

Student Name: Kieran Dellow

Student ID number: 2102899

Module Title: Analytical Methods

Module Code: ACHA4000

Date: 16th May 2022

Time: 3 pm

Analytical methods

16/05/2022

$$1) p(x) = \sqrt{\frac{1}{2}x^3 + (2x)^3 - \frac{6}{7}}$$

$$\begin{aligned} &= \left(\frac{1}{2}x^3 + (2x)^3 - \frac{6}{7} \right)^{1/2} \\ &= \left(\frac{1}{2}x^3 + 8x^3 - \frac{6}{7} \right)^{1/2} \\ &= \left(8\frac{1}{2}x^3 - \frac{6}{7} \right)^{1/2} \\ &= \left(25\frac{1}{2}x^3 \right)^{1/2} \\ &= \left(\frac{25\frac{1}{2}x^3}{2} \right)^{-1/2} \end{aligned}$$

$$= \sqrt{\frac{1}{2}x^3 + 8x^3 - \frac{6}{7}}$$

$$= \left(\frac{1}{2}x^3 + 8x^3 - \frac{6}{7} \right)^{1/2}$$

$$= (1/2) \left(\frac{1}{2}x^3 + 8x^3 - \frac{6}{7} \right)^{-1/2} \times \frac{-3}{2}x^2 + 2(8x^2)$$

2

$$6) g(x) = \frac{5x+3}{2x+1}$$

$$g(x) = \frac{5}{2x+1} - \frac{2(5x+3)}{(2x+1)^2}$$

$$g(x) = -\frac{1}{(2x+1)^2}$$

$$g(x) = \frac{d}{dx} \left(\frac{1}{(2x+1)^2} \right)$$

$$= -\frac{d}{dx} \left((2x+1)^{-2} \right)$$

$$= -\frac{2}{(2x+1)^3} \frac{d}{dx} (2x+1)$$

$$= -\left(-\frac{2}{(2x+1)^3} \times 2 \right)$$

$$= 4/(2x+1)^3$$

22

2)

$$y = \frac{(3x+1) \cdot \sin^3(x)}{e^{2x}}$$

$$\ln(y) = \ln\left(\frac{(3x+1) \cdot \sin^3(x)}{e^{2x}}\right)$$

$$\ln(y) = \ln(3x+1) + \ln(\sin^3(x)) - \ln(e^{2x})$$

~~Y'~~

$$\ln(y) = \ln(3x+1) + \ln(\sin^3(x)) - \ln(e^{2x})$$

$$= \ln(3x+1) + \ln(\sin^3(x)) - \ln(e^{2x})$$

$$= \ln(3x+1) + 3\ln(\sin(x)) - 2x \ln(e)$$

$$= \ln(3x+1) + 3\ln(\sin(x)) - 2x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{3}{3x+1} - 3 \cdot \frac{\cos(x)}{\sin(x)} - 2$$

$$\frac{dy}{dx} = y \cdot \left(\frac{3}{3x+1} - 3 \cdot \frac{\cos(x)}{\sin(x)} - 2 \right)$$

$$\frac{dy}{dx} = \frac{(3x+1) \cdot \sin^3(x)}{e^{2x}} \cdot \left(\frac{3}{3x+1} - 3 \cdot \frac{\cos(x)}{\sin(x)} - 2 \right)$$

3) Let,

$$g(x, y) = x^2 \sin(2x+3y)$$

$$a) \text{ calc. } \frac{\partial g(x, y)}{\partial y}$$

$$\frac{\partial g(x, y)}{\partial y} = \frac{\partial x^2}{\partial y} \cdot \sin(2x+3y) + x^2 \cdot \frac{\partial \sin(2x+3y)}{\partial y}$$

$$= 0 \cdot \sin(2x+3y) + x^2 \cdot (3) \cdot \cos(2x+3y)$$

$$= 3x^2 \cdot \cos(2x+3y)$$

$$b) \frac{d^2 g(x, y)}{dy dx} = \frac{d}{dx} \left(\frac{\partial g(x, y)}{\partial y} \right)$$

$$= \frac{d}{dx} (3x^2 \cdot \cos(2x+3y)) = \frac{d}{dx} (3x^2) \cdot \cos(2x+3y) + x^2 \cdot \frac{\partial \cos(2x+3y)}{\partial x}$$

$$= \frac{-6x}{2x} \cdot \cos(2x+3y) - x^2 \cdot (2x) \sin(2x+3y)$$

$$= -6x \cdot \cos(2x+3y) - 2x^3 \cdot \sin(2x+3y)$$

7)



a) $-2x^2 + x^2 + 8x - 10 = -6x^2 + 2x + 8$

b) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 6 \cdot (-8)}}{2 \cdot (-6)}$

$$x = -1$$

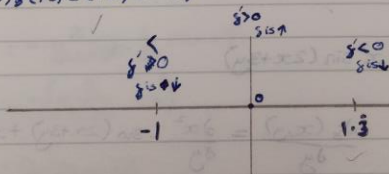
$$x = 1.3$$

$(-1, f(-1)) = (-1, 5)$

$(1.3, f(1.3))$

$(1.3, f(1.3)) = (1.3, -2.296)$

c)



$-6(0)^2 + 0 + 8 = 8$

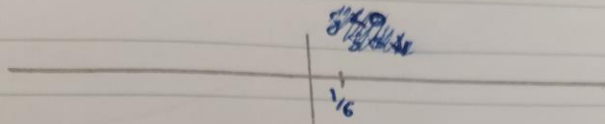
$-6(1.3)^2 + 3 + 8 = -25.5$

$-6(-1.5) - 3 + 8 = -8.5$

d) $f'' = -12x + 2$

e) $-12x + 2 = 0, \therefore -12x = -2, \therefore x = \frac{1}{6}$

8)



$$-12 \times 0 + 2 = 2$$

$$-12 \times 1 + 2 = -10$$

$$f''(0) > 0$$
$$f = 0$$

$$f''(1) < 0$$
$$f = 0$$

