

University of Wales Trinity Saint David

Evan Morcom 1802426

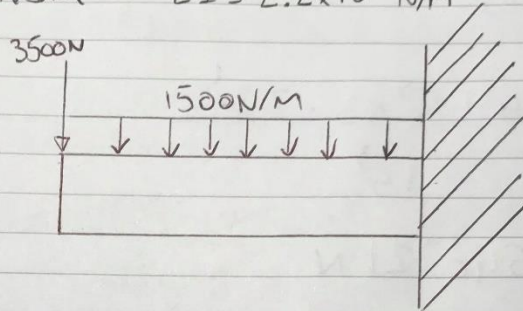
Stress Analysis Exam ACHA5015

11th January 2021

Pages 1 to 8 below

Q1.

$$L = 1.5\text{m} \quad EI = 2.2 \times 10^6 \text{ N/m}^2$$



$$a) y_{\max} = y_{\text{UDL}} + y_{\text{PL}}$$

$$y_{\max} = \frac{\omega L^4}{8EI} + \frac{FL^3}{3EI}$$

$$y_{\max} = \frac{1500 \times 1.5^4}{8 \times 2.2 \times 10^6} + \frac{3500 \times 1.5^3}{3 \times 2.2 \times 10^6}$$

$$y_{\max} = 431.46 \times 10^{-6} + 1.789 \times 10^{-3}$$

$$y_{\max} = 2.22 \times 10^{-3} \text{ m}$$

$$b) y_{\max} = y_{\text{PL}} + y_{\text{UDL}}$$

$$y_{\text{PL}} = y_{\max} - y_{\text{UDL}} = -5 \times 10^{-3} - (431.46 \times 10^{-6})$$

$$y_{\text{PL}} = -5.431 \times 10^{-3}$$

$$y_{PL} = \frac{FL^3}{3EI} \quad \therefore F = \frac{y_{PL} \times 3EI}{L^3}$$

$$F = \frac{-5.431 \times 10^{-3} \times 3 \times 2.2 \times 10^6}{1.5^3}$$

$$F = -3540.21 \text{ N}$$

$$\Delta F = 40.21 \text{ N}$$

Q2

$$d_o = 45 \times 10^{-3} \text{ m}$$

$$d_i = 35 \times 10^{-3} \text{ m}$$

$$r_o = 22.5 \times 10^{-3} \text{ m}$$

$$r_i = 17.5 \times 10^{-3} \text{ m}$$

$$P = 115 \times 10^5 \text{ N/m}^2$$

$$d_o = 0.045 \text{ m} \quad d_i = 0.035 \text{ m}$$

$$r_o = 0.0225 \text{ m} \quad r_i = 0.0175 \text{ m}$$

$$@ r_o = 0.0225 \text{ m} \quad \sigma_R = 0$$

$$\sigma_R = a - \frac{b}{r^2}$$

$$0 = a - \frac{b}{0.0225^2}$$

$$0 = b - (0.0225)^2 a \quad - (1)$$

$$@ r_i = 0.0175 \text{ m} \quad \sigma_R = -115 \times 10^5$$

$$\sigma_R = a - \frac{b}{r^2}$$

$$-115 \times 10^5 = a - \frac{b}{0.0175^2}$$

$$3521.875 = b - (0.0175)^2 a \quad - (2)$$

② - ①

$$3521.875 = -(0.0175)^2 a + (0.0225)^2 a$$

$$a = \frac{3521.875}{0.0225^2 - 0.0175^2} \quad a = 17.61 \times 10^6 \text{ N/m}^2$$

$$b = \frac{-(0.0175)^2 \times 17.61 \times 10^6}{3521.875} \quad b = +1.53 \text{ N/m}^2$$

$$b = -0.0175^2 \times 17.61 \times 10^6 \times 3521.875 = 18.99 \times 10^6 \text{ N/m}^2$$

@r_i $\sigma_c = a + b = \frac{17.61 \times 10^6}{r^2} + \frac{18.99 \times 10^6}{0.0175}$

$$\sigma_c = 1.103 \text{ GPa} \quad \sigma_c = 1103 \text{ MPa}$$

$$\text{@r}_o \quad \sigma_c = a + b = \frac{17.61 \times 10^6}{r^2} + \frac{18.99 \times 10^6}{0.0225}$$

$$\sigma_c = 861.61 \text{ MPa}$$

Please Look @ Pg 7 and 8
 For Q2.

Evan Morcom 1802426
Stress Analysis
ACHA SOLS 11/1/21
pg 4 of 8

$$Q3 \quad \epsilon_x = -10 \times 10^{-6} \quad \epsilon_y = 13 \times 10^{-6}$$

$$\nu = 0.20$$

$$E = 165 \times 10^9 \text{ N/m}^2$$

$$a) \quad \sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$$

$$\sigma_x = \frac{165 \times 10^9}{1-0.2^2} (-10 \times 10^{-6} + (0.20 \times 13 \times 10^{-6}))$$

$$\sigma_x = -1.272 \text{ MPa} \quad \therefore \text{Compressive}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$\sigma_y = \frac{165 \times 10^9}{1-0.2^2} (13 \times 10^{-6} + (0.20 \times -10 \times 10^{-6}))$$

$$\sigma_y = 1.891 \text{ MPa} \quad \therefore \text{Tensile}$$

$$b) \quad \sigma_x = 20 \times 10^6 \text{ N/m}^2 \quad \sigma_y = -15 \times 10^6 \text{ N/m}^2$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{165 \times 10^9} (20 \times 10^6 - (0.20 \times -15 \times 10^6))$$

$$\epsilon_x = 139.39 \times 10^{-6}$$

Evan Morcom 1802426
 Stress Analysis
 ACH95015 11/1/21
 Pg 5 of 8

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{165 \times 10^9} (-15 \times 10^6 - (0.20 \times 20 \times 10^6))$$

$$\epsilon_y = -115.15 \times 10^{-6}$$

Q4 $d = 0.2m$ $L = 0.39m$ $t = 0.0015m$ ($1.5 \times 10^{-3}m$)

$$\sigma_{max} = 100 \times 10^6 N/m^2 \quad \eta_L = 0.9 \quad \eta_c = 0.6$$

$$P = ?$$

$$\sigma_c = \frac{Pd}{2t\eta_L}$$

$$\sigma_L = \frac{Pd}{4t\eta_c}$$

$$\frac{2t\eta_L \sigma_c}{d} = P$$

$$\frac{4t\eta_c \sigma_L}{d} = P$$

$$\frac{2 \times 0.15 \times 10^{-3} \times 100 \times 10^6}{0.2} = 1.5 \times 10^6 N/m^2$$

$$\frac{2 \times 1.5 \times 10^{-3} \times 0.9 \times 100 \times 10^6}{0.2} = 1.35 \times 10^6 N/m^2$$

$$\frac{4 \times 1.5 \times 10^{-3} \times 0.6 \times 100 \times 10^6}{0.2} = 1.8 \times 10^6 N/m^2$$

$$\therefore P_{max} = 1.8 \times 10^6 N/m^2 \quad 1.35 \times 10^6 N/m^2$$

Q5

$$L = 0.35 \text{ m} \quad P = 50 \times 10^6 \text{ N/m}^2$$

$$\nu = 0.3 \quad E = 165 \times 10^9 \text{ N/m}^2$$

$$a) K = \frac{E}{3(1-2\nu)} = \frac{165 \times 10^9}{3(1-(2 \times 0.3))}$$

$$K = 137.5 \times 10^9 \text{ N/m}^2$$

$$K = \frac{\sigma_1}{\epsilon_v} = \frac{-P}{\epsilon_v}$$

$$\epsilon_v = \frac{-P}{K} = \frac{-50 \times 10^6}{137.5 \times 10^9}$$

$$\epsilon_v = -363.636 \times 10^{-6}$$

$$\epsilon_v = \frac{\Delta v}{v} \therefore \Delta v = \epsilon_v \cdot v = -363.636 \times 10^{-6} \times (0.35)^3$$

$$\Delta v = -1.56 \times 10^{-5} \text{ m}^3$$

- b) Orthotropic materials have different elastic constants in different directions.

This maybe due to grain structure of fibre directions in composites etc.

An example would be ~~Carbon Fibre~~
wood

Q2.

$$d_o = 45 \times 10^{-3} \text{ m}$$

$$r_o = 22 \times 10^{-3} \text{ m}$$

$$d_i = ~~35~~ 35 \times 10^{-3} \text{ m}$$

$$r_i = 17.5 \times 10^{-3} \text{ m}$$

$$P = 115 \times 10^5 \text{ N/m}^2$$

$$@ r_o = 22 \times 10^{-3} \quad \sigma_R = 0$$

$$\sigma_R = a - \frac{b}{r^2}$$

$$0 = a - \frac{b}{(22 \times 10^{-3})^2}$$

$$0 = b - (22 \times 10^{-3})^2 a \quad - (1)$$

$$@ r_i = 17.5 \times 10^{-3} \quad \sigma_R = -115 \times 10^5$$

$$\sigma_R = a - \frac{b}{r^2} \quad -115 \times 10^5 = a - \frac{b}{(17.5 \times 10^{-3})^2}$$

$$3521.88 = b - (17.5 \times 10^{-3})^2 a \quad - (2)$$

$$(2) - (1) \quad 3521.88 = -(17.5 \times 10^{-3})^2 a + (22 \times 10^{-3})^2 a$$

$$a = \frac{3521.88}{(22 \times 10^{-3})^2 - (17.5 \times 10^{-3})^2}$$

$$a = 17.61 \times 10^6 \text{ N/m}^2$$

$$b = \frac{(-115 \times 10^5) \times (22 \times 10^{-3})^2 \times (17.5 \times 10^{-3})}{(22 \times 10^{-3}) - (17.5 \times 10^{-3})}$$

$$b = -983.89 \times 10^3$$

$$@r_i \quad \sigma_c = a + \frac{b}{r^2} = 17.61 \times 10^6 \times \left(\frac{-983.89 \times 10^3}{17.5 \times 10^{-3}} \right)$$

$$\sigma_c = -38.61 \text{ MPa}$$

$$@r_o \quad \sigma_c = a + \frac{b}{r^2} = 17.61 \times 10^6 \times \left(\frac{-983.89 \times 10^3}{22 \times 10^{-3}} \right)$$

$$\sigma_c = -27.11 \text{ MPa}$$