

ACHA 4006

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Q1) i) $y = 7x^3 - 2x^2 + x + 2$

$$y' = \frac{d}{dx} (7x^3 - 2x^2 + x + 2)$$

$$y' = \frac{d}{dx} (7x^3) + \frac{d}{dx} (-2x^2) + \frac{d}{dx} (x) + \frac{d}{dx} (2)$$

$$y' = 7x^3 \cdot 2 - 2x^2 \cdot 1 + 1 + 0$$

$$y' = 21x^2 - 4x + 1$$

ii) $y = (x+3)(x+2)$

$$y' = \frac{d}{dx} ((x+3)(x+2))$$

$$y' = \frac{d}{dx} (x^2 + 2x + 3x + 6)$$

$$y' = \frac{d}{dx} (x^2) + \frac{d}{dx} (5x) + \frac{d}{dx} (6)$$

$$y' = 2x + 5$$

iii) $y = \frac{1}{x^3} + \sin(3x)$

$$y' = \frac{-3x^2}{(x^3)^2} + \cos(3x) \cdot 3$$

$$y' = \frac{d}{dx} \left(\frac{1}{x^3} + \sin(3x) \right)$$

$$y' = \frac{-3}{x^4} + 3\cos(3x)$$

$$y' = \frac{d}{dx} \left(\frac{1}{x^3} \right) + \frac{d}{dx} (\sin(3x))$$

$$\text{iv)} \quad y = x^4 \tan 2x$$

$$y' = \frac{d}{dx} (x^4 \times \tan(2x))$$

$$y' = \frac{d}{dx} (x^4) \times \tan(2x) + x^4 \times \frac{d}{dx} (\tan(2x))$$

$$y' = 4x^3 \times \tan(2x) + x^4 \times \sec(2x)^2 \times 2$$

$$y' = \frac{2x^3 \times \sin(4x) + 2x^4}{\cos(2x)^2}$$

$$\text{v)} \quad y = \frac{e^{2x}}{\cos(2x)} \quad y' = \frac{d}{dx} \left(\frac{e^{2x}}{\cos(2x)} \right)$$

$$y' = \frac{\frac{d}{dx} (e^{2x}) \times \cos(2x) - e^{2x} \times \frac{d}{dx} (\cos(2x))}{\cos(2x)^2}$$

$$y' = \frac{2e^{2x} \times \cos(2x) + 2e^{2x} \times \sin(2x)}{\cos(2x)^2}$$

$$\text{vi)} \quad y = (5x^2 + 2x + 1)^5$$

$$y' = \frac{d}{dx} ((5x^2 + 2x + 1)^5)$$

$$y' = \frac{d}{dg} (g^5) \times \frac{d}{dx} (5x^2 + 2x + 1)$$

$$y' = 5g^4 \times (5 \times 2x + 2)$$

$$y' = 5(5x^2 + 2x + 1)^4 \times (5 \times 2x + 2)$$

$$y' = (50x + 10) \times (5x^2 + 2x + 1)^4$$

Q2) i) $\sin x = 0.65 \Rightarrow \sin(x) = \frac{13}{20}$

$$\sin x = \frac{13}{20}$$

$$x \approx \begin{cases} 0.707584 + 2k\pi, & k \in \mathbb{Z} \\ 2.43401 + 2k\pi, & k \in \mathbb{Z} \end{cases}$$

$$\sin(x) = \frac{13}{20}$$

$$x = \arcsin\left(\frac{13}{20}\right)$$

$$x \approx \begin{cases} 40.5^\circ + k \times 360^\circ, & k \in \mathbb{Z} \\ 139.5^\circ + k \times 360^\circ, & k \in \mathbb{Z} \end{cases}$$

$$\pi - x = \arcsin\left(\frac{13}{20}\right)$$

$$x = \arcsin\left(\frac{13}{20}\right) + 2k\pi, k \in \mathbb{Z}$$

$$\pi - x = \arcsin\left(\frac{13}{20}\right)$$

$$\pi - x = \arcsin\left(\frac{13}{20}\right) + 2k\pi, k \in \mathbb{Z}$$

$$x = \arcsin\left(\frac{13}{20}\right) + \pi - 2k\pi, k \in \mathbb{Z}$$

$$x = -\arcsin\left(\frac{13}{20}\right) + \pi - 2k\pi, k \in \mathbb{Z}$$

$$x = \begin{cases} \arcsin\left(\frac{13}{20}\right) + 2k\pi \\ -\arcsin\left(\frac{13}{20}\right) + \pi + 2k\pi, & k \in \mathbb{Z} \end{cases}$$

iii) $\tan x = -0.22 \quad x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$$\tan x = -\frac{11}{50}$$

$$x = \arctan\left(-\frac{11}{50}\right) + k\pi, k \in \mathbb{Z}$$

$$x = -\arctan\left(\frac{11}{50}\right) + k\pi, k \in \mathbb{Z}, \quad x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

$$x = -\arctan\left(\frac{11}{50}\right) + k\pi, k \in \mathbb{Z}$$

$$x = -\arctan\left(\frac{11}{50}\right) + k\pi, k \in \mathbb{Z}$$

$$x \approx -0.21655 + k\pi, k \in \mathbb{Z}$$

$$x \approx -12.4^\circ + k \times 180^\circ, k \in \mathbb{Z}$$

$$ii) 7\cos(x) = 4$$

$$\cos(x) = \frac{4}{7}$$

$$\cos(2\pi - x) = \frac{4}{7}$$

$$x = \arccos\left(\frac{4}{7}\right)$$

$$\cos(2\pi - x) = \frac{4}{7}$$

$$2\pi - x = \arccos\left(\frac{4}{7}\right)$$

$$x = \arccos\left(\frac{4}{7}\right) + 2k\pi, k \in \mathbb{Z}$$

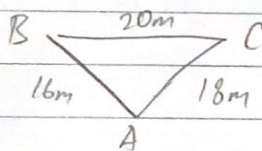
$$2\pi - x = \arccos\left(\frac{4}{7}\right) + 2k\pi, k \in \mathbb{Z}$$

$$x = \begin{cases} \arccos\left(\frac{4}{7}\right) + 2k\pi \\ -\arccos\left(\frac{4}{7}\right) + 2k\pi, k \in \mathbb{Z} \end{cases}$$

$$x = \begin{cases} 0.962551 + 2k\pi \\ -0.962551 + 2k\pi, k \in \mathbb{Z} \end{cases}$$

$$x \approx \begin{cases} 55.2^\circ + k \times 360^\circ \\ -55.2^\circ + k \times 360^\circ, k \in \mathbb{Z} \end{cases}$$

Q3)



$$Q4) p = \sqrt{\frac{xy}{T}} + 3$$

$$p^2 = \frac{xy}{T} + 3^2$$

$$\frac{xy}{T} = 3^2 - p^2$$

$$xy = T(3^2 - p^2)$$

$$x = \frac{T(3^2 - p^2)}{y}$$

Q5) i) $5^{x+2} = 2x$
 $x+2 = \log_5 2x$
 $x - \log_5 2x = -2$
 $(1 - \log_5 2x) = -2$
 $x = -\frac{2}{1 - \log_5 2}$

ii) $\log_{10}(x+2) + \log_{10}(x+3) = 2\log_{10}(x+1), x \in (-1, +\infty)$
 $\log_{10}((x+2)(x+3)) = \log_{10}((x+1)^2)$
 $\log_{10}(x^2 + 3x + 2x + 6) = \log_{10}((x+1)^2)$
 $x^2 + 3x + 2x + 6 = (x+1)^2$
 $x^2 + 3x + 2x + 6 = x^2 + 2x + 1$
 $3x + 6 = 1$
 $3x = 1 - 6$
 $3x = -5$
 $x = -\frac{5}{3}, x \in (-1, +\infty)$
 $x \notin \mathbb{R}$

Q6) i) $\int x^4 + 3x^2 - 3x + 1 dx$

$\int x^4 dx + \int 3x^2 dx - \int 3x dx + \int 1 dx \quad n \neq -1$
 $\frac{x^5}{5} + \int 3x^2 dx - \int 3x dx + \int 1 dx$
 $\frac{x^5}{5} + \frac{3x^3}{3} - \frac{3x^2}{2} + x + C, C \in \mathbb{R}$

$$ii) \int (x-1)(x-3) dx$$

$$\int x^2 - 3x - x + 3 dx$$

$$\int x^2 - 4x + 3 dx$$

$$\underbrace{\int x^2 dx}_{\frac{x^3}{3}} - \underbrace{\int 4x dx}_{2x^2} + \underbrace{\int 3 dx}_{3x} \quad n \neq -1$$

$$\frac{x^3}{3} - 2x^2 + 3x + C, \quad C \in \mathbb{R}$$

$$iii) \int 3 \sec^2 2x dx$$

$$3 \times \int \sec(2x)^2 dx$$

$$3 \times \frac{1}{2} \times \int \sec(t)^2 dt$$

$$\frac{3}{2} \times \underbrace{\int \sec(t)^2 dt}_{\tan(t)}$$

$$\frac{3}{2} \times \tan \overset{t=2x}{(2x)}$$

$$\frac{3 \tan(2x)}{2}$$

$$\frac{3 \tan(2x)}{2} + C, \quad C \in \mathbb{R}$$

$$(iv) \int_0^4 (x+3)(x+2) dx \quad \left(\frac{x^3}{3} + \frac{5x^2}{2} + 6x \right) \Big|_0^4$$

$$\int (x+3)(x+2) dx$$

$$\int x^2 + 5x + 6 dx$$

$$\underbrace{\int x^2 dx}_{\frac{x^3}{3}} + \underbrace{\int 5x dx}_{\frac{5x^2}{2}} + \underbrace{\int 6 dx}_{6x} \quad n \neq -1$$

$$\frac{4^3}{3} + \frac{5 \times 4^2}{2} + 6 \times 4 \left(\frac{0^3}{3} + \frac{5 \times 0^2}{2} + 6 \times 0 \right) = \frac{256}{3} \approx 85.33$$

$$v) \int_2^3 x(x+1) + e^x dx$$

$$\int x(x+1) + e^x dx$$

$$\int x^2 + x + e^x dx$$

$$\int x^2 dx + \int x dx + \int e^x dx$$

$$\frac{x^3}{3} \quad \frac{x^2}{2} \quad e^x$$

$$\left(\frac{x^3}{3} + \frac{x^2}{2} + e^x \right) \Big|_2^3$$

$$\frac{3^3}{3} + \frac{3^2}{2} + e^3 - \left(\frac{2^3}{3} + \frac{2^2}{2} + e^2 \right)$$

$$\frac{53}{6} + e^3 - e^2$$