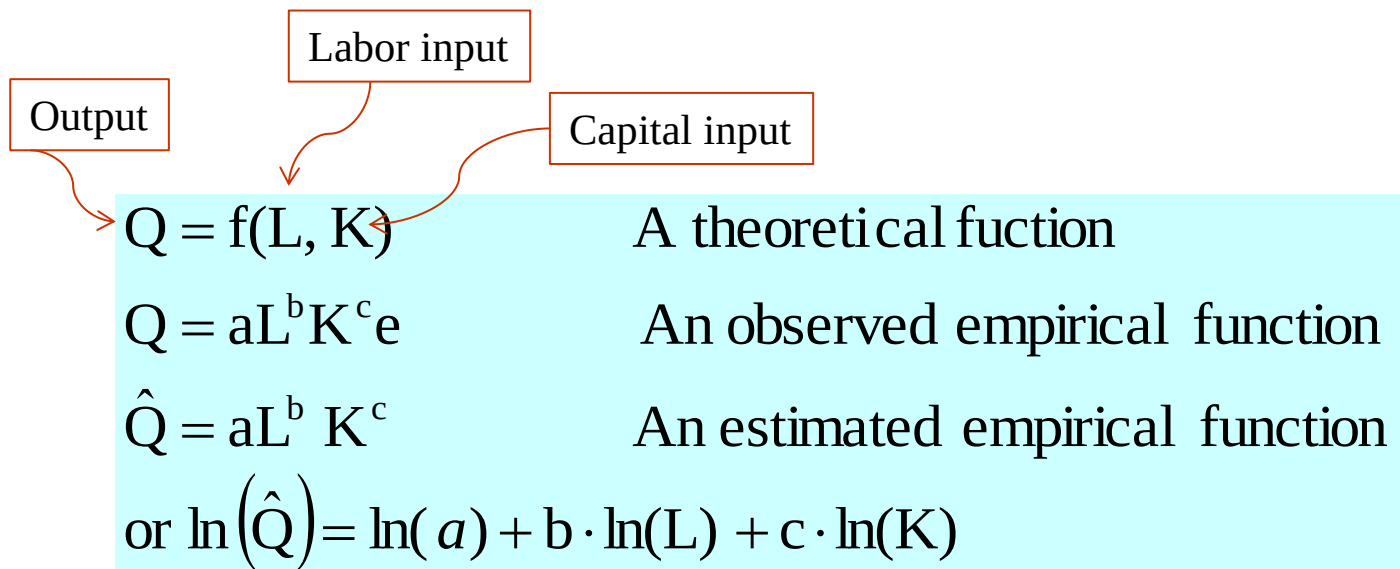
An estimated empirical function

Note:

For population observations, we use $\alpha, \beta, \gamma, \eta, \dots$ etc.; for sample observations, we use $a, b, c, d, \dots$ etc.

It is easy to estimate the 3 parameters (a, b , and c) by a suitable regression model.

An example for a 3-variable non-linear function (Cobb-Douglas production function)



If $b + c = 1$, the production function displays constant returns to scale.

If $b + c > 1$, the production function displays increasing returns to scale.

If $b + c < 1$, the production function displays decreasing returns to scale.

Empirical example: Airline ticket pricing

Assumptions:

1. You are a manager for a regional airline offering non-stop service between Houston, TX and Orlando, FL.
2. Your airline makes one departure from each city per day (2 flights total).
3. One rival airline offers non-stop service on this route.
4. We ignore first class service and focus on the demand for coach-class travel.

The theoretical demand function:

$$Q_d = f(P, P^0, Y) \dots\dots\dots (1)$$

Equation (1) can be read as follows: The number of your airline's coach seats sold per flight (Q_d) is a function of the your airline's coach fare (P), its rival's fare (P^0), and wage index in the region (Y)

The estimated demand function:

$$Q_d = 25 + 3Y + P^o - 2P \dots\dots\dots (2a)$$

$$P = 12.5 + 1.5Y + 0.5P^o - 0.5Q_d \dots\dots\dots (2b)$$

Effect of changes in the exogenous variables:

1. For each one point increase in the wage index (Y), 3 additional seats will be sold, ceteris paribus.
2. For each \$10 increase in the airline's fare, 20 fewer seats will be sold, ceteris paribus.
3. For each \$10 increase in the competitor's fare, 10 additional seats will be sold, ceteris paribus.

The demand function:

Example:

Let $Y = 105$ and $P^o = 240$. Our quantity demand function is given by:

$$Q_d = 25 + 3(105) + 1(240) - 2P = \mathbf{580 - 2P \dots\dots\dots (3a)}$$

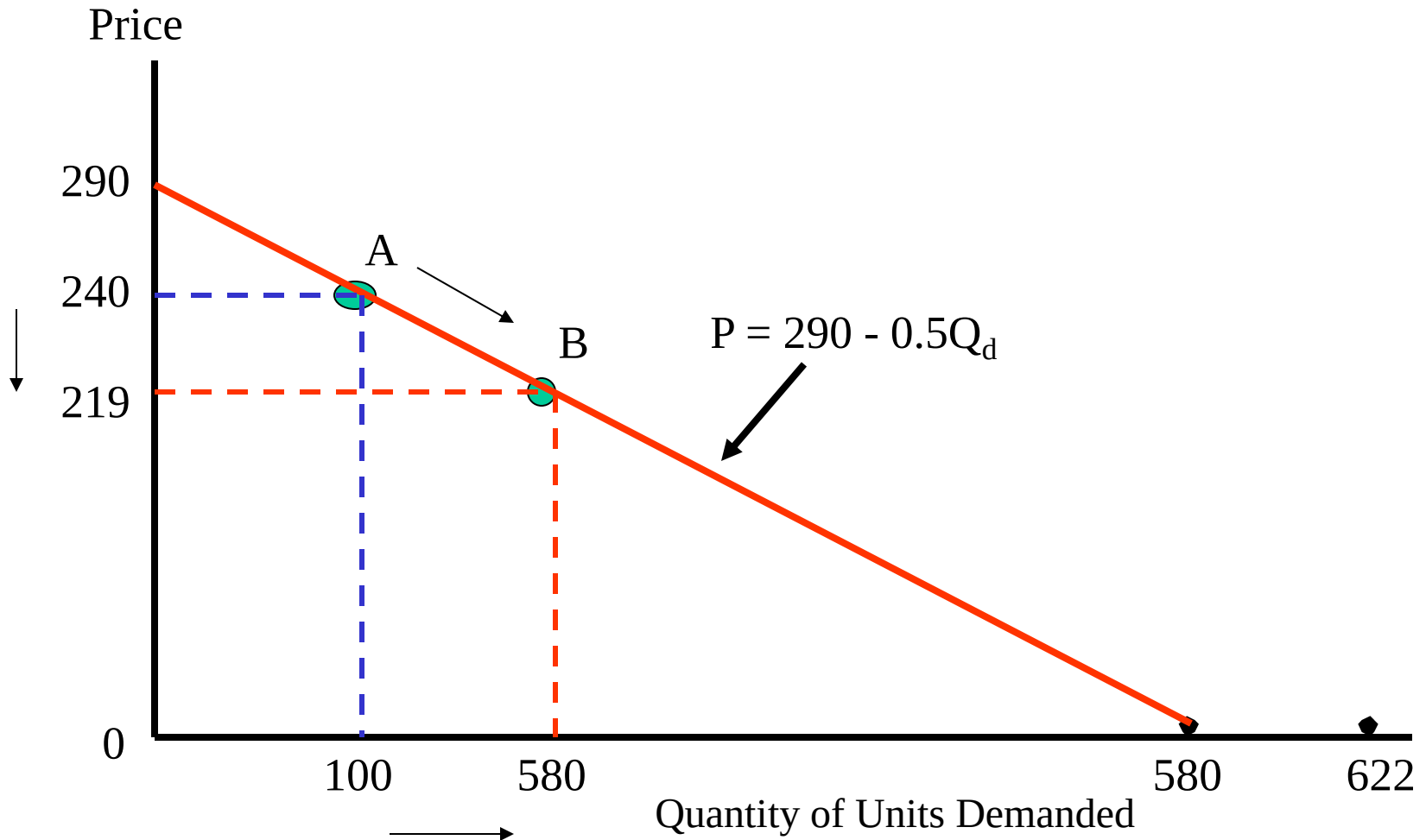
Our price demand function is given by:

$$P = 12.5 + 1.5(105) + 0.5(240) - 0.5Q_d \dots\dots\dots (3b)$$

$$\mathbf{P = 290 - 0.5Q_d}$$

Ceteris paribus:

(We hold all other things constant. In our case, this means $Y = 105$ and $P^o = \$240$.)





The initial demand function is given by:

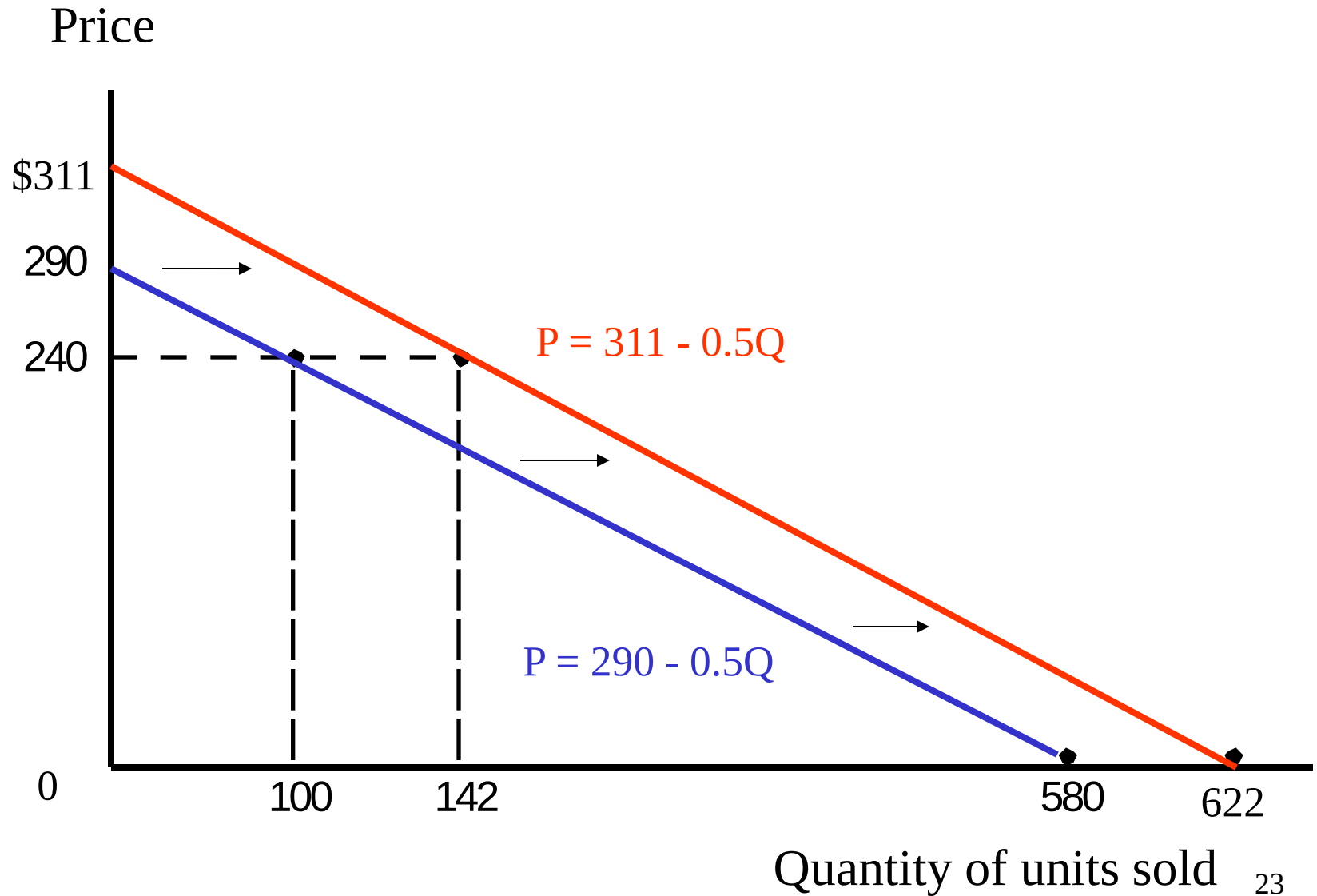
$$P = 290 - 0.5Q$$

What would happen on the demand function if, ceteris paribus, Y increased from 105 to 119?

Let $Y = 119$ and $P^O = \$240$. Our demand function is given by:

$$P = 12.5 + 1.5(119) + 0.5(240) - 0.5Q_d = 311 - 0.5P$$

Shifts of the demand curve

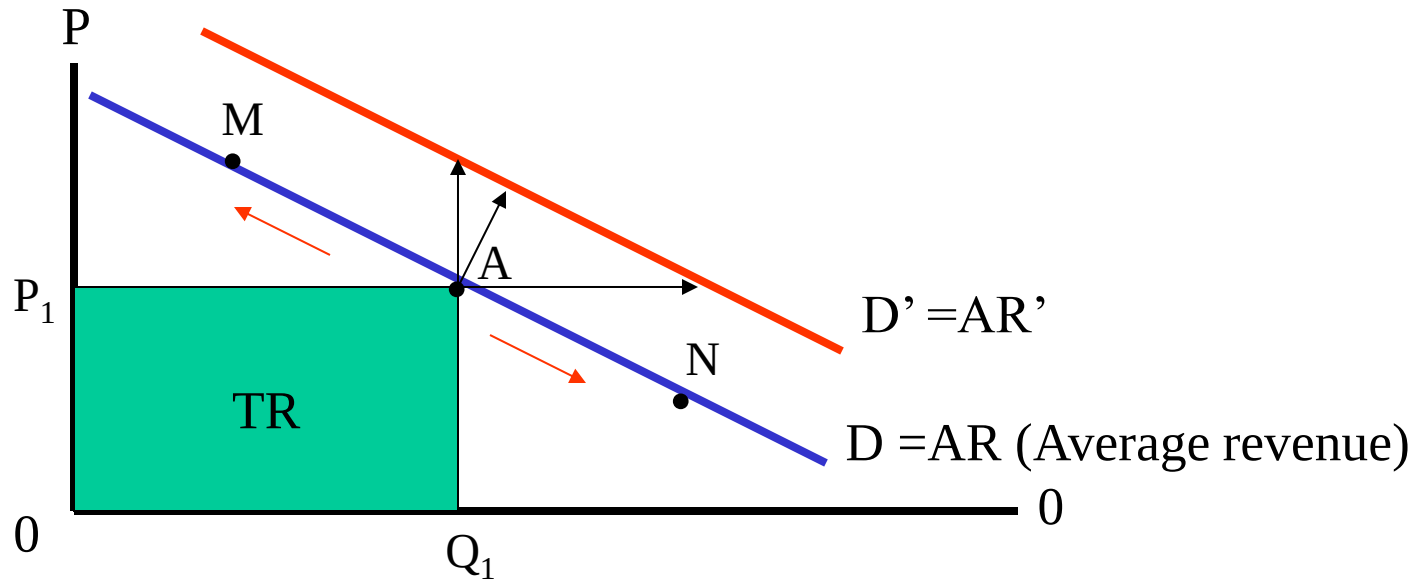




How responsive is the demand for air service to changes in fares, *ceteris paribus*?

The Elasticity Concept

$$TR = P \times Q$$



$$P \uparrow; Q \uparrow \Rightarrow TR \uparrow$$

$$P \uparrow; \bar{Q} \Rightarrow TR \uparrow$$

$$\bar{P}; Q \uparrow \Rightarrow TR \uparrow$$

$$P \uparrow; Q \downarrow \Rightarrow TR ?$$

Elasticity is a measure of the responsiveness of one variable (the dependent variable) to changes in another variable (the independent variable).



Coil spring for heavy machine

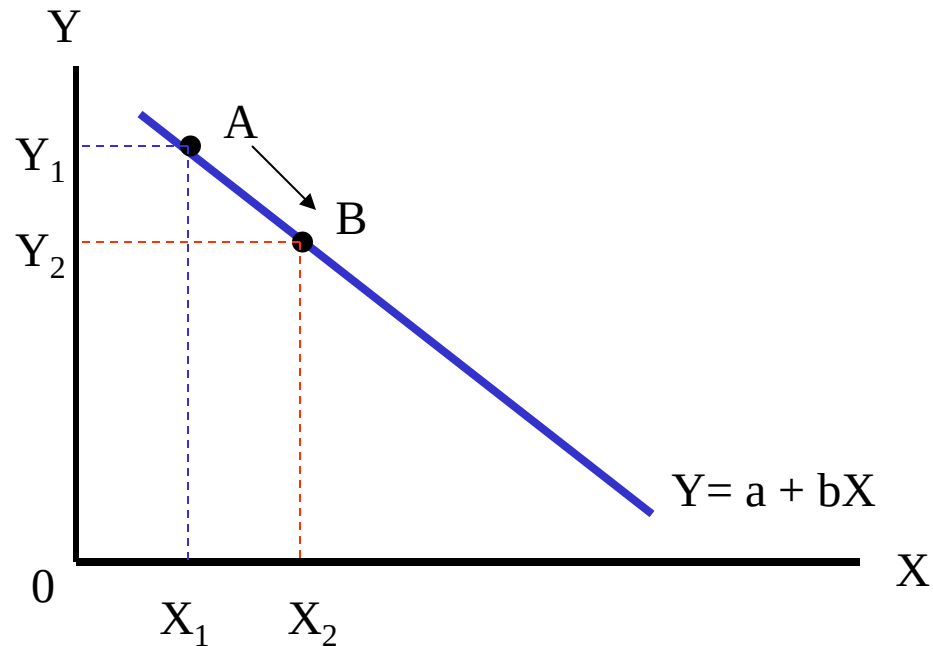


Coil spring for ball point pens

Measurement of elasticity

$$\text{The coefficient of elasticity} = \frac{\text{The percentage change in the dependent variable}}{\text{A given percentage change in the independent variable}}$$

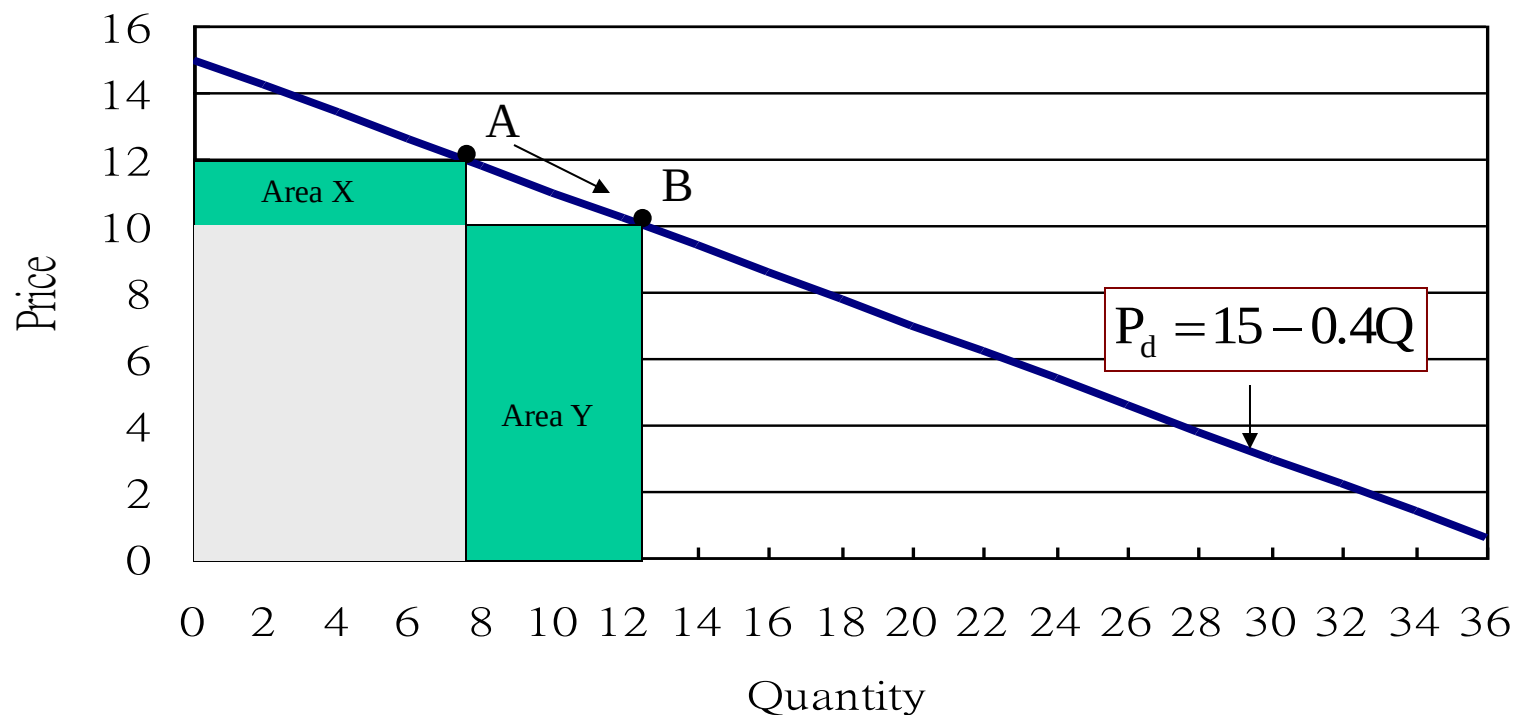
Arc Elasticity



$$\left| E_{\frac{Y}{X}} \right| = \frac{\frac{\Delta Y}{Y} \cdot 100\%}{\frac{\Delta X}{X} \cdot 100\%} = \frac{X}{Y} \cdot \frac{\Delta Y}{\Delta X} = \frac{\frac{X_2 + X_1}{2}}{\frac{Y_2 + Y_1}{2}} \cdot \frac{\Delta Y}{\Delta X}$$

Example

The Demand Curve



$$\left| E_{\frac{Y=Q_d}{X=P}} \right|_{A \rightarrow B} = \frac{\frac{P_2 + P_1}{2}}{\frac{Q_{d2} + Q_{d1}}{2}} \cdot \frac{\Delta Q_d}{\Delta P} = \frac{\frac{10 + 12}{2}}{\frac{12.5 + 7.5}{2}} \cdot \frac{5}{2} = 2.75$$

Point Elasticity

Note: In economic analysis, we often use point elasticity (instead of arc elasticity) to measure the responsiveness of the dependent variable to a change in the independent variable from an initial value point to a new one.

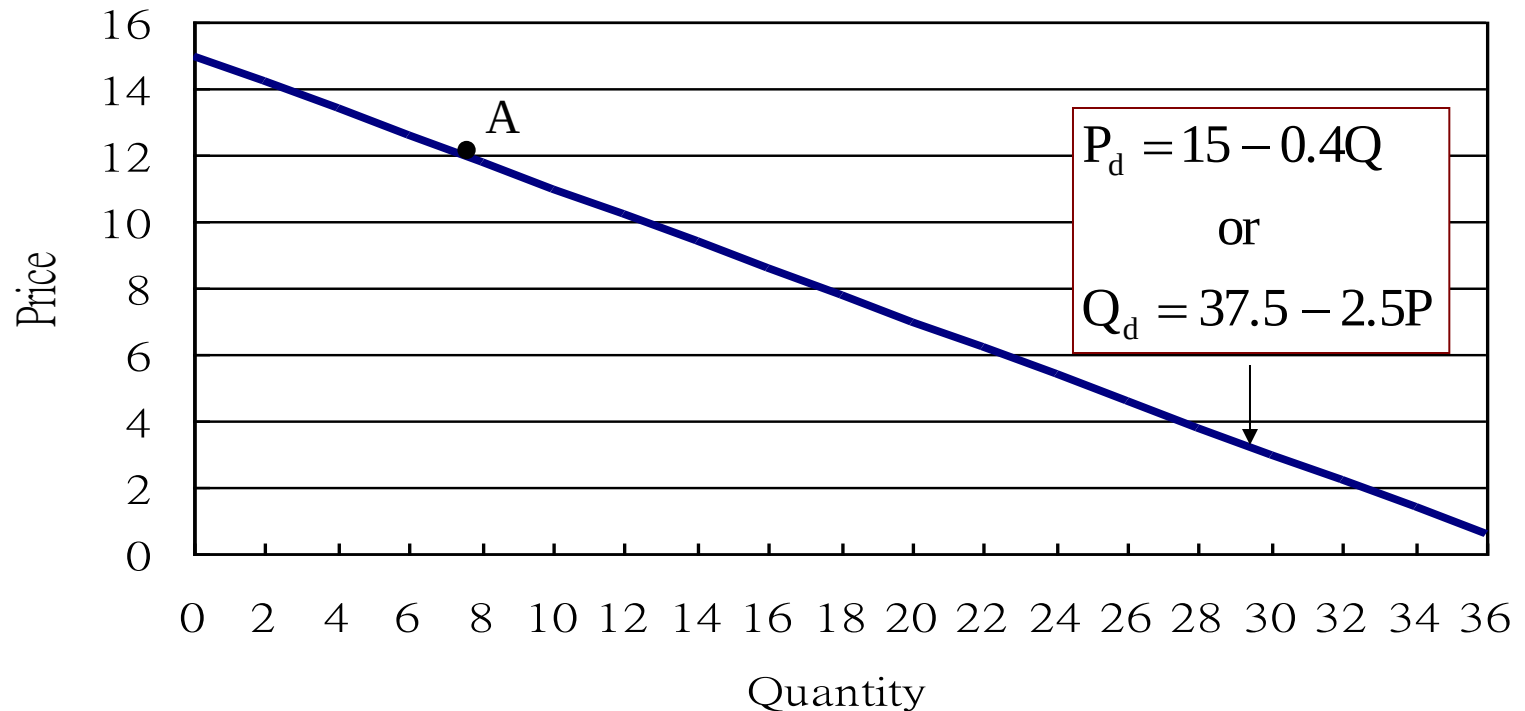
$$\varepsilon_{\frac{Y}{X}} = \frac{X}{Y} \cdot \frac{dY}{dX}$$

Where $\frac{dY}{dX}$ or $\frac{d}{dX} Y$

is the slope of the function of Y in terms of X , that is, $Y = f(X)$

Example

The Demand Curve



$$\begin{aligned}\varepsilon_{\frac{Y=Q_d}{X=P}} &= \frac{P}{Q_d} \cdot \frac{d}{dP} Q_d = \frac{12}{7.5} \cdot \frac{d}{dp} (37.5 - 2.5P) \\ &= \frac{12}{7.5} \cdot (-2.5) = -4\end{aligned}$$

Own Price Elasticity of Demand

Definition

$$\frac{\Delta P_X}{P_X} \times 100\% \longrightarrow \frac{\Delta Q_X}{Q_X} \times 100\%$$

10% ? %

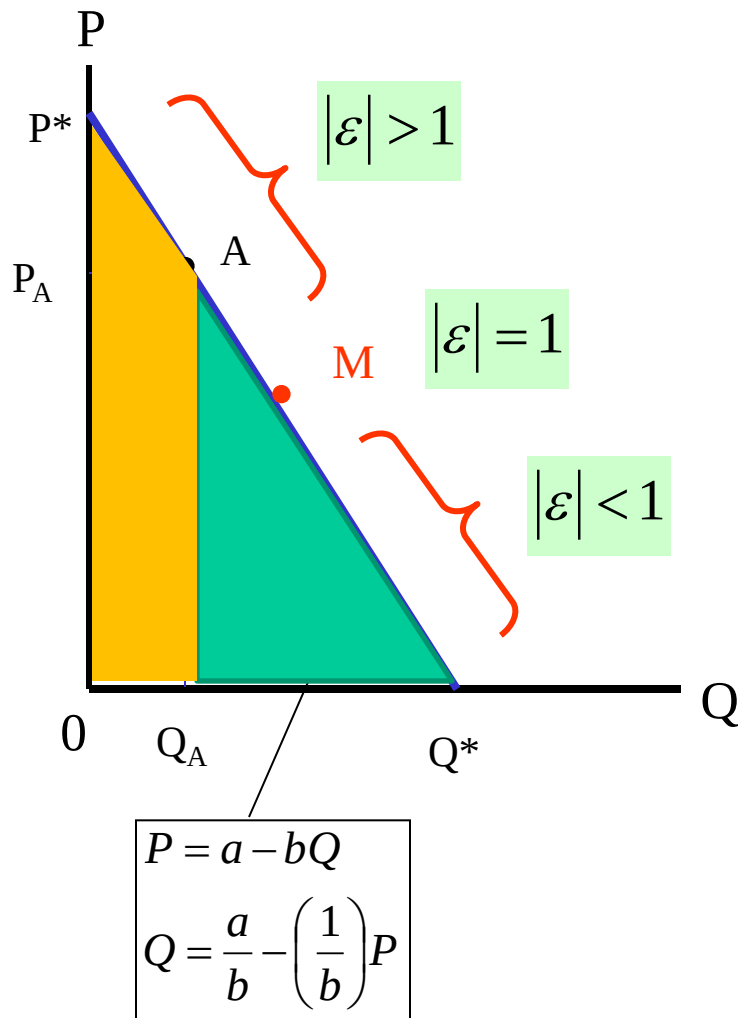
Own price elasticity is a measure of responsiveness of the quantity demanded of good X to a change in its price.



The coefficients of point elasticity along a linear demand curve are identical. True or false?

How about an hyperbolic function in the form of $Y=k/X$ (where k is a constant) ?

The determination of Point Demand elasticity: Geometric approach

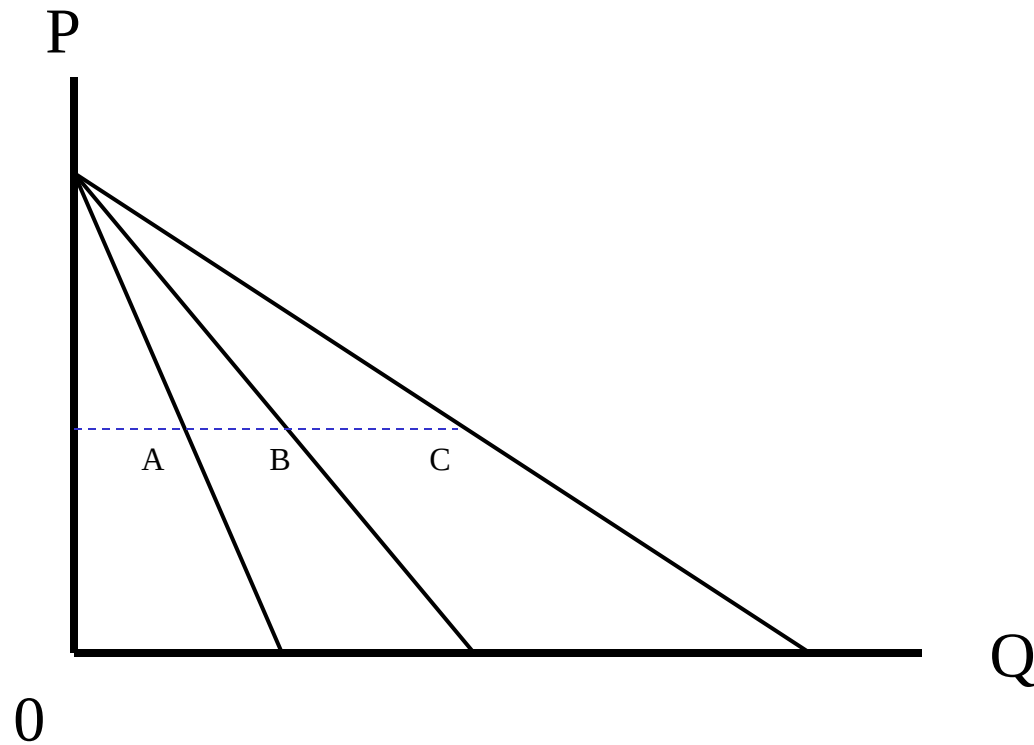


$$\begin{aligned} \left| \varepsilon_{\frac{Q}{P}} \right| &= \frac{p}{Q} \cdot \frac{d}{dP} Q \\ &= \frac{0P_A}{0Q_A} \cdot \frac{1}{\frac{AQ_A}{Q^*Q_A}} \\ &= \frac{0P_A}{0Q_A} \cdot \frac{Q^*Q_A}{AQ_A} \\ &= \frac{0P_A}{0Q_A} \cdot \frac{Q^*Q_A}{0P_A} \quad \text{since } AQ_A = 0P_A \\ &= \frac{Q^*Q_A}{0Q_A} = \frac{AQ^*}{AP^*} = \frac{0P_A}{P_A P^*} \end{aligned}$$

(In $\triangle P^*OQ^*$: lines AQ_A and OP^* are parallel, lines AP_A and OQ^* are parallel.)



The figure below shows three points (A, B, and C) located on three demand curves with different slopes. Please compare the coefficients of the point demand elasticity at points A, B, and C, that is, ϵ_A , ϵ_B , and ϵ_C

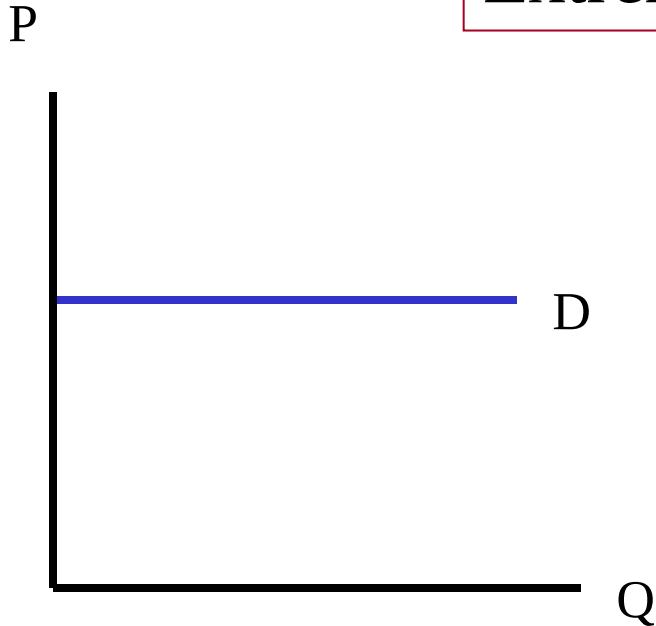


$$\left| E_{Q_X, P_X} \right| > 1 \quad \text{Elastic: (sensitive)}$$

$$\left| E_{Q_X, P_X} \right| = 1 \quad \text{Unitary elastic}$$

$$\left| E_{Q_X, P_X} \right| < 1 \quad \text{Inelastic: (insensitive)}$$

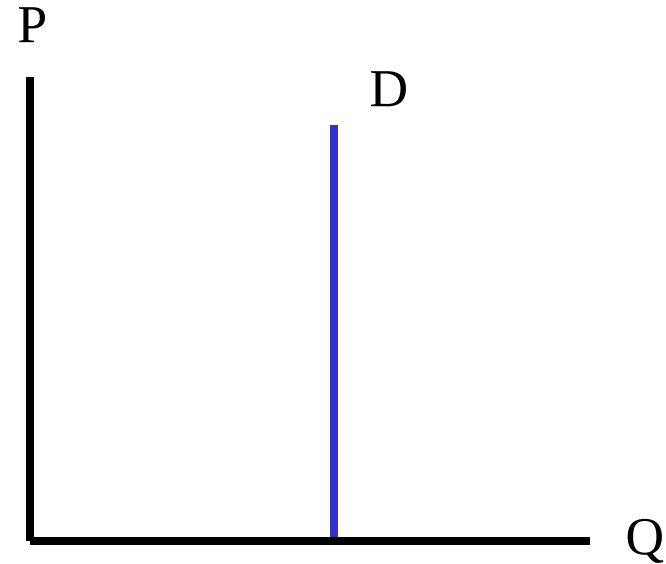
Extreme cases



$$E_d = \infty$$

Perfectly elastic demand:

Demand is perfectly elastic if the own price elasticity is infinite in absolute value. In this case the demand curve is horizontal.



$$E_d = 0$$

Perfectly inelastic demand:

Demand is perfectly inelastic if the own price elasticity is zero. In this case the demand curve is vertical.

Own-Price Elasticity and Total Revenue

- Demand is inelastic
 - Increase in price leads to an increase in total revenue.
- Demand is unitary elastic
 - Total revenue is maximized at the point where demand is unitary elastic.
- Demand is elastic
 - Increase in price leads to a decrease in total revenue.

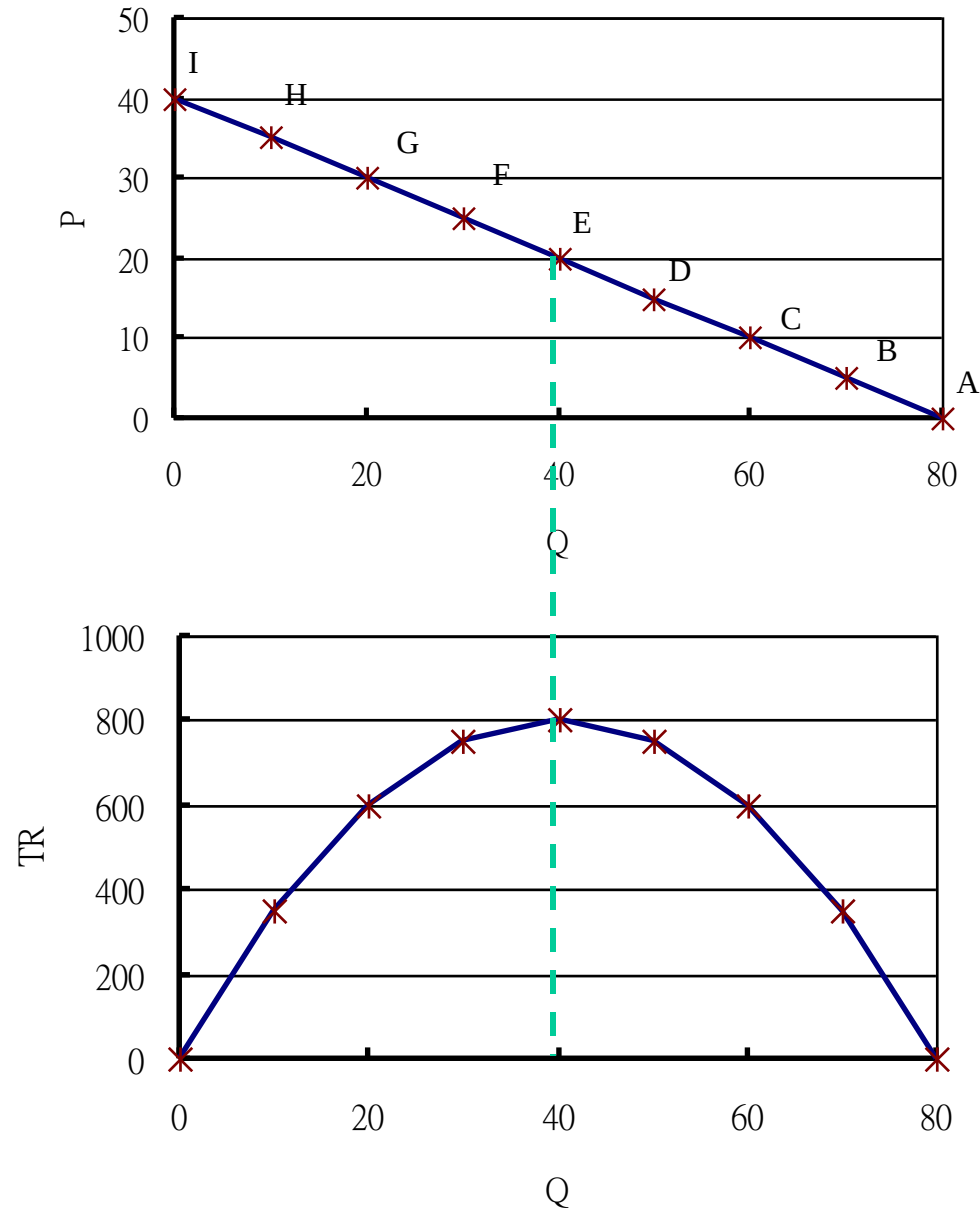
Example

$$P_X = 40 - 0.5Q_X$$

$$Q_X = 80 - 2P_X$$

$$\varepsilon_{\frac{Q_X=70}{P_X=5}} = \frac{p_X}{Q_X} \cdot \frac{d}{dP} Q = \frac{5}{70} \cdot (-2) = -0.14$$

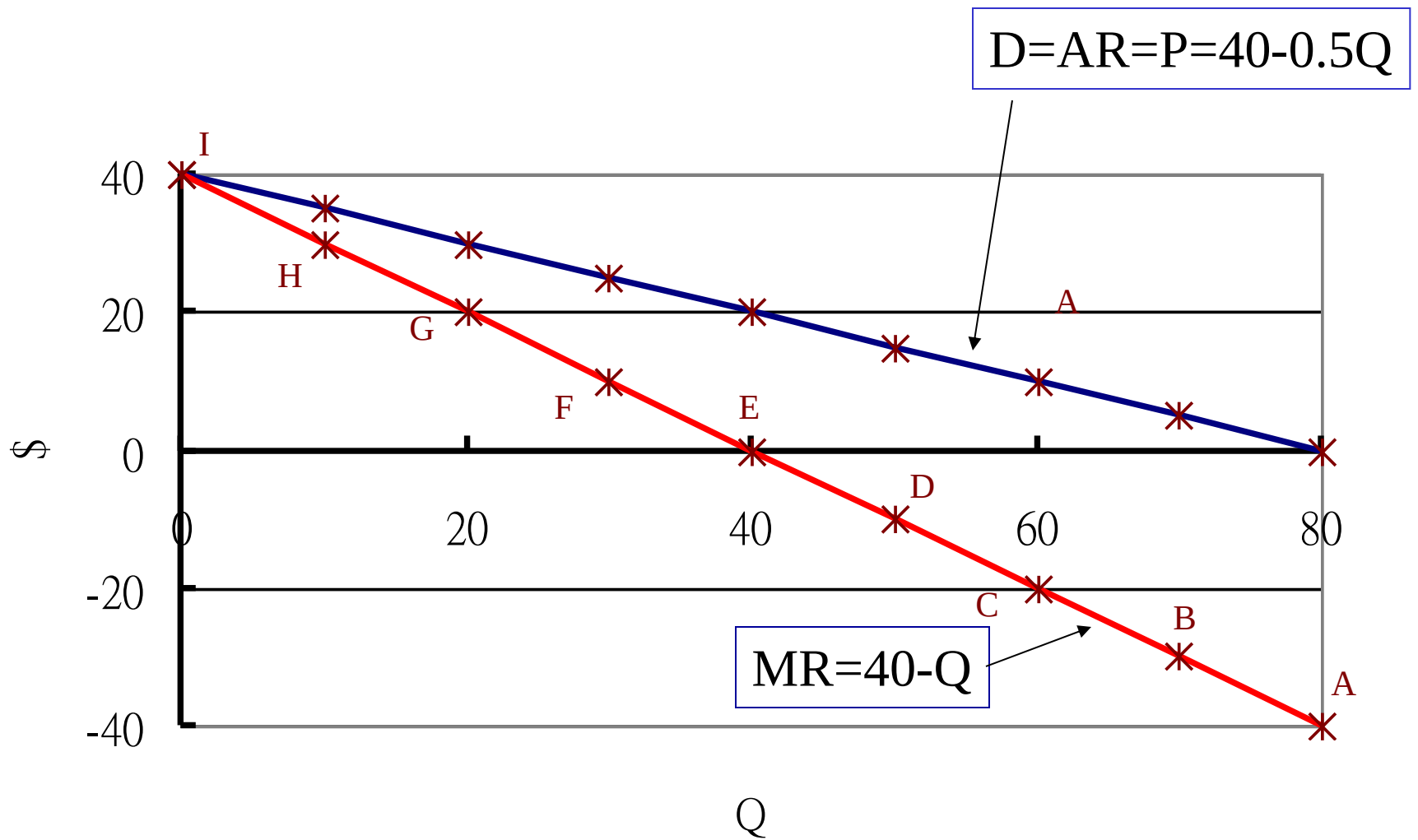
Obs. point	Price of Software (P_X)	Quantity of Software sold (Q_X)	Own Price Elasticity $\varepsilon_{\frac{Q_X}{P_X}}$	Total Revenue $TR = P_X \times Q_X$	Marginal Revenue $MR = \frac{\Delta TR}{\Delta Q_X}$	Average Revenue $AR = \frac{TR}{Q_X} = P_X$
A	0	80	0	0	-40	0
B	5	70	-0.14	350	-30	5
C	10	60	-0.3.3	600	-20	10
D	15	50	-0.60	750	-10	15
E	20	40	-1.00	800	0	20
F	25	30	-1.67	750	10	25
G	30	20	-3.0	600	20	30
H	35	10	-7.0	350	30	35
I	40	0	∞	0	40	40



$(\epsilon < 1) : A - E$

$(\epsilon > 1) : E - I$

$(\epsilon = 1) : E$



Manager's Problem

If $E_d = -1.6$, how much should a manager lower price to increase total sales by 20%?

$$\% \Delta Q / \% \Delta P = -1.6$$

$$(0.2 \div x = -1.6 ; x = -.125 \text{ or } -12.5\%)$$

Same Question with $E_d = -0.6$?

$$(0.2 \div x = -0.6 ; x = -.333 \text{ or } -33.3\%)$$

Factors Affecting Price Elasticity

- Available Substitutes
 - The more substitutes available for the good, the more elastic the demand.
- Time
 - Demand tends to be more inelastic in the short term than in the long term.
- Expenditure (income) Share
 - Goods that comprise a small share of consumer's budgets tend to be more inelastic than goods for which consumers spend a large portion of their incomes.



Why do luxury goods have high elastic demand?

why do necessities have inelastic demand?

Other Coefficients of Elasticity and their applications

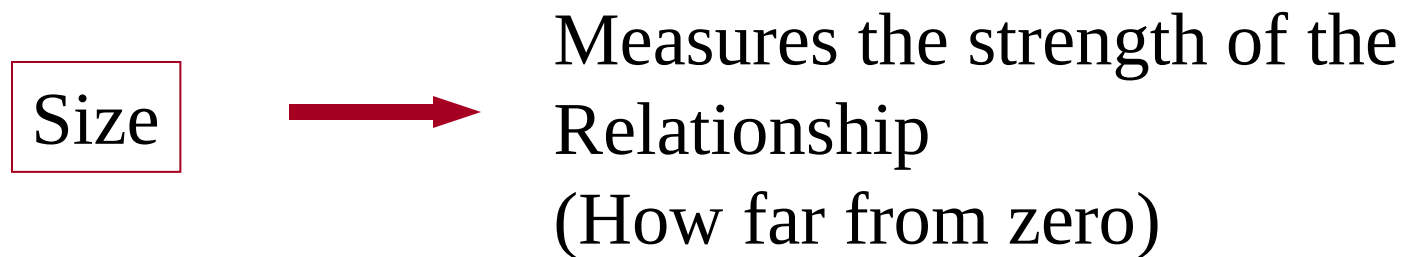
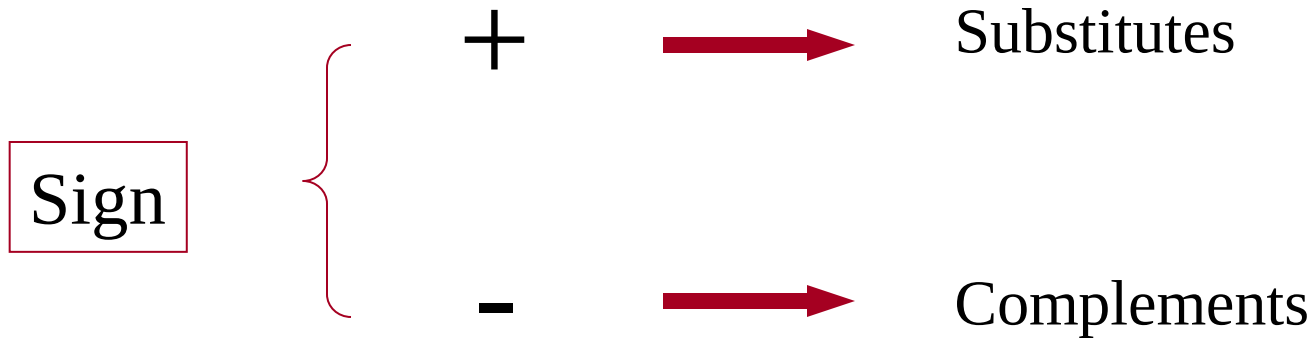
Cross-Price Elasticity

It is a measure of the responsiveness of the demand for a good to changes in the price of a related good (substitute or complement).

Assuming an autonomous change in the price of Good Y, other things being equal, the responsiveness of quantity demanded for good X can be expressed as:

$$E_{\frac{Q_X^d}{P_Y}} = \frac{\% \Delta Q_X^d}{\% \Delta P_Y}$$

$$E_{\frac{Q_X^d}{P_Y}} = \frac{P_Y}{Q_X} \cdot \frac{\Delta Q_X}{\Delta P_Y}$$



Own price elasticity of supply

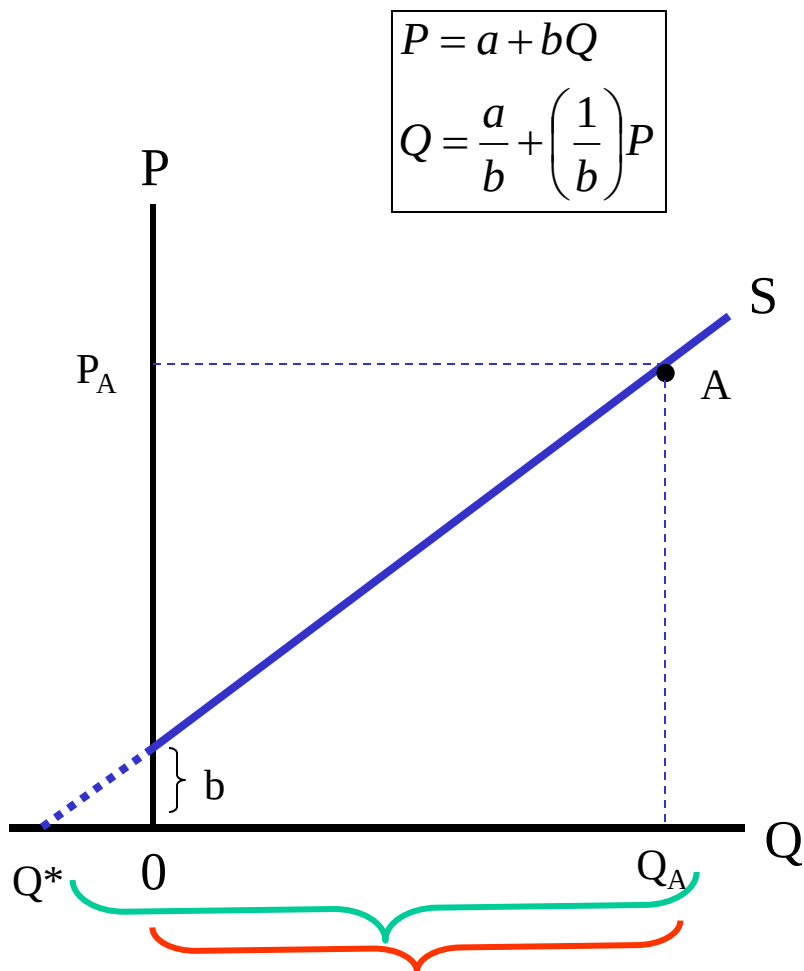
Own price elasticity of supply is a measure of responsiveness of the quantity supplied of good X to a change in its price.

$$\varepsilon_{\frac{Q_X^s}{P}} = \frac{P}{Q_X^s} \cdot \frac{\partial Q_X^s}{\partial P} = \beta_P \cdot \frac{P}{Q_X^s}$$



Are the coefficients of point elasticity of supply along a linear supply curve identical?

Formula for point price elasticity of a linear supply curve



$$\begin{aligned}
 \left| \varepsilon_{\frac{Q}{P}} \right| &= \frac{P}{Q} \cdot \frac{d}{dP} Q \\
 &= \frac{0P_A}{0Q_A} \cdot \frac{1}{\frac{AQ_A}{Q^*Q_A}} \\
 &= \frac{0P_A}{0Q_A} \cdot \frac{Q^*Q_A}{AQ_A} \\
 &= \frac{0P_A}{0Q_A} \cdot \frac{Q^*Q_A}{0P_A} \quad \text{since } AQ_A = 0P_A \\
 &= \frac{Q^*Q_A}{0Q_A}
 \end{aligned}$$

$$|\varepsilon| = \frac{Q^*Q_A}{0Q_A} > 1$$

$$\text{If } b = 0; |\varepsilon| = 1$$

$$\text{If } b < 0; |\varepsilon| < 1$$

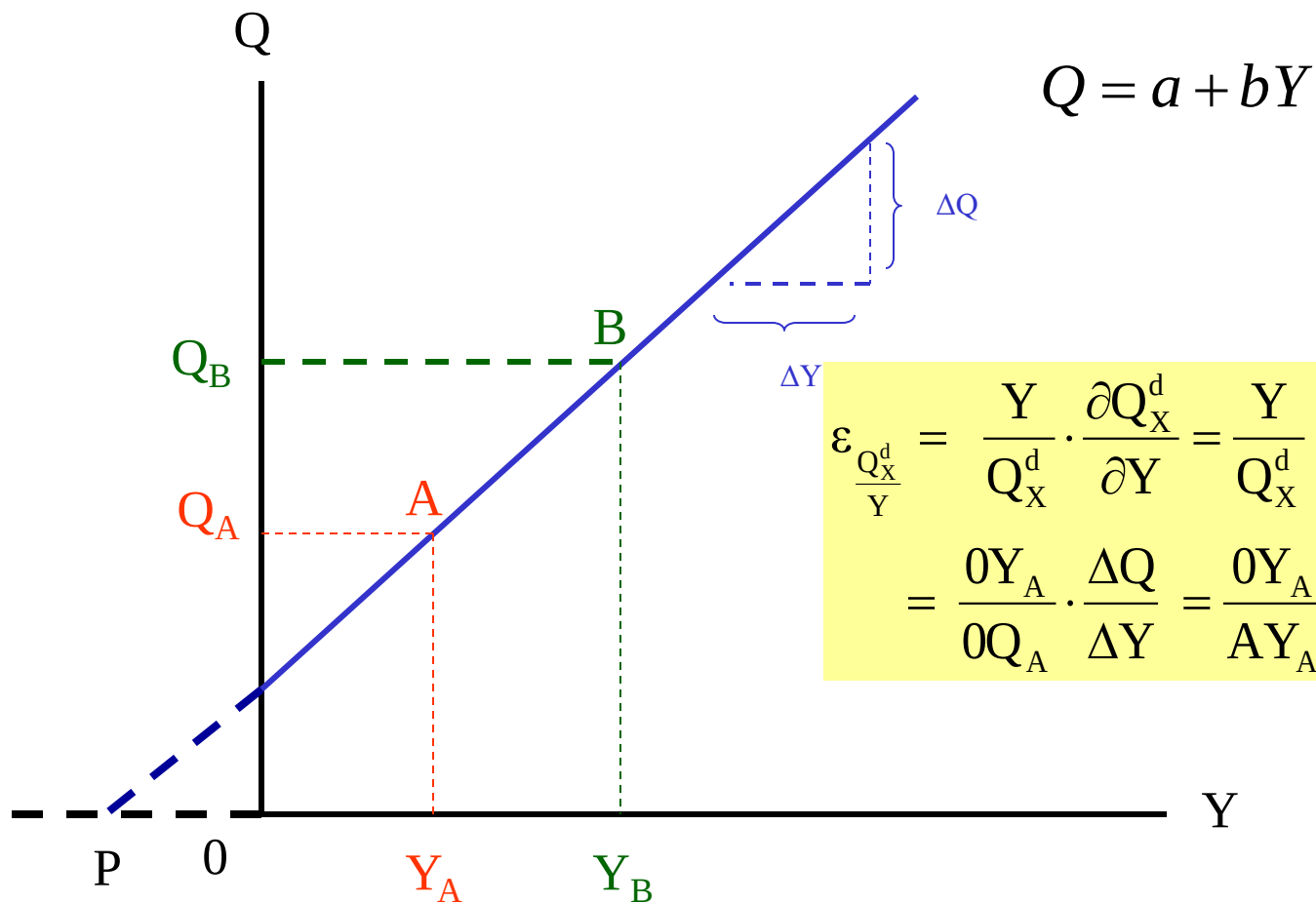
Income Elasticity of demand

The coefficient of income elasticity = $\frac{\% \text{ change in the quantity demanded}}{\% \text{ change in income}}$

$$\varepsilon_{\frac{Q_X^d}{Y}} = \frac{Y}{Q_X^d} \cdot \frac{\partial Q_X^d}{\partial Y} = \frac{Y}{Q_X^d} \cdot \beta_Y$$

$\varepsilon_{\frac{Q_X^d}{Y}} > 1$; X is a superior good

$\varepsilon_{\frac{Q_X^d}{Y}} < 1$; X is a inferior good



$$\begin{aligned}\epsilon_{\frac{Q_X^d}{Y}} &= \frac{Y}{Q_X^d} \cdot \frac{\partial Q_X^d}{\partial Y} = \frac{Y}{Q_X^d} \cdot \beta_Y \\ &= \frac{\partial Y_A}{\partial Q_A} \cdot \frac{\Delta Q}{\Delta Y} = \frac{\partial Y_A}{\Delta Y_A} \cdot \frac{\Delta Y_A}{\Delta Y_A} = \frac{\partial Y_A}{\Delta Y_A}\end{aligned}$$



Are the coefficient of income elasticity at points A and B identical? If not, which one is larger?

Interest rate elasticity of investment

$$\hat{I} = f(r; Y, \pi^e)$$

$$\hat{I} = \beta_0 - \beta_r r + \beta_Y Y + \beta_{\pi^e} \pi^e$$

The coefficient of interest rate elasticity = $\frac{\% \text{ change in the investment expenditure}}{\% \text{ change in interest rate}}$

$$\varepsilon_{\frac{I}{r}} = \frac{\tilde{r}}{\tilde{I}} \cdot \frac{\partial I}{\partial r} = \frac{\tilde{r}}{\tilde{I}} \cdot \beta_r$$

Interest rate elasticity of money demand

$$\hat{I} = f(r; Y, \pi^e)$$

$$\hat{M}^d = \beta_0 - \beta_r r + \beta_Y Y$$

The coefficient of interest rate elasticity = $\frac{\% \text{ change in money demand}}{\% \text{ change in interest rate}}$

$$\varepsilon_{\frac{M^d}{r}} = \frac{\tilde{r}}{\tilde{M}^d} \cdot \frac{\partial M^d}{\partial r} = \frac{\tilde{r}}{\tilde{M}^d} \cdot \beta_r$$



(1) When the economic cycle is at the upturn phase

a. $\left| \varepsilon_{\frac{I}{r}} \right| > 1$

b. $\left| \varepsilon_{\frac{I}{r}} \right| < 1$

(2) When $\left| \varepsilon_{\frac{M^d}{r}} \right| \rightarrow \infty$

a. The economy is in depression .

b. The economy is in prosperity .

Import/Export price elasticity

The coefficient of import/export price elasticity = $\frac{\begin{array}{c} \% \text{ change in the quantity} \\ \text{imported/ exported} \end{array}}{\% \text{ change in import/export price}}$

$$\epsilon_{\frac{Q_X^d}{P_X}} = \frac{P_X}{Q_X^d} \cdot \frac{\partial Q_X^d}{\partial P_X} = \frac{P_X}{Q_X^d} \cdot \beta_{P_X}$$

$$\epsilon_{\frac{Q_M^d}{P_M}} = \frac{P_M}{Q_M^d} \cdot \frac{\partial Q_M^d}{\partial P_M} = \frac{P_M}{Q_M^d} \cdot \beta_{P_M}$$

Where P_X and Q_X^d is export prices in foreign currency and foreign demand for domestic exports respectively.

Where P_M and Q_M^d is import prices in domestic currency and domestic demand for imports respectively.



(1) Which of the following is the export price elasticity of China to the US?

a. $\left| \varepsilon_{\frac{Q_X^d}{P_X}} \right| > 1$

b. $\left| \varepsilon_{\frac{Q_X^d}{P_X}} \right| < 1$

(2) Which of the following is the import price elasticity of US's imports from China

a. $\left| \varepsilon_{\frac{Q_M^d}{P_M}} \right| > 1$

b. $\left| \varepsilon_{\frac{Q_M^d}{P_M}} \right| < 1?$

Thank You
for Your Attention