

# 1. ADVANCED ENGINE DESIGN

## M6X00410

LECTURE NOTES 24 MARCH.

### TORSIONAL VIBRATIONS

JUST LIKE A MASS ON A SPRING WILL VIBRATE AT THE FREQUENCY OF EXCITATION, THE SAME PHENOMENON OCCURS WITH A TORSIONAL SYSTEM OF TORSIONAL SPRINGS AND TORSIONAL INERTIA WITH AN EXCITATIONAL TORSIONAL FREQUENCY.

IN THE CASE OF AN I.C. ENGINE THE RISING AND FALLING CYLINDER PRESSURE CREATES AN EXCITATION TORQUE TO THE CRANKSHAFT AND FLYWHEEL

2.

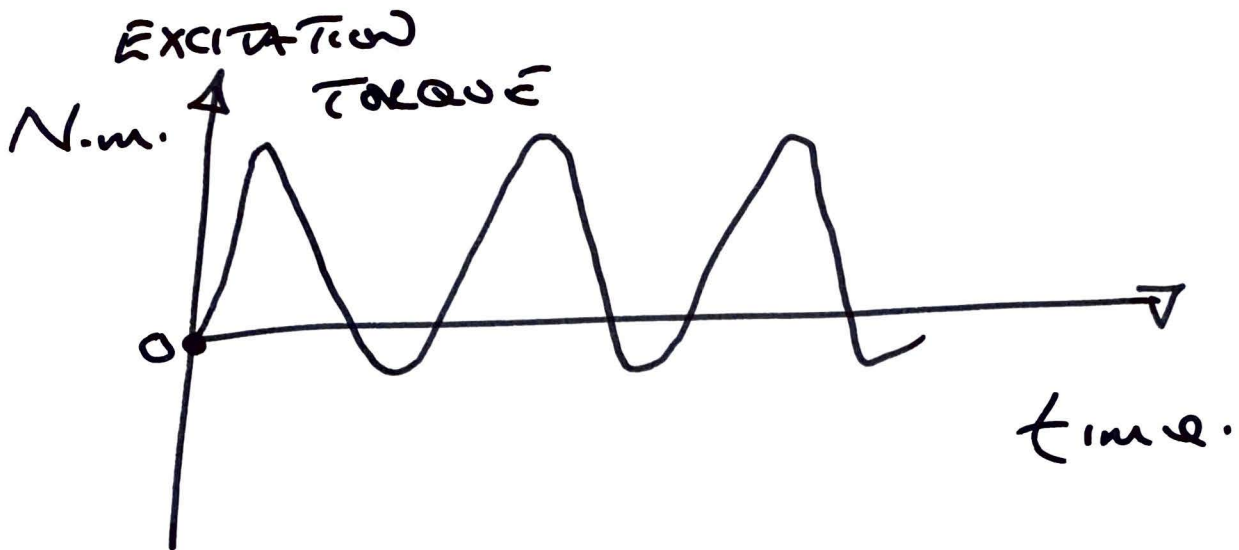
## ASSEMBLY.

THE REASON FOR THE FLYWHEEL IS TO INCREASE THE MOMENT OF INERTIA OF THE SYSTEM TO REDUCE THE AMPLITUDE OR ANGLE OF THE TORSIONAL RESPONSE OF THE END OF CLANKSHAFT AND FLYWHEEL TO REDUCE THE TORSIONAL VIBRATION INPUT INTO THE GEARBOX.

IF YOU TAKE AN IN-LINE 4 CYLINDER ENGINE (4 STROKE) THE TORQUE PULSES OCCUR EVERY  $180^\circ$ .

A TORQUE GRAPH ALONG THE CLANKSHAFT WILL LOOK ROUGHLY AS BELOW:

3.



THIS CAN BE ALTERED BY  
CALCULATING THE INDIVIDUAL  
TORQUE PER CYLINDER FROM  
THE GAS PRESSURE, ADDING  
THEM UP IN THE CORRECT  
PHASE DIFFERENCE OF  $180^\circ$   
AND KNOWING THAT THE  
POWERS REPEAT THEMSELVES  
TWICE PER REVOLUTION.

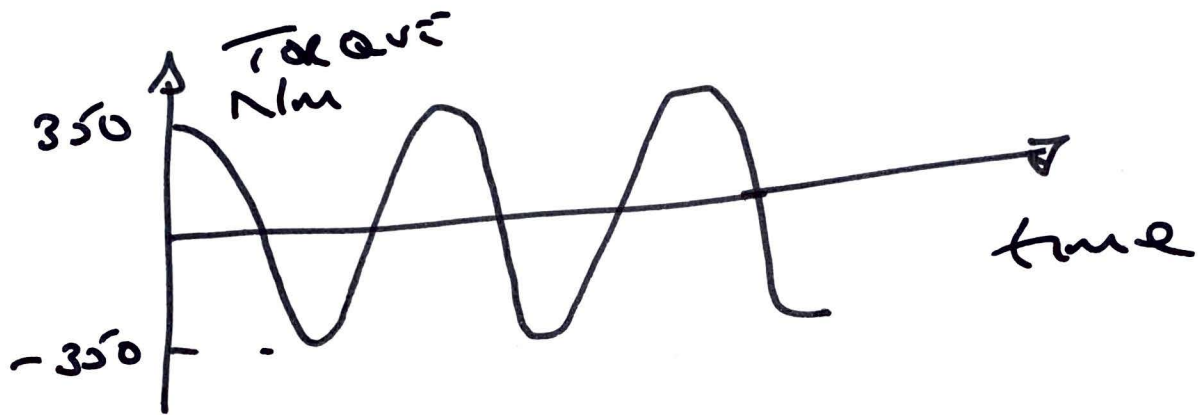
IN THE 1.6 GDI FORD, THE  
TORQUE PER CYLINDER MAY BE  
AROUND 700 Nm.

4.

ALTHOUGH A FULL FREQUENCY ANALYSIS COULD BE CALCULATED VIA A FOURIER TRANSFORM, FOR INITIAL CALCULATIONS IT IS USEFUL TO SIMPLIFY THE DYNAMIC OR CHANGING PART OF THE TORQUE TO A COSINE FUNCTION OF FUND FREQUENCY AS SHOWN BELOW



5. Now a vibration point of view it is equivalent to a torque of amplitude 350 Nm changing at firing frequency as shown below:



This can be represented by

A function:

$$T = T_0 \cos 2\omega t$$

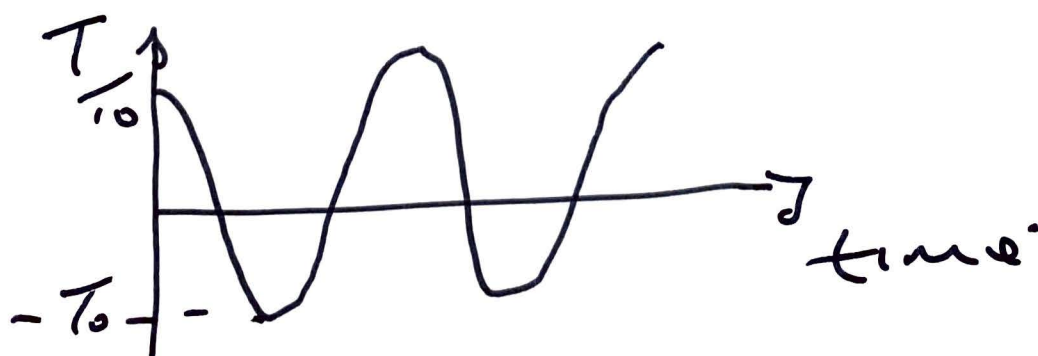
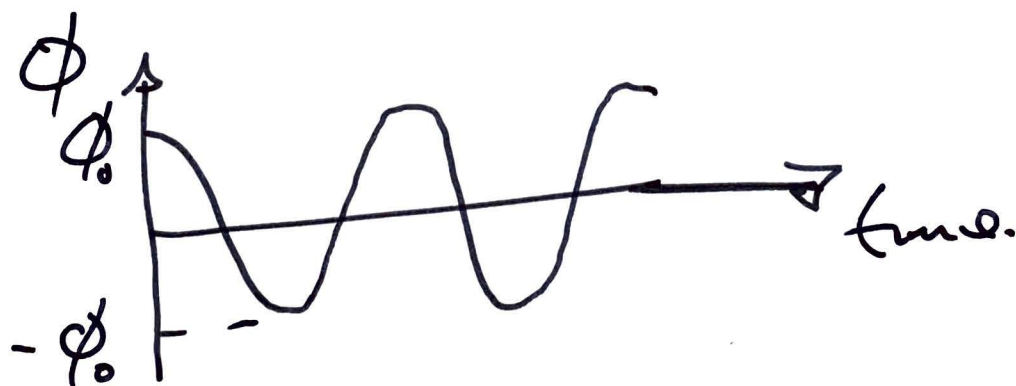
$\omega = \text{Crank Angular Velocity}$

$2\omega = \text{Firing Angular Velocity on IN-LINE 4 cylinder, 4 stroke.}$

6. LET  $I$  = MOMENT OF  
INERTIA OF  
CENTERSHAFT/  
FLYWHEEL  
ASSEMBLY.

$$\text{LET } \phi = \phi_0 \cos 2\omega t$$

WHERE  $\phi$  IS THE ANGULAR  
OSCILLATION AMPLITUDE IN  
RADIAN AS SHOWN BELOW:



7. THIS MEANS THAT AS THE CRANKSHAFT IS ROTATING IT IS ALSO OSCILLATION BACKWARDS AND FORWARDS AT FIRENG RESONANCE/ BY A SMALL ANGLE AMPLITUDE.

FROM NEWTON'S LAW

$$F = M \cdot \frac{d^2x}{dt^2}$$

$$\text{AND } T = I \cdot \frac{d^2\phi}{dt^2} \quad \text{--- (1)}$$

(PROOF AT  
END)

$$\text{PUT } \phi = \phi_0 \cos 2\omega t$$

$$\frac{d\phi}{dt} = -2\omega \phi_0 \sin 2\omega t$$

$$\frac{d^2\phi}{dt^2} = -4\omega^2 \phi_0 \cos 2\omega t \quad \text{--- (2)}$$

8. Put into eqn 1 to

give

$$\overline{T}_0 \cos \omega t = -4\omega^2 I \phi_0 \cos \omega t$$

Dividing throughout by  $\cos \omega t$   
GIVES:

$$\overline{T}_0 = -4\omega^2 I \phi_0$$

$$\phi_0 = -\frac{\overline{T}_0}{4\omega^2 I}$$

THE MINUS SIGN MEANS  
THAT THE ANGULAR OSCILLATION  
IS OUT OF PHASE WITH THE  
EXCITATION TORQUE AS THE  
TORQUE IS BEING USED TO  
STOP THE OSCILLATION FROM

9. GOING TOO FAR EXACTLY  
LIKE THE ENGINE ON ITS  
MOUNTS ABOVE THE RESONANT  
FREQUENCY/.

### EXAMPLE

ASSUME ON AN IN-LINE  
4 CYLINDER YOU CAN REPRESENT  
THE EXCITATION TORQUE AS  
A COSINE FUNCTION AT THREE  
ENGINE SPEED OR FIRING  
FREQUENCY/ WITH AMPLITUDE  
350 Nm AT 3000 RPM.

IF MOMENT OF INERTIA  
OF CRANKSHAFT / FLYWHEEL  
ASSEMBLY IS  $0,2 \text{ kg.m}^2$

WHAT IS ANGULAR OSCILLATION AMPLITUDE  
IN DEGREES?

10.

$$\tau = 350 \cos 2\omega t$$

$$\phi = \phi_0 \cos 2\omega t$$

$$I = 0,2 \text{ kg.m}^2$$

$$\tau = I \cdot \frac{d^2\phi}{dt^2}$$

$$350 = -4\omega^2 \cdot I \cdot \phi_0$$

$$\phi_0 = - \frac{350}{4 \cdot \left( \frac{2\pi \cdot 3000}{60} \right)^2 \cdot 0,2}$$

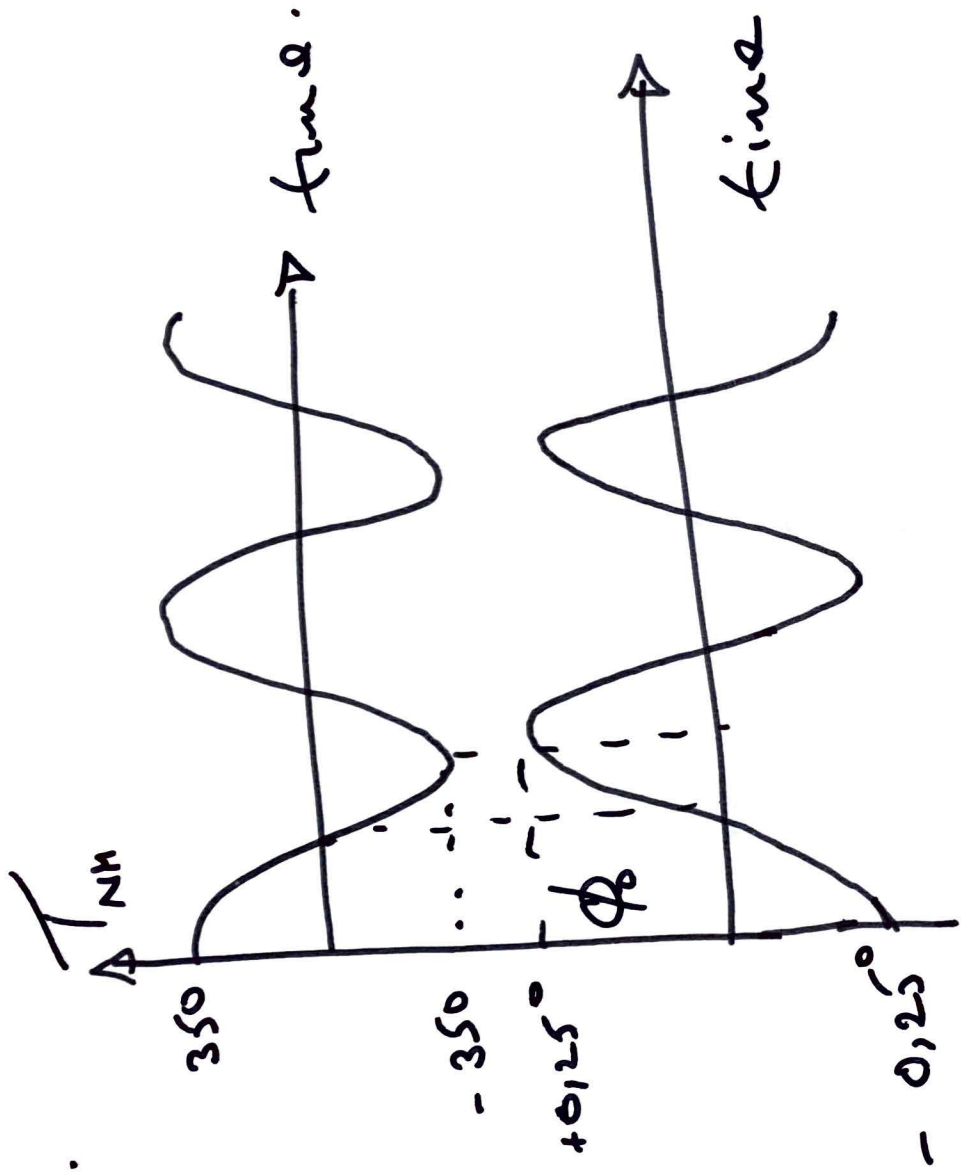
$$= -0,0044 \text{ RAD}$$

$$= -0,0044 \times \frac{360}{2\pi}^\circ$$

$$= \underline{\underline{-0,25^\circ}}$$

VIBRATION AMPLITUDE LOOKS  
LIKE FIGURE BELOW:

11.



THIS GIVES AN IDEA OF  
THE VIBRATION AMPLITUDE GROWS  
INTO THE CASEBOX.

12.

## DUAL MASS FLYWHEELS

THE AM OF A DUAL MASS FLYWHEEL IS TO SIGNIFICANTLY REDUCE THE VIBRATION AND AURAL AMPLITUDE TRANSMITTED TO THE GEARBOX.

THIS IS ACHIEVED BY MOUNTING A SECOND FLYWHEEL TO THE FIRST FLYWHEEL AND COUPLE THEM WITH TORSIONAL SPRINGS.

THE MODEL CAN BE

DEVELOPED AS BELOW ALTHOUGH FOLLOWING PART OF THE EXAM.

$$\frac{I_1 \ddot{\phi}_1}{E H_T} = \frac{I_2 \ddot{\phi}_2}{E H_T}$$

LET  $\phi_1$  = TWIST ANGLE IN RAD'S OF CLAMSHAFT / 1<sup>ST</sup> FLYWHEEL.

13.

$\phi_2 =$  Twist Angle of  
Shaft fix where in radians.

The max applied torque to  
Shaft fix where comes from  
the deflection of the coupling  
torsional spring  $k_T$ .

Torque applied to shaft  
fix where

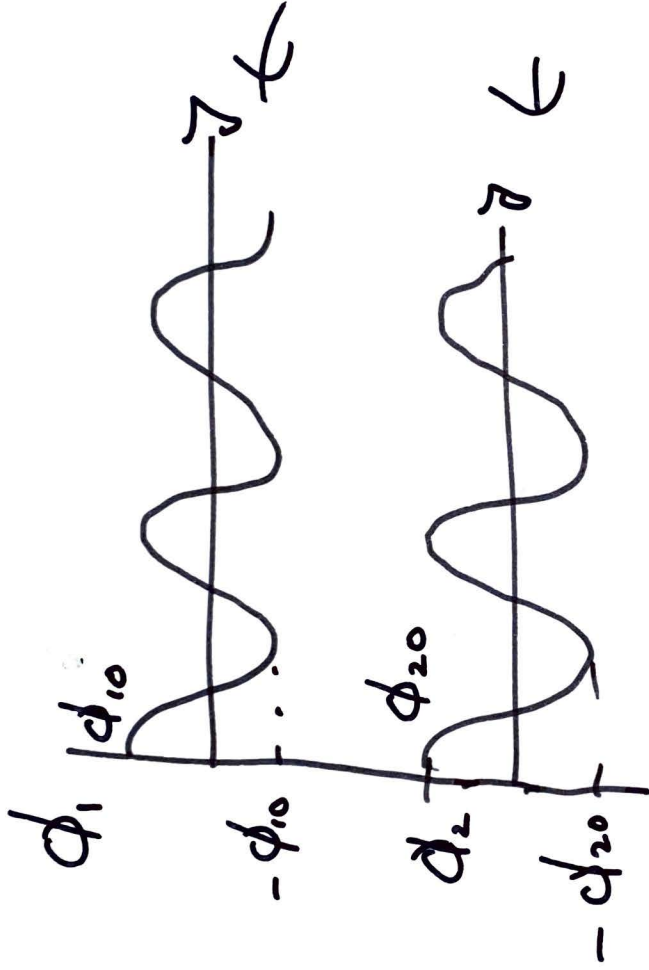
$$= k_T (\phi_1 - \phi_2)$$

$\uparrow$   $\downarrow$  NET. Torsional  
Spring Torque Deflection of  
spring shaft

$$\therefore k_T (\phi_1 - \phi_2) = I_2 \frac{d^2 \phi_2}{dt^2} \quad \text{--- (3)}$$

14.

Put  $\phi_1 = \phi_{10} \cos 2\omega t$   
 $\phi_2 = \phi_{20} \cos 2\omega t$ .



$$\frac{d^2\phi_2}{dt^2} = -4\omega^2 \phi_{20} \cos 2\omega t.$$

Put into Newton's Law

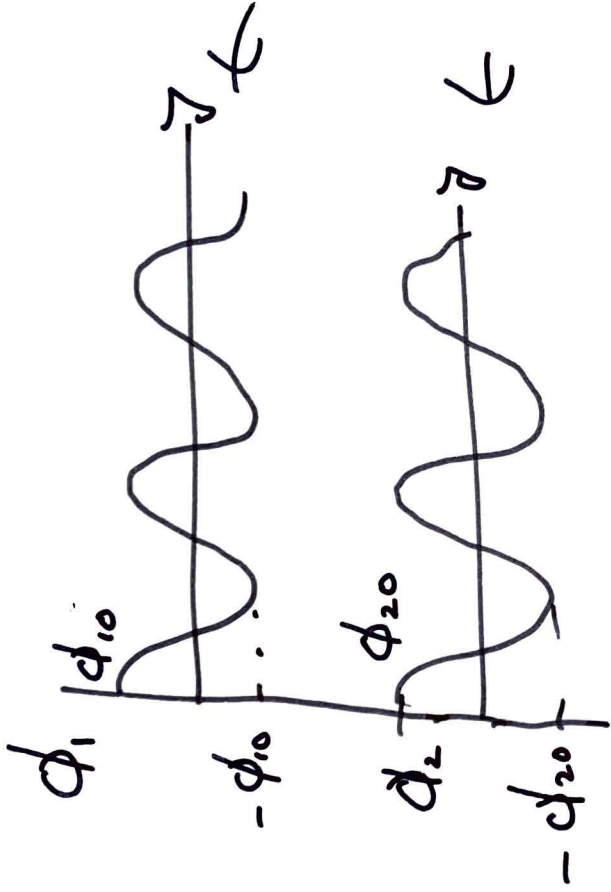
$$T = \frac{I_2 \cdot d^2\phi_2}{dt^2}$$

$$T = (\phi_{10} \cos 2\omega t - \phi_{20} \cos 2\omega t)$$

$$= -4\omega^2 I_2 \phi_{10} \cos 2\omega t$$

14.

Put  $\phi_1 = \phi_{10} \cos \omega t$   
 $\phi_2 = \phi_{20} \cos 2\omega t$ .



$$\frac{d^2 \phi_2}{dt^2} = -4\omega^2 \phi_{20} \cos 2\omega t.$$

Put into Newton's law

$$T = \frac{I_2 \cdot d^2 \phi_2}{dt^2}$$

$$\begin{aligned} \tau &= (\phi_{10} \cos \omega t - \phi_{20} \cos 2\omega t) \\ &= -4\omega^2 I_2 \phi_{20} \cos 2\omega t \end{aligned}$$

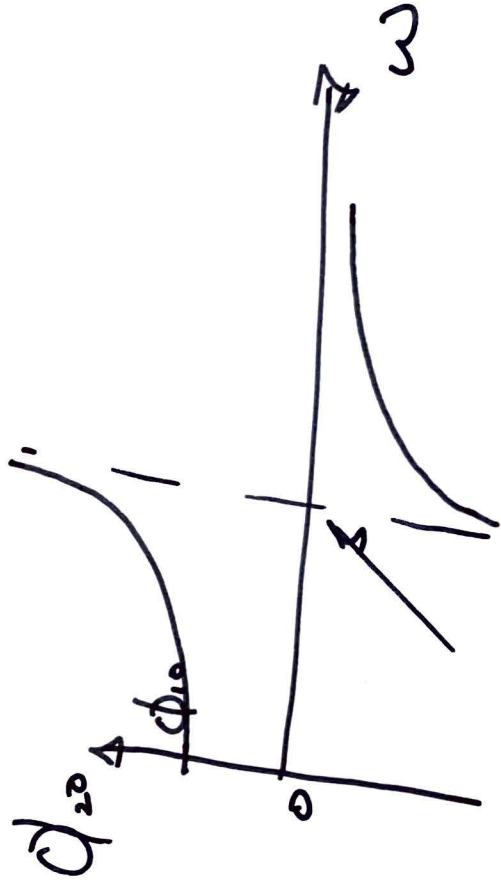
15. Divide everything by  $\cos 2\omega t$  gives

$$k_{1T} (\phi_{10} - \phi_{20}) = -4\omega^2 I_2 \phi_2$$

REARRANGING

$$\begin{aligned} k_{1T} \phi_{10} &= k_{1T} \phi_{20} - 4\omega^2 I_2 \phi_{20} \\ &= \phi_{20} \{ k_{1T} - 4\omega^2 I_2 \} \end{aligned}$$

$$\phi_{20} = \frac{k_{1T} \phi_{10}}{\{ k_{1T} - 4\omega^2 I_2 \}}$$



RESONANCE WHEN

$$k_{1T} - 4\omega^2 I_2 = 0$$

16.

SET RESONANCE UP AT AROUND  
250 RPM TO AVOID ANY/  
HAGAROOING OR JERKING WHEN  
DRIVING.

THAT IS, CALCULATE  $\tau_{17}$  BY  
PUTTING  $\omega = \frac{2\pi}{60} \times 250 \text{ RAD/sec.}$

RESONANCE IS WHEN

$$\tau_{17} = 4\omega^2 I_2$$

ASSUME  $I_2 = 0.1 \text{ kg.m}^2$

$$\begin{aligned} \therefore \tau_{17} &= 4 \cdot \left( \frac{2\pi}{60} \times 250 \right)^2 \times 0.1 \\ &= 2744 \text{ Nm/RAD} \\ &= \end{aligned}$$

17.

AT 3000 RPM

$$\text{IF } \phi_1 = 0,25^\circ$$

$$\begin{aligned}\phi_2 &= \frac{K_T \cdot 0,25}{\{K_T - 4\omega^2 J_2\}} \\ &= \frac{274 \times 0,25}{\{274 - 4\left(\frac{2\pi \times 3000}{60}\right)^2 \times 0,1\}}\end{aligned}$$

$$\approx \underline{\underline{0,0017^\circ}}$$

You can see how significantly  
reduced the vibration amplitude  
now is.

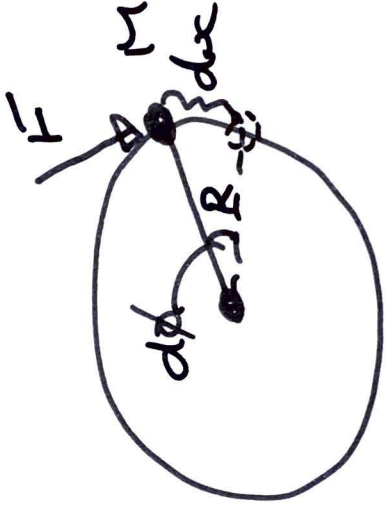
The trick with both  
engine mountings & your  
mass flywheels is to  
have the resonant frequency

18.

BELOW THE IDLE SPEED.



$$\underline{\text{Proof of } T = I \cdot \frac{d^2\phi}{dt^2}}$$



IMAGINE MASS GOING ROUND CIRCLE.

IF DISTANCE TRAVELLED IS  $dx$  AND ANGLE ROTATED IS  $d\phi$

$$dx = R \cdot d\phi$$

NEWTON'S LAW

$$F = M \cdot \frac{d^2x}{dt^2} = MR \cdot \frac{d^2\phi}{dt^2}$$

19.

$$\begin{aligned}\tau_{\text{torque}} &= I \cdot R \\ &= M \cdot R \cdot \frac{d\phi}{dt^2} \cdot R \\ &= MR^2 \cdot \frac{d^2\phi}{dt^2}.\end{aligned}$$

CALL  $MR^2 = \text{MOMENT OF INERTIA } I$ .

IF THERE IS MORE THAN 1 MASS

$$I = \sum MR^2$$



EXAM:

BASED ON WHAT

WE DID IN CLASS & SOME

OF THESE NOTES. (UP TO YOUR

MASS FURTHER)