

Complex numbers worksheet

Q1) Given the following complex numbers:

$$z_1 = 5 + 2j \quad z_2 = 1 - j \quad z_3 = 3 + 5j$$

Calculate the following:

$$(i) z_1 + z_2 \quad (ii) z_1 - z_2 \quad (iii) \frac{z_1}{z_2} \quad (iv) z_1 z_2 z_3 \quad (v) \overline{z_1} - z_2 \quad (vi) (z_1)^2 - z_2$$

Q2) Express the following in polar form:

$$(i) -2 - 7j \quad (ii) -5 + 2j$$

Q3) Solve the following quadratic equation, giving your answers in polar form:

$$7x^2 - 3x + 6 = 0$$

Q4)

Calculate the following, expressing your answers in rectangular form:

$$(i) (3 + 7j)^5$$

$$(ii) (-4 + 5j)^6$$

$$Q1) \quad z_1 = 5 + 2j \quad z_2 = 1 - j \quad z_3 = 3 + 5j$$

$$(i) \quad z_1 + z_2 = (5 + 2j) + (1 - j) \\ = 6 + j$$

$$(ii) \quad z_1 - z_2 = (5 + 2j) - (1 - j) \\ = 4 + 3j$$

$$(iii) \quad z_1 z_2 z_3 = (5 + 2j)(1 - j)(3 + 5j) \\ = (5 - 5j + 2j - 2j^2)(3 + 5j) \\ = (7 - 3j)(3 + 5j) \\ = 21 + 35j - 9j - 15j^2 \\ = 36 + 26j$$

$$(iv) \quad \frac{z_1}{z_2} = \frac{(5 + 2j)}{(1 - j)} \times \frac{(1 + j)}{(1 + j)} \\ = \frac{5 + 5j + 2j + 2j^2}{1 + j - j - j^2} \\ = \frac{3 + 7j}{2} \\ = \frac{3}{2} + \frac{7}{2}j$$

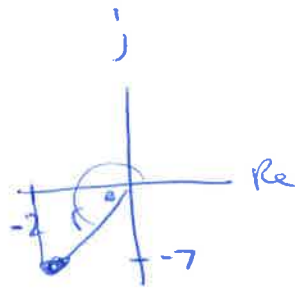
$$(v) \quad \overline{z_1} - z_2 = (5 - 2j) - (1 - j) \\ = 4 - j$$

↑
conjugate

$$(vi) \quad (z_1)^2 - z_2 = (5 + 2j)(5 + 2j) - (1 - j) \\ = (25 + 20j + 4j^2) - (1 - j) \\ = (24 + 20j) - (1 - j) \\ = 23 + 21j$$

Q2)

(i) $-2 - 7j$



$$r = \sqrt{2^2 + 7^2}$$

$$r = \sqrt{53} = 7.28$$

$$\tan \phi = \frac{7}{2}$$

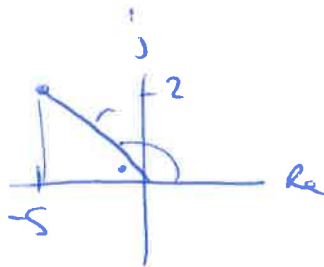
$$\phi = 74.05^\circ$$

$$\theta = 180 + 74.05^\circ$$

$$\theta = 254.05^\circ$$

$$= 7.28 (\cos 254.05^\circ + j \sin 254.05^\circ)$$

(ii) $-5 + 2j$



$$r = \sqrt{5^2 + 2^2}$$

$$r = \sqrt{29} = 5.39$$

$$\tan \phi = \frac{2}{5}$$

$$\phi = 21.80^\circ$$

$$\theta = 180 - 21.80^\circ$$

$$\theta = 158.2^\circ$$

$$= 5.39 (\cos 158.2^\circ + j \sin 158.2^\circ)$$

Q3)

$$7x^2 - 3x + 6 =$$

$$x = \frac{-(-3) \pm \sqrt{(3)^2 - 4(7)(6)}}{2(7)}$$

$$x = \frac{3 \pm \sqrt{9 - 168}}{14}$$

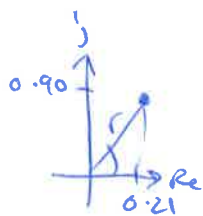
$$x = \frac{3 \pm \sqrt{-159}}{14}$$

$$x = \frac{3 \pm \sqrt{159} \sqrt{-1}}{14}$$

$$x = \frac{3 \pm 12.61j}{14}$$

$$x = \frac{3}{14} + \frac{12.61}{14}j$$

$$x = 0.21 + 0.90j$$



$$r = \sqrt{(0.21)^2 + (0.90)^2}$$

$$r = \sqrt{0.04 + 0.81}$$

$$r = 0.92$$

$$\tan \theta = \frac{0.90}{0.21}$$

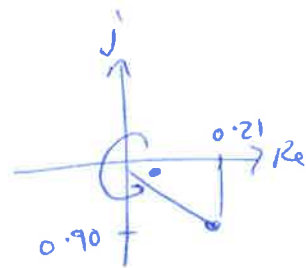
$$\theta = 76.87^\circ$$

$$0.92 (\cos 76.87^\circ + j \sin 76.87^\circ)$$

$$x = \frac{3}{14} - \frac{12.61}{14}j$$

$$x = 0.21 - 0.90j$$

$$r = 0.92$$



$$\tan \theta = \frac{0.90}{0.21}$$

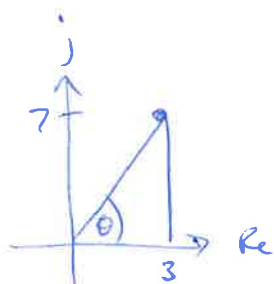
$$\theta = 76.87^\circ$$

$$\theta = 283.13^\circ$$

$$0.92 (\cos 283.13^\circ + j \sin 283.13^\circ)$$

Q4) $(3+7j)^5$

(i)



$$r = \sqrt{3^2 + (7)^2}$$

$$\tan \theta = 7/3$$

$$r = \sqrt{58} = \underline{7.62}$$

$$\theta = \underline{66.80^\circ}$$

$$DMT = r^n (\cos n\theta + j \sin n\theta)$$

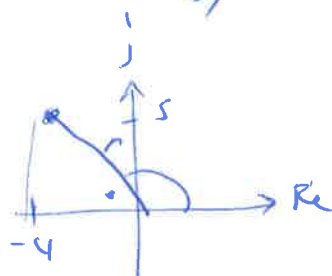
$$(7.62)^5 (\cos 334^\circ + j \sin 334^\circ)$$

$$25690.64 (\cos 334^\circ + j \sin 334^\circ)$$

Multiply back out for rectangular form:

$$\underline{23090.59 - 11262.04j}$$

(ii) $(-4+5j)^6$



$$r = \sqrt{(-4)^2 + (5)^2}$$

$$\tan \theta = 5/4$$

$$r = \sqrt{41} = \underline{6.40}$$

$$\theta = 51.34^\circ$$

$$\theta = 180 - 51.34^\circ$$

$$\underline{\underline{\theta = 128.66^\circ}}$$

$$DMT = (6.40)^6 (\cos 771.96^\circ + j \sin 771.96^\circ)$$

$$68719.48 (\cos 771.96^\circ + j \sin 771.96^\circ)$$

$$\underline{= 42345.73 + 54122.14j}$$