

Control + Automation ExamLewis Christopher
1909572Question 4

$$a) \quad 7.5 \frac{d^2 \theta_o}{dt^2} + 40 \frac{d\theta_o}{dt} + 300 \theta_o = 800 \theta_i$$

$$\frac{d^2 \theta_o}{dt^2} + \frac{40}{7.5} \frac{d\theta_o}{dt} + \frac{300}{7.5} \theta_o = \frac{800}{7.5} \theta_i$$

Natural frequency

$$\omega_n^2 = \frac{300}{7.5}$$

$$\omega_n = \sqrt{40}$$

$$\omega_n = 6.32 \text{ Hz}$$

Damping ratio

$$2 \zeta \omega_n = \frac{40}{7.5}$$

$$\zeta = \frac{5.3}{(2)(6.32)}$$

$\zeta = 0.422$, $\therefore$ the system is underdamped.

$$\begin{aligned} b) \quad \% \text{ Peak overshoot} &= \exp \left(\frac{-\zeta \pi}{\sqrt{1-\zeta^2}} \right) \times 100 \\ &= \exp \left(\frac{-0.422 \pi}{\sqrt{1-(0.422^2)}} \right) \times 100 \\ &= (0.232) \times 100 \\ &= 23.2 \% \end{aligned}$$

Rise time

$$t_r = \frac{\pi}{\omega_n} = \frac{\pi}{6.32}$$