

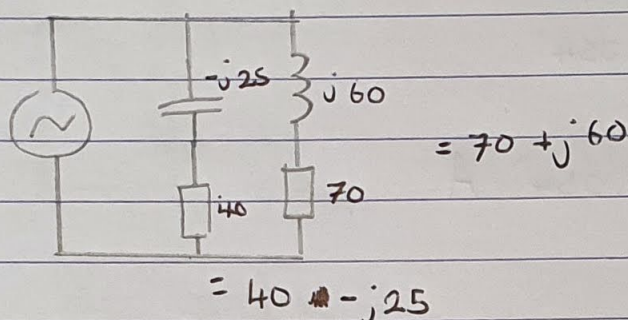
Electrical Principles Exam - TAY17837698 - 21.5.21

Question One

69%

Q1=[20/20]

a)



Q2=[13/20]

Q3=[20/20]

Q4=[0/10]

Q5=[16/30]

$$\frac{\text{product}}{\text{sum}} = \frac{(40 - j25)(70 + j60)}{(40 - j25) + (70 + j60)}$$

$$= \frac{2800 + j2400 - j1750 - j^2 1500}{110 + j35}$$

$$= \frac{2800 + j650 - (1500 \times -1)}{110 + j35}$$

$$= \frac{2800 + 1500 + j650}{110 + j35}$$

$$= \frac{4300 + j650}{110 + j35} \Rightarrow \frac{4348.85 \angle 8.596^\circ}{115.434 \angle 17.65^\circ}$$

$$= 37.674 \angle -9.054^\circ \Omega \quad [8/8]$$

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b) $V = IZ$

$$I = \frac{V}{Z}$$

$$= \frac{100 \angle 0^\circ}{37.674 \angle -9.054^\circ}$$

$$= 2.654 \angle 9.054^\circ \text{ A}$$

[2/2]

c) $I = \frac{V}{Z}$

$$= \frac{100 \angle 0^\circ}{70 + j60} > \frac{100 \angle 0^\circ}{92.195 \angle 40.601^\circ}$$

$$= 1.085 \angle -40.601^\circ \text{ A}$$

[5/5]

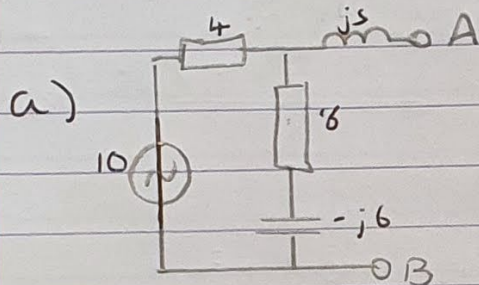
d) $I = \frac{V}{Z}$

$$= \frac{100 \angle 0^\circ}{40 - j25} > \frac{100 \angle 0^\circ}{47.17 \angle -32.005^\circ}$$

$$= ~~2.12~~ 2.12 \angle 32.005^\circ \text{ A}$$

[5/5]

Question Two



$$V_2 = \left(\frac{R_2}{R_1 + R_2} \right) V_s$$

$$= \left(\frac{8 - j6}{4 + 8 - j6} \right) 10 \angle 0^\circ$$

$$= \left(\frac{10 \angle -36.87^\circ}{13.416 \angle -26.565^\circ} \right) 10 \angle 0^\circ$$

$$= 0.745 \angle -10.305^\circ \times 10 \angle 0^\circ$$

$$= \underline{7.45 \angle -10.305^\circ \text{ V}} \quad [5/5]$$

$$j5 + 4 \parallel 8 - j6$$

Product	$(4)(8 - j6)$
Sum	$4 + 8 - j6$

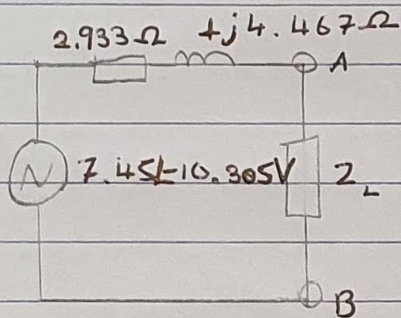
$$= \frac{32 - j24}{12 - j6} \rightarrow \frac{40 \angle -36.87^\circ}{13.416 \angle -26.565^\circ}$$

$$= 2.981 \angle -10.305^\circ + j5$$

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$$= 2.933 - 0.533j + 5j$$

$$= 2.933 + j4.467 \Omega \quad [5/5]$$



$$\frac{5.344}{56.711}$$

b) for max power transfer Z_L needs to be equal to the complex conjugate of Z_{th}

$$Z_L = Z_{th}^* = 2.933 - j4.467 \Omega \quad [3/3]$$

c) ~~$S = I^* V$~~ $V = I Z$
 $I = \frac{V}{Z}$

$$= \frac{7.45 \angle -10.305^\circ}{2.933 + j4.467} \rightarrow \frac{7.45 \angle -10.305^\circ}{5.344 \angle 56.711^\circ}$$

$$= 1.394 \angle -67.016^\circ \text{ A}$$

$$S = I^* V$$

$$= 1.394 \angle 67.016^\circ \times 7.45 \angle -10.305^\circ$$

$$= 10.385 \angle 56.711^\circ \text{ VA}$$



Question Three [20/20]

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^0 f(x) dx + \frac{1}{\pi} \int_0^{\pi} f(x) dx$$

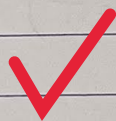
$$= \frac{1}{\pi} \int_{-\pi}^0 1 dx + \frac{1}{\pi} \int_0^{\pi} 0 dx$$

~~$$= \frac{1}{\pi} \int$$~~

$$= \frac{1}{\pi} [x]_{-\pi}^0$$

$$= \frac{1}{\pi} (0 - (-\pi))$$

$$= 1 \therefore a_0 = 1$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$= \frac{1}{\pi} \int_{-\pi}^0 1 \cos nx dx + \frac{1}{\pi} \int_0^{\pi} 0 \cos nx dx$$

$$= \frac{1}{\pi} \left[\frac{\sin nx}{n} \right]_{-\pi}^0$$

$$= \frac{1}{n\pi} [\sin nx]_{-\pi}^0$$

$$= \frac{1}{n\pi} [\sin 0 - \sin(-n\pi)]$$

$$= \frac{1}{n\pi} [0 + \sin(n\pi)]$$

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$$= \frac{1}{n\pi} [0+0]$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^0 f(x) \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^0 1 \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} 0 \sin nx \, dx$$

$$= \frac{1}{\pi} \left[-\frac{\cos nx}{n} \right]_{-\pi}^0$$

$$= \frac{1}{n\pi} [\cos nx]_{-\pi}^0$$

$$= -\frac{1}{n\pi} (1 - \cos(-n\pi))$$

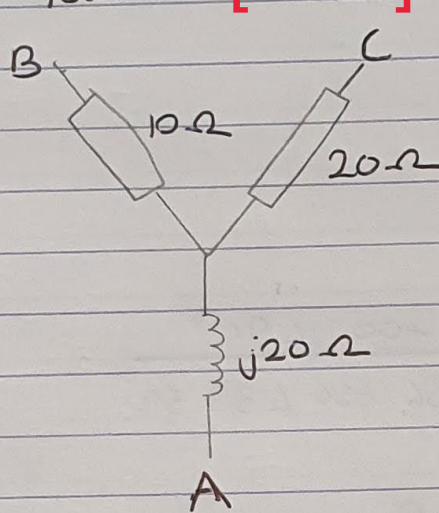
$$b_n = -\frac{1}{n\pi} (1 - (-1)^n)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

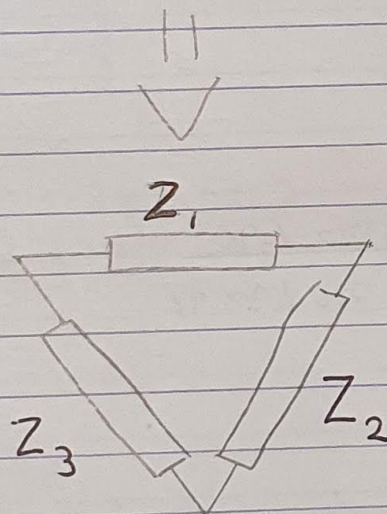
$$a_0 = 1, \quad a_n = 0, \quad b_n = -\frac{1}{n\pi} (1 - (-1)^n)$$

$$f(x) = \frac{1}{2} - \frac{2}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \frac{1}{7} \sin 7x + \frac{1}{9} \sin 9x \right)$$

Question Four [0/10]



$$\begin{aligned} Z_a + Z_b + Z_c &= 10 + 20 + j20 \\ &= 30 + j20 = 36.056 \angle 33.69^\circ \end{aligned}$$



$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$= \frac{10 \times 20}{36.056 \angle 33.69^\circ}$$

$$= 5.547 \angle -33.69^\circ \Omega$$

$$Z_2 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

$$= \frac{20(+j20)}{36.656 \angle 33.69}$$

$$\frac{j400}{36.656 \angle 33.69} > \frac{400 \angle 90^\circ}{36.656 \angle 33.69^\circ}$$

$$= 11.094 \angle 56.31^\circ \Omega$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$

$$= \frac{10(+j20)}{36.656 \angle 33.69} > \frac{200 \angle 90^\circ}{36.656 \angle 33.69^\circ}$$

$$= 5.547 \angle 56.31^\circ \Omega$$

Question five

a) $L \frac{di}{dt} + Ri = V$

$3 \frac{di}{dt} + 6 = 30e^{-t}$ [1/1]

$3 \int \left[\frac{di}{dt} \right] + 6 \int [i] = 30 \int [e^{-t}]$

$3sI(s) - i(0) + 6I(s) = \frac{30}{s+1}$ [2/2]

$3sI(s) + 6I(s) = \frac{30}{s+1}$

$I(s) \{3s+6\} = \frac{30}{s+1}$ [1/1]

$I(s) = \frac{30}{(3s+6)(s+1)} = \frac{10}{(s+2)(s+1)}$ [1/1]

$\frac{16.667}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$ [0/2]
 $10 \frac{16.667}{(s+2)(s+1)} = A(s+1) + B(s+2)$
 $s = -2$

$\downarrow$
 $16.667 = A(s+2) + B(s+1)$
 $s = -1$
 $10 \frac{16.667}{(s+2)(s+1)} = A(-2+2) + B(-2+1)$
 $10 \frac{16.667}{(s+2)(s+1)} = -1B$

$B = -16.667$ [2/2]

$16.667 = A(-1+2) + B(0)$

$16.667 = A$ [1/1]

$I(s) = \frac{10}{(s+2)} + \frac{-10}{(s+1)}$ [1/1]

$= I(t) = 10e^{-2t} - 10e^{-1t}$
 $= I(t) = 16.667e^{-2t} - 16.667e^{-1t}$ [1/1]

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b)

$$\text{find } \int^{-1} \left\{ \frac{s+3}{(s+1)(s-1)} \right\}$$

$$\frac{s+3}{(s+1)(s-1)} = \frac{A(s+3)}{(s+1)} + \frac{B(s+3)}{(s-1)} \quad \times$$

c)

i) $e^{3t} + 7$

$$= \frac{1}{s-3} + \frac{7}{s} \quad [2/2]$$

ii) $\cos(3t) + 2\sin(2t)$

$$= \frac{s}{s^2+3^2} + \frac{2 \times 2}{s^2+2^2}$$

$$= \frac{s}{s^2+9} + \frac{4}{s^2+4} \quad [2/2]$$

iii) $(t^3+4t)e^{3t}$

iv) $5e^{st} \cos(2t) - e^{3t}$

$$= \frac{s+(-s)}{(s+(-s))^2+2^2} - \frac{1}{s+(-s)} \quad [1/2]$$

$$= \frac{s-5}{(s-5)^2+4} - \frac{1}{s-5}$$

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$$V) 2e^{-2t} \sin 4t$$

$$= \frac{4}{(s+2)^2 + 4^2}$$

$$= \frac{4}{(s+2^2) + 16} \quad [1/2]$$

$$= \frac{4}{s^2 + 2^2 + 16}$$

$$= \frac{4}{s^2 + 20}$$

$$= \frac{4}{s^2} + \frac{4}{20}$$

$$= \frac{4}{s^2} + 5$$

END