

University of Wales Trinity Saint David

Calculation on Piston, Rod, Crankshaft Assemblies

Advanced Engine Design

M6X00410

Cornelius May

1701329

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Gudgeon Pin

In order to investigate the performance of the gudgeon pin, the stresses throughout the part are often investigated and compared to the maximum stress that it is rated to withstand. This shows if the part is likely to fail under the loads it is expected to see. As long as there is a reasonable safety margin for error included, the acceptable performance of the part can be guaranteed with some confidence.

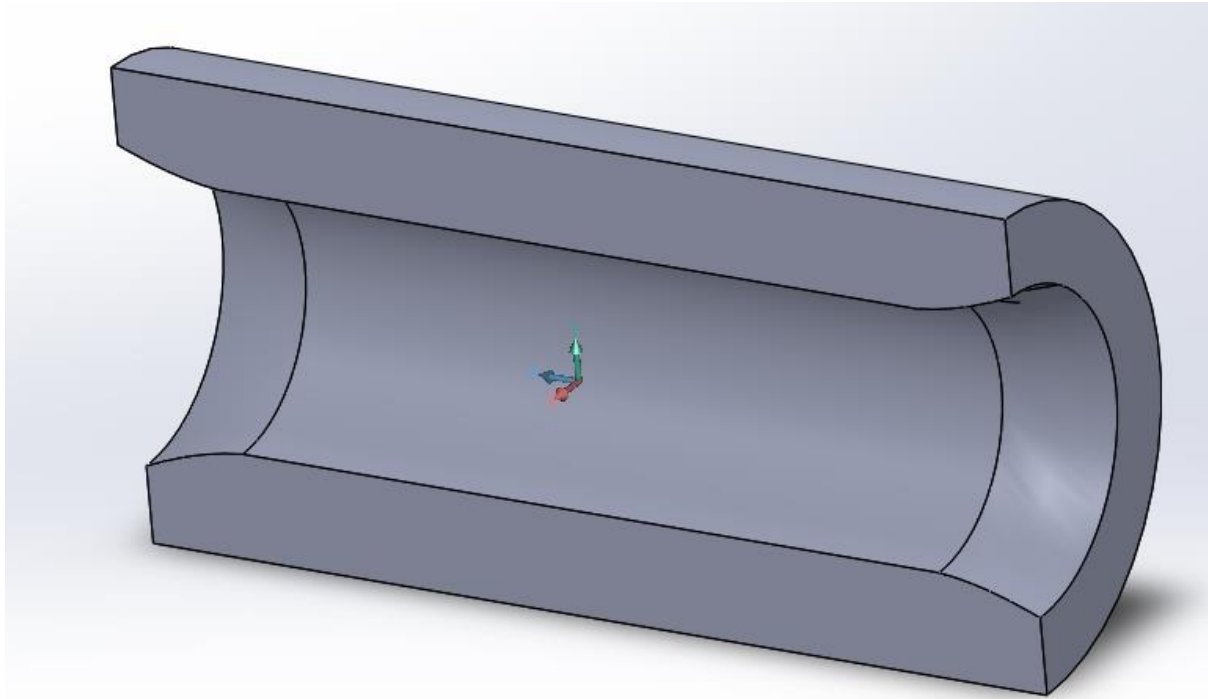


Figure 1: Gudgeon Pin

- In the case of the gudgeon pin, the shear stress is caused by the force of the con-rod pushing or pulling on the gudgeon pin, as can be seen in the figure below. The stress is parallel to the direction of the force, through the geometry of the part (*Shear stresss / physics / Britannica*, no date). The equation used to find the value for shear stress can be seen in $\tau = \text{Shear Stress}$
- F_{Net} = Net force acting on the Gudgeon Pin
- $F_{Inertia\ pin}$ = Force due to inertia acting on the Gudgeon Pin
- D_1 = Outside diameter of the Gudgeon Pin
- D_2 = Inside diameter of the Gudgeon Pin

Equation 1.

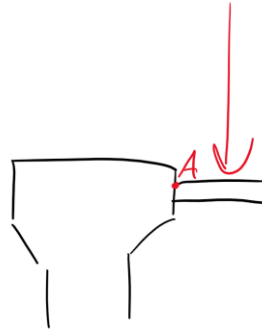


Figure 2: Shear Stress

$$\tau = \frac{\frac{F_{Net}}{2} - F_{Inertia Pin}}{\frac{\pi}{4}(D_1^2 - D_2^2)}$$

- τ = Shear Stress
- F_{Net} = Net force acting on the Gudgeon Pin
- $F_{Inertia Pin}$ = Force due to inertia acting on the Gudgeon Pin
- D_1 = Outside diameter of the Gudgeon Pin
- D_2 = Inside diameter of the Gudgeon Pin

Equation 1: Shear Stress

The bending stress is caused by the opposing forces on the gudgeon pin, not acting at the same point. These cause opposing moments within the component that cause this stress. An example of this can be seen in Figure 3. In the case of the gudgeon pin, the single force (F) is caused by the Con-Rod whilst the stationary points are caused by the piston.

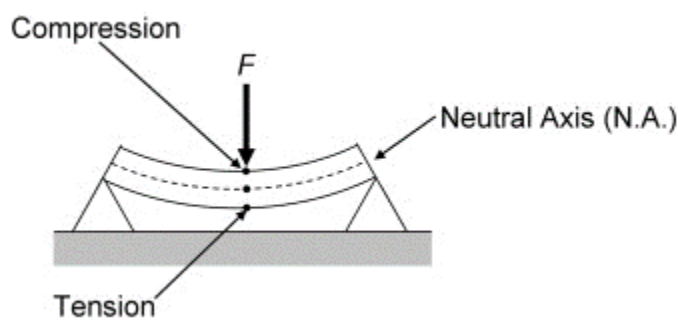


Figure 3: Bending Stress

$$\sigma = \frac{M \cdot y}{I}$$

- σ = Bending Stress
- M = Bending Moment
- y = Neutral axis
- I = Moment of Inertia of the pin

Equation 2: Bending Stress

$$M = \left(\frac{F_{Net}}{2} \times L_1 \right) - (F_{Inertia} \times L_2)$$

- M = Bending Moment
- L = Length Sticking out

Equation 3: Bending Moment

In the case of the gudgeon pin, the total stress, or combined stress, is made up of two components. These are the shear stress, and the bending stress. Two methods for combining the individual stresses were considered before the combined stress method (Equation 7) was used. The alternative Von Mises stress is often used in FEA (Lombard, 2013) as a method for combining stress, however it was not applicable in this scenario. For this area of investigation, the Combined stress needs to be found when the Net Force is at its maximum and minimum in order to find the range of Combined Stresses, as well as the mean Combined Stress. All inertia forces were calculated as seen at the same degree of crank rotation as the maximum net force.

Using the information calculated above, two different methods can be applied to determine the fatigue safety factor for the gudgeon pin. The first is to produce a Goodman Diagram, as seen in Figure 4. This method uses the maximum, minimum and mean combined stresses in order to determine the “allowable range”. In the figure below, the allowable range is represented by the length of the green line (mean stress) between the points where it intersects the maximum and minimum stress lines. The other method uses a mathematical approach to calculate the allowable range. This can be done using Equation 4 to calculate the allowable range. Once either method has been used to calculate the allowable range, this must be divided by the actual range, a constant for the material, in order to find the fatigue safety factor. In the case of this engine, a result of 0.85 was found. A result of less than 1 suggests that the part is likely not to have an infinite fatigue lifetime. For short term use this might be acceptable, however in commercial vehicles, a minimum value of 1 would usually be expected (*Safety factor and fatigue life - effective design measures - CAEbay, no date*).

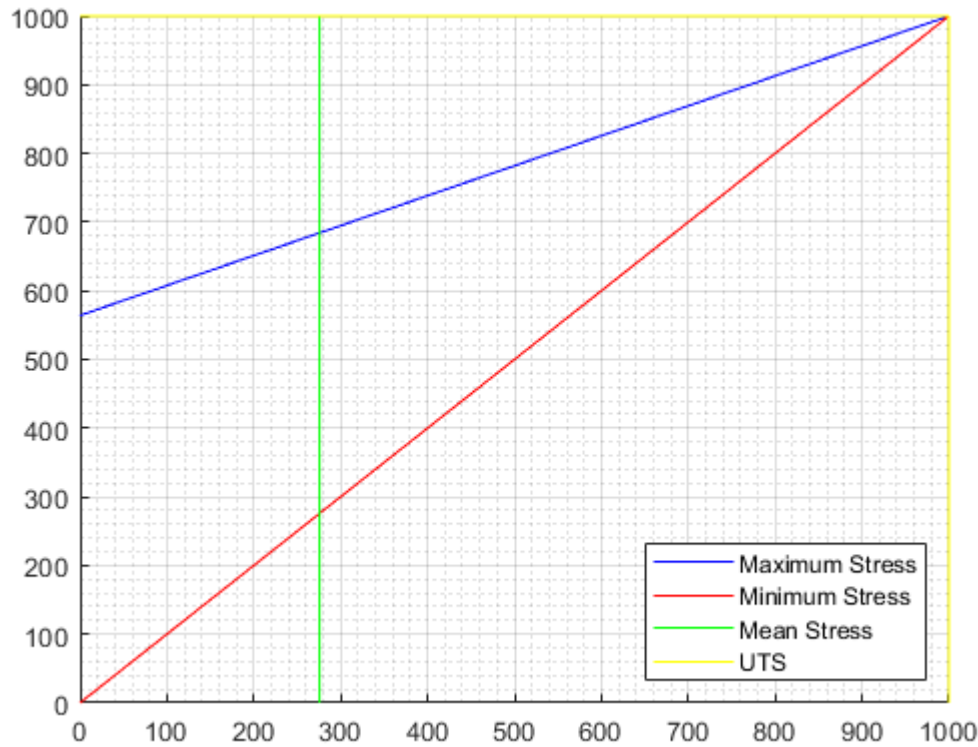


Figure 4: Goodman Diagram - Gudgeon Pin

$$Allowable\ Range = 2 \times Endurance\ Limit \times \left(1 - \frac{Mean\ Combined\ Stress}{UTS}\right)$$

Equation 4: Allowable Range

Looking at Figure 5 shows how the fatigue safety factor varies with each RPM of the Crank Shaft. It is obvious that this is inversely proportional to the maximum forces acting on the pin. This correlates to expectation as the higher the forces on the pin become, the more likely it is that the part will fail (Gillam, 1969).

Figure 6 shows how the combined stress varies at each degree of crank rotation. Since the Fatigue safety factor is proportional to the combined stress the part sees, this also shows at which degree of crank the part is most likely to fail. The graph shows that maximum stress is seen at around 360° of crank rotation. This coincides with top dead centre on the combustion stroke where the pressure in the cylinder is at its highest.

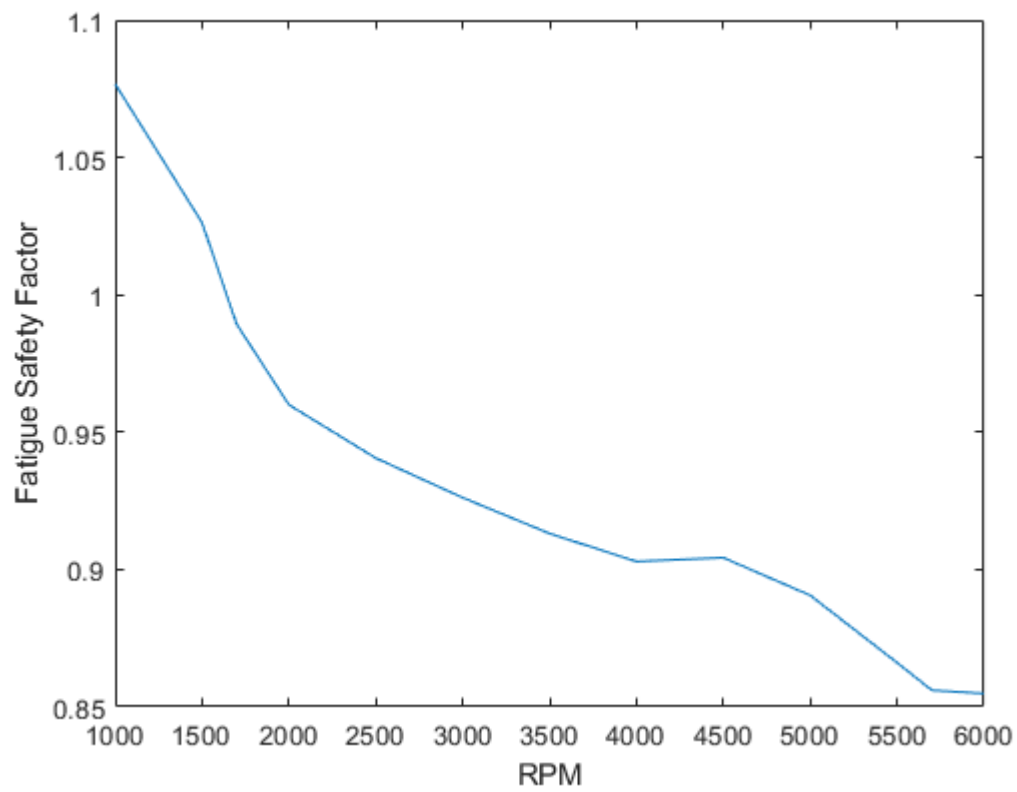


Figure 5: Gudgeon Pin Fatigue Safety Factor

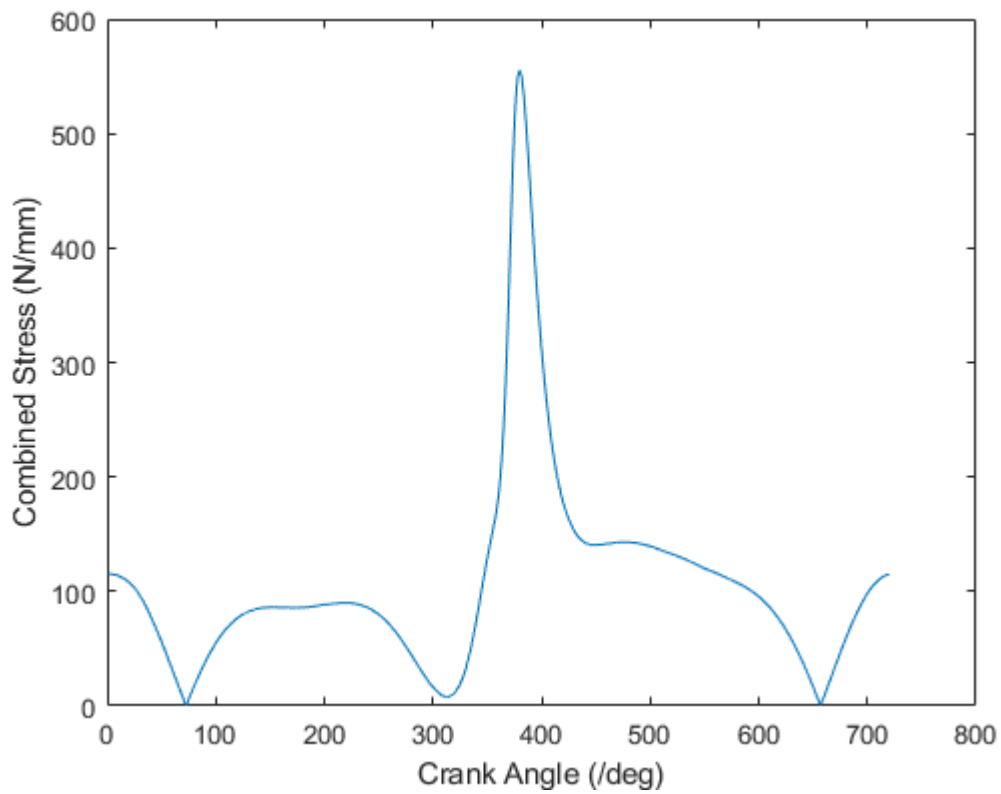


Figure 6: Combined Stress vs. Crank Angle

Con-Rod

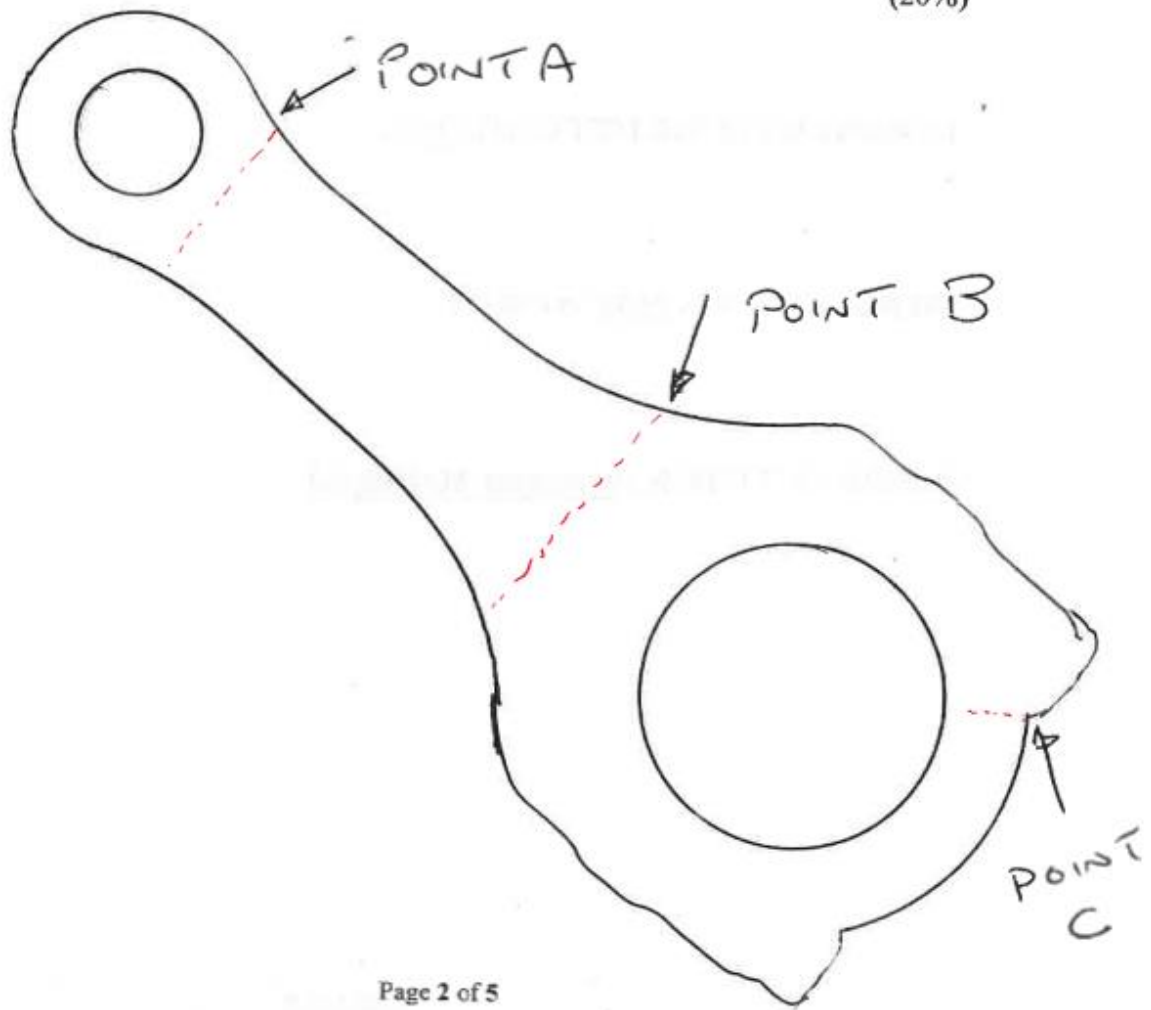
Using similar methods to the ones used for the gudgeon pin, the fatigue safety factor can be established for the con-rod. Due to the way in which the forces act on the con-rod, there is no bending moment that needs to be taken into account. Calculating the shear stress alone and using it instead of the combined stress used on the gudgeon pin, the fatigue safety factor can be calculated.

$$Shear\ Stress = \frac{\left(\frac{F_{Net}}{2}\right)}{Surface\ Area}$$

Figure 7: Shear Stress - Con-Rod

Three areas of the con-rod were investigated. These can be seen in Figure 8. Each area shows a different point at which a stress concentration, potentially leading to failure, might occur. At points A and B, the stresses through the con-rod consist solely of a shear stress, caused by tension and compression during the four-stroke cycle. Whilst the Gas Forces acting on the component remains the same in both areas, the component of the con-rod taken into consideration when calculating Inertia Forces changes, as well as the cross-sectional surface area, used to calculate the stress. Due to the low radii in these areas, the magnitude of the stress concentrations becomes so small that they have a negligible effect on the overall Fatigue Safety Factor (Pilkey and Peterson, 1997). The same however, can not be said for point C. Here, a stress concentration of 1.5 caused the fatigue safety factor to become reduced.

(20%)



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Figure 8: Con-Rod

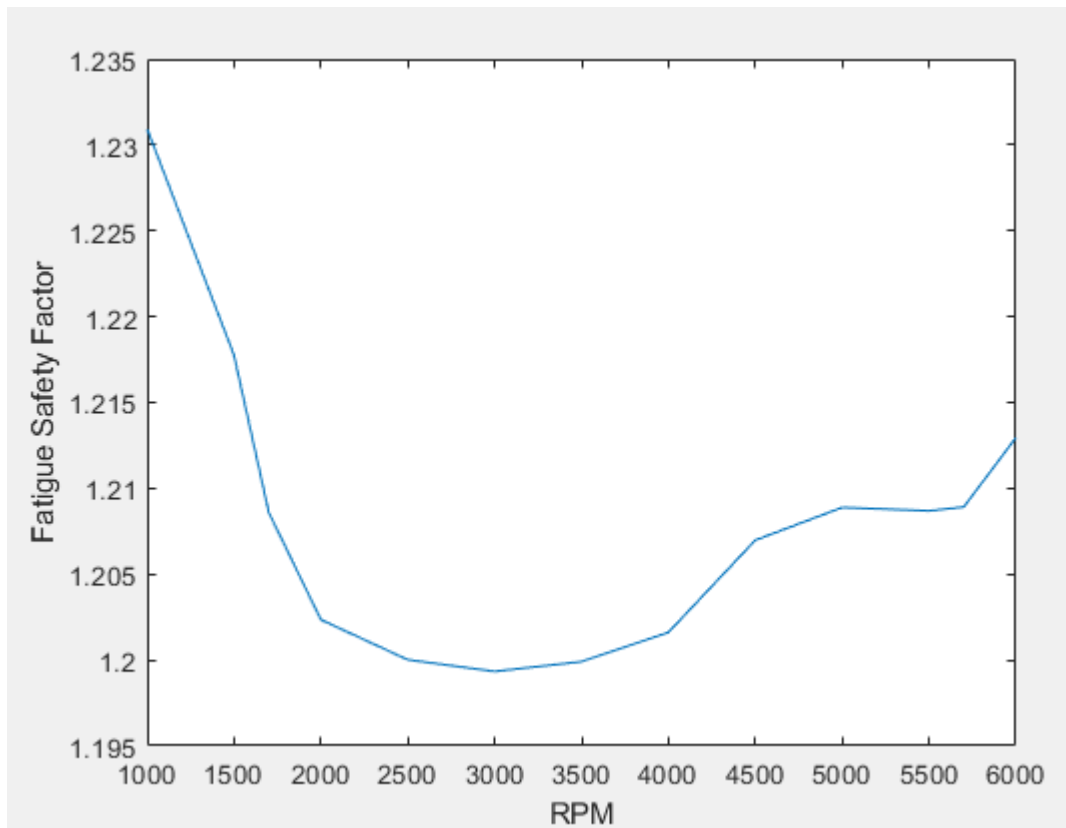


Figure 9: Point A - Fatigue Safety Factor vs. RPM

As the figure above shows, the fatigue safety factor varies depending on the RPM. The point where the lowest fatigue safety factor was found to occur was at 3000 RPM. Since it is the worst-case scenario, all fatigue safety factors for the con-rod were investigated at this point. Usually it might be expected that the worst-case scenario might occur at the maximum RPM being looked at, or where the gas forces are at their highest. The reason this is not the case for the scenario being investigated was determined to be as a result of the Inertial forces acting on the part.

Looking at points A and B shows that the con-rod is more likely to fail at point A. The lower fatigue safety factor of 1.19 at this point compared to the value of 1.25 is to be expected due to the lower cross-sectional surface area in this region. This however is unlikely to cause any concerns as even this low value is above 1, suggesting that the part should have an infinite fatigue life, provided that the forces remain within the range investigated.

Point C on the other hand raises some concerns due to its low fatigue safety factor. The low value of only 0.37 suggests that within normal operating conditions, the part will fail despite the forces acting on the part, not exceeding the yield stress of the part. Depending on the intended use for this part, this low fatigue life might be acceptable. If it is intended for commercial use, it would be advisable to choose a different material or to alter the geometry of the part, in order to ensure its function.

Con-Rod Bolt

In order to approximate the ratio of stiffnesses between the con-rod cap and the con-rod bolt, the ratio of surface areas is often used. This ratio helps to ensure that the bolts in the con-rod will not loosen themselves over the lifetime of the engine. It is important to ensure that the stiffness of the bolt is lower than that of the cap, as this way it can be tightened into yield, allowing it to stretch with the cap, rather than coming undone. When material properties are unknown or yet to be determined, the surface area ratio can be used instead of the stiffness. This allows for an approximation. If the material properties are known, the equations below can be used in order to determine the correct stiffness ratio, for the same purpose.

$$Stiffness = \frac{Young's Modulus \times Surface Area}{Length}$$

Equation 5: Material Stiffness

$$Ratio\ of\ Bolt\ Stiffness = \frac{Stiffness_{Bolt}}{Stiffness_{Cap}}$$

Equation 6: Ratio of Bolt Stiffness

The alternating load shows the range between the minimum and maximum stress acting on a part over the combustion cycle. In the case of the con-rod cap, although the individual loads acting on the part are not enough to bring the part to failure, or even within its yield range, a constant, oscillating force can still cause the part to fail. Comparing this load to the actual load gives the fatigue safety factor. If this value is greater than 1, the part will never fail, as long as the forces remain within the range of the alternating load. However, if the fatigue safety factor is less than 1, the part will eventually fail.

In order to ensure that the bolts act the correct clamping force on the con-rod, a pre-defined tightening torque has to be determined. This ensures that during the assembly process, the correct clamping force is acting on the bolt and the con-rod. The bolt is tightened to the point where it has gone into 90% of yield.

At this point, the 90% yield can be used as the minimum value for stress within the bolt, as this is the amount of stress put into the bolt, before any further stress is introduced through the combustion cycle of the engine. The range of stresses in the bolt can be defined as the alternating load divided by the surface area of the bolt (i.e. Force/Surface Area). These values can be used to determine the fatigue safety factor and to produce Goodman Diagrams.

Now that the safety factor of the standard bolt is known, it is possible to begin modifying it. The safety factor helps to show if it is possible to take away material from the bolt. This would make it possible to reduce the overall mass of the part. However, it might not be possible to remove much material, without reducing the safety factor, below an acceptable threshold. In that situation, other materials with higher UTS values might be considered, as these will return the safety factor to a suitable value.



Figure 10: Original Bolt Geometry

The original bolt used in the con-rod has a UTS of 1100 and its geometry can be seen in Figure 10. Using this bolt results in a Fatigue Safety Factor of 2.7 suggesting that there is no risk of failure. Despite this high safety factor within the usual RPM range of the engine, it is advisable to extend the investigation to include RPM ranges above the maximum. Although situations where this might occur are rare, they are not impossible. Although gas forces beyond the real RPM range are not known, the increased force on the parts due to inertia can be calculated. For the purpose of this investigation the maximum RPM was extended to 9000 RPM. The calculated inertia forces were combined with the real maximum gas forces to gain a fatigue safety factor of 1.3, so still within acceptable limits. This “traditional” style of bolt uses threads reaching from the flange all the way to the end of the bolt. Although this method of manufacturing bolts is cheaper than a more complex geometry, there is a large amount of wasted material, resulting in the part weighing more than necessary.

Apart from the area in which the thread that is engaged to the con-rod, the thread can be removed all the way up to the flange at the head of the bolt. In these areas, the diameter of the bolt could be reduced even further, provided that the overall stiffness of the bolt does not fall below a certain threshold in order for the Fatigue Safety Factor to be maintained. Equation 8 to Equation 11 show how the fatigue safety factor is related to the overall stiffness of the bolt. Equation 12 further shows how the overall stiffness can be separated into the three individual stiffnesses across the individual areas as seen in Figure 11. Finally, Equation 13 relates the stiffness of the waisted section of the bolt, to its surface area and therefore its diameter. Furthermore, a material with a higher UTS of 1862 was chosen, allowing the bolt to be further lightened without causing it to fail. This meant that a final diameter of only 5.65mm versus the original 7.35mm was needed to maintain a safety factor of 1.4 at 9000 RPM.

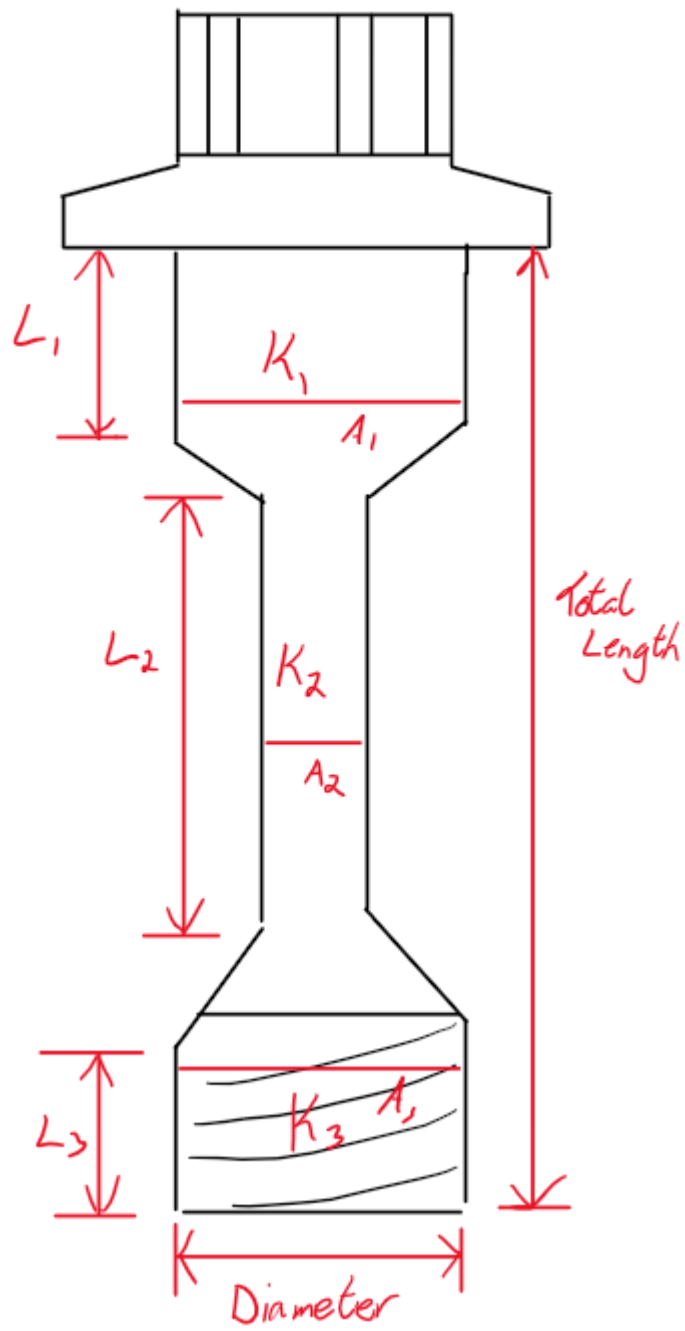


Figure 11: Modified Bolt

Crank Shaft

The final component of the engine that is liable to failure, is the crank shaft itself. In order to determine which areas of the crank shaft to investigate, an FEA analysis can be used to find areas where stress concentrations might occur. This analysis can also be used to find what stresses are expected to be found throughout various sections of the crank shaft, however the accuracy of these values is limited by the choices of material available.

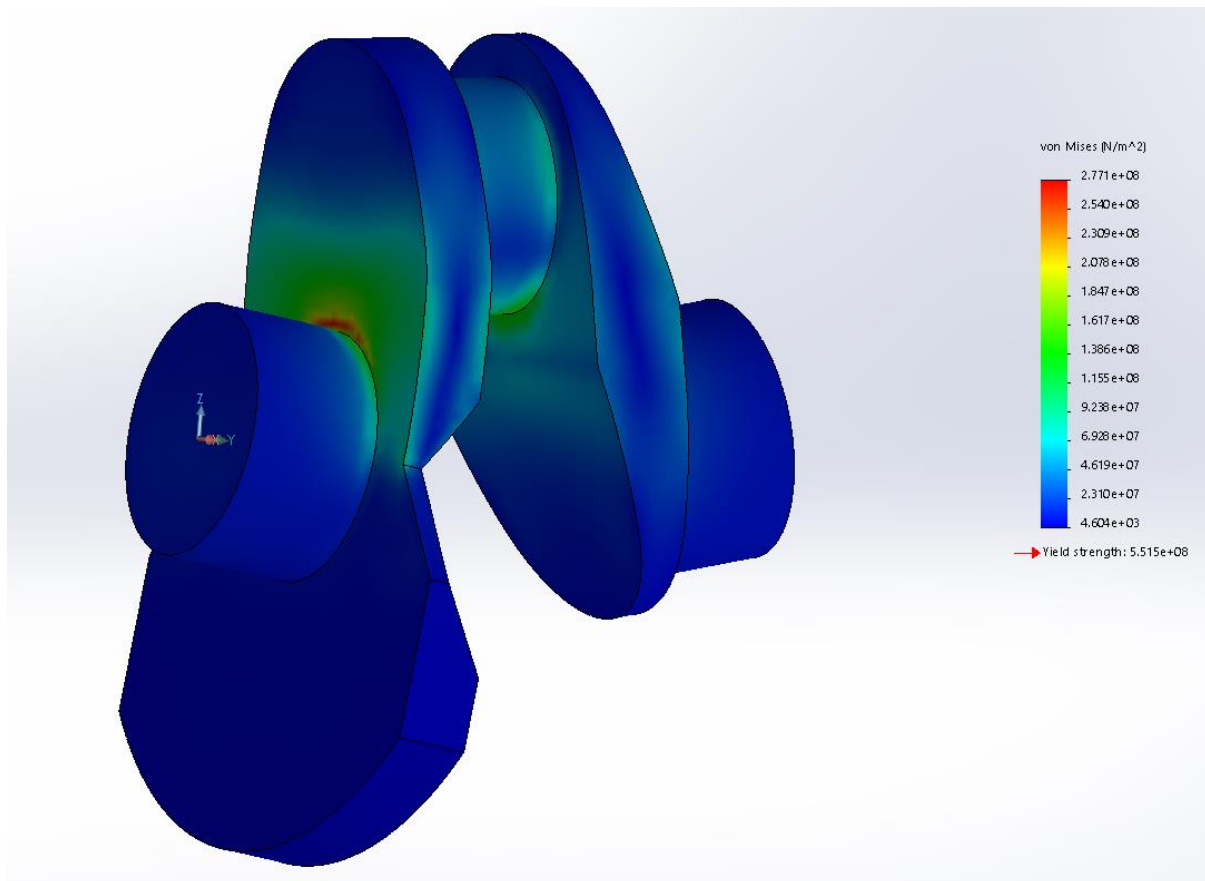


Figure 12: Crank Shaft FEA

The FEA produced on the crank shaft shows that within normal use, the forces it experiences would not be enough to cause any part of it to go into yield, as the highest stress it sees, is still less than 50% of its yield strength. However, this simulation does show where the yield strength would be expected, meaning that a fatigue safety factor can be investigated at this point since, as mentioned previously, the part can still fail through repeated use, even if forces do not exceed the elastic limit of the material.

A minimum fatigue safety factor of 1.153 is achieved. Although this value is lower than desirable, as it does not leave a lot of room for error, it still confirms that the crank should have an infinite fatigue life within normal use.

In order to calculate the fatigue safety factor, similar equations were used as previously. In this case the line of the second moment of inertia was taken in the location shown in the figure below.

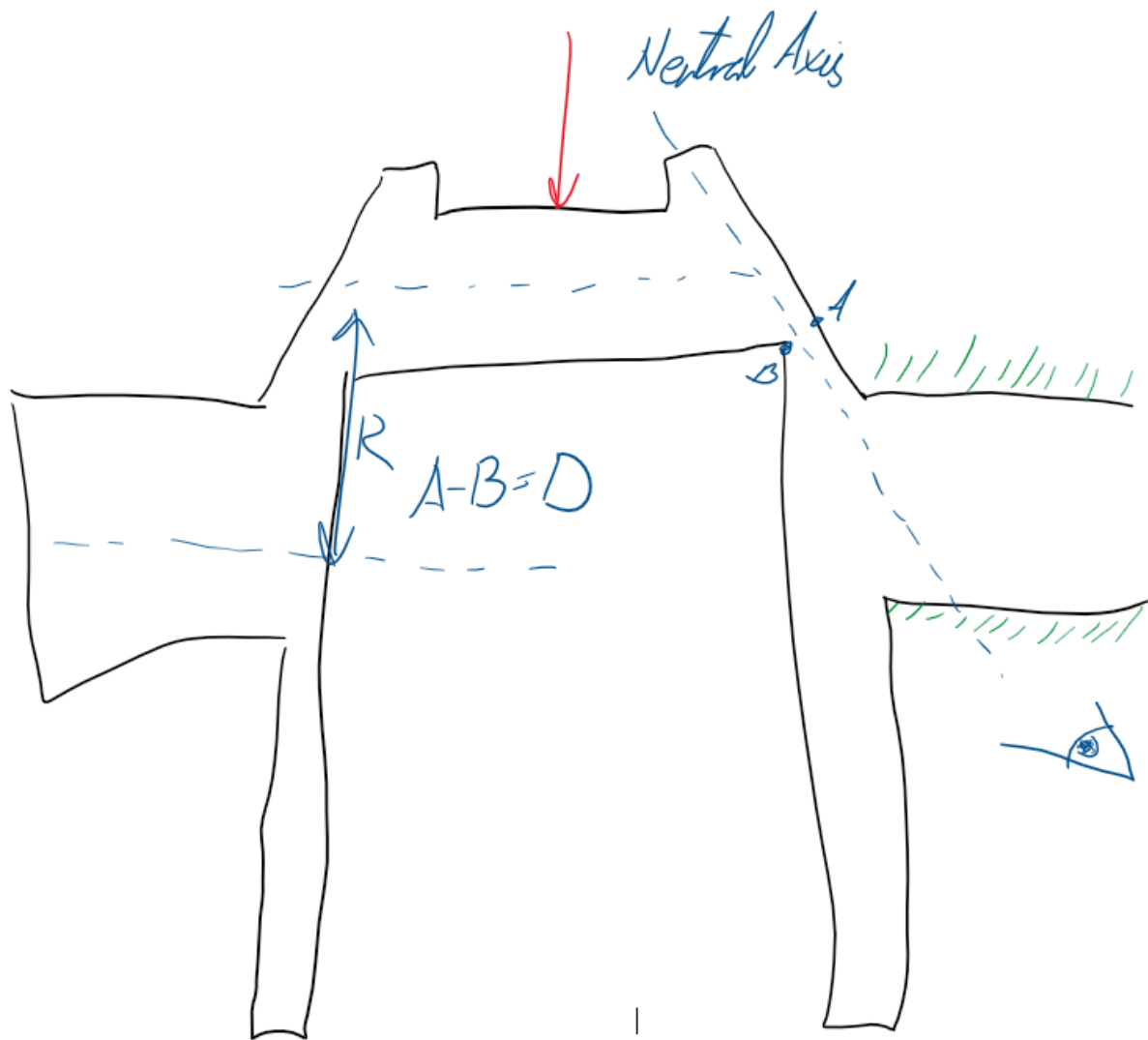


Figure 13: Crank Shaft

These calculations however do not take into account two important aspects of the crank shaft. The first is that the structure of the component is compromised by the holes passing through the material, allowing oil to flow to various parts of the crank shaft during normal use. Further FEA analysis including these holes might reveal a different stress concentration point that requires investigating. On the other hand, the FEA already completed does not take into account any radii that the actual geometry includes, in an attempt to relieve the stress concentrations that are already present. In actuality, the stress concentration already identified, might not be present on the real component. Future work might include updating the CAD model with these features before repeating the FEA on the component.

Conclusion

This investigation has shown that each component in its current design is likely to survive the task it is supposed to complete throughout and beyond the expected lifetime of the engine they were designed for. The investigation has highlighted various areas for improvement, that might reduce cost, reduce weight or increase performance of the parts, however these improvements are likely to increase the cost of manufacture of these components. This increased cost is likely the reason why the original manufacturer dismissed the idea of implementing the same design changes.

Finally, although the calculations carried out give an indication of the performance ability of the components, they are never perfect. Ideally, real world testing should be carried out to confirm their accuracy.

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Appendix

$$\text{Combined Stress} = \sqrt{\sigma^2 + (2 \times \tau)^2}$$

- σ = Bending Stress (N/mm²)
- τ = Shear Stress (N/mm²)

Equation 7: Combined Stress

$$\text{Fatigue Safety Factor} = \frac{\text{Actual Range}}{\text{Alternating Stress}}$$

Equation 8: Fatigue Safety Factor

$$\text{Actual Range} = 2 \times \text{Elastic Limit} \times \left(1 - \frac{\text{Shear Stress}}{\text{UTS}}\right)$$

Equation 9: Actual Range

$$\text{Alternating Stress} = \frac{\text{Alternating Load}}{\text{Surface Area}}$$

Equation 10: Alternating Stress

$$\text{Alternating Load} = \frac{1}{1 + \frac{\text{Stiffness}_{\text{Cap}}}{\text{Stiffness}_{\text{Bolt}}}} \times \frac{F_{\text{Inertia}}}{2}$$

Equation 11: Alternating Load

$$K_{\text{Total}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

Equation 12: Bolt Stiffness

$$K_x = \frac{\text{Young's Modulus} \times \text{Cross Sectional Surface Area}}{\text{Length}}$$

Equation 13: Stiffness

$$\text{Minimum Safe Diameter} = \sqrt{\frac{\text{Old UTS}}{\text{New UTS}}} \times \text{Old Diameter}^2$$

Equation 14: Minimum Safe Bolt Diameter

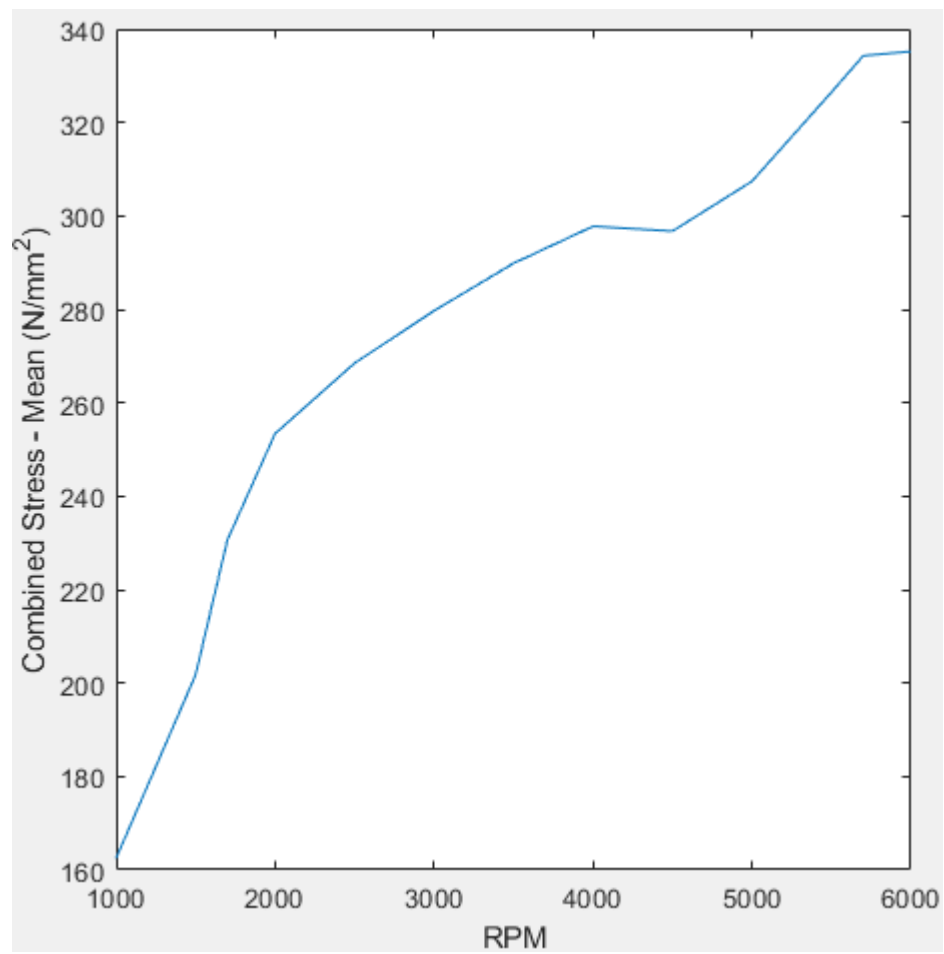


Figure 14: Combined Stress Mean vs. RPM

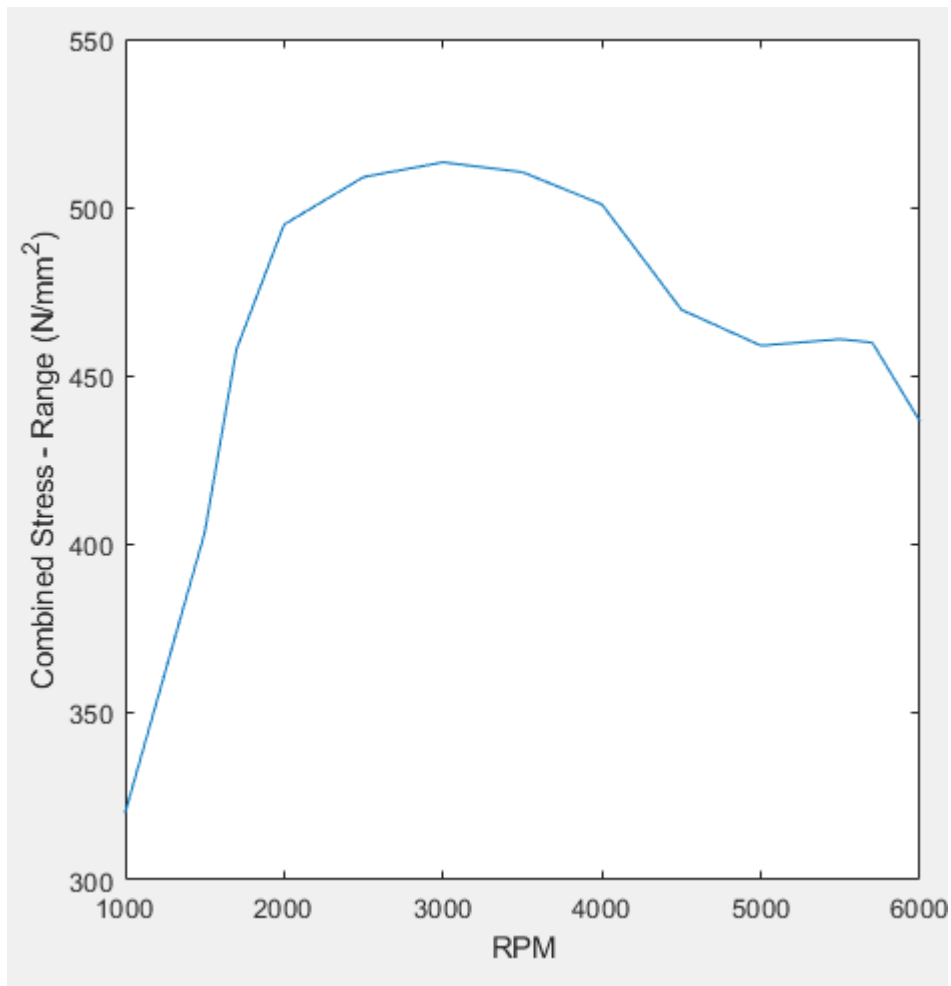


Figure 15: Combined Stress Range vs. RPM

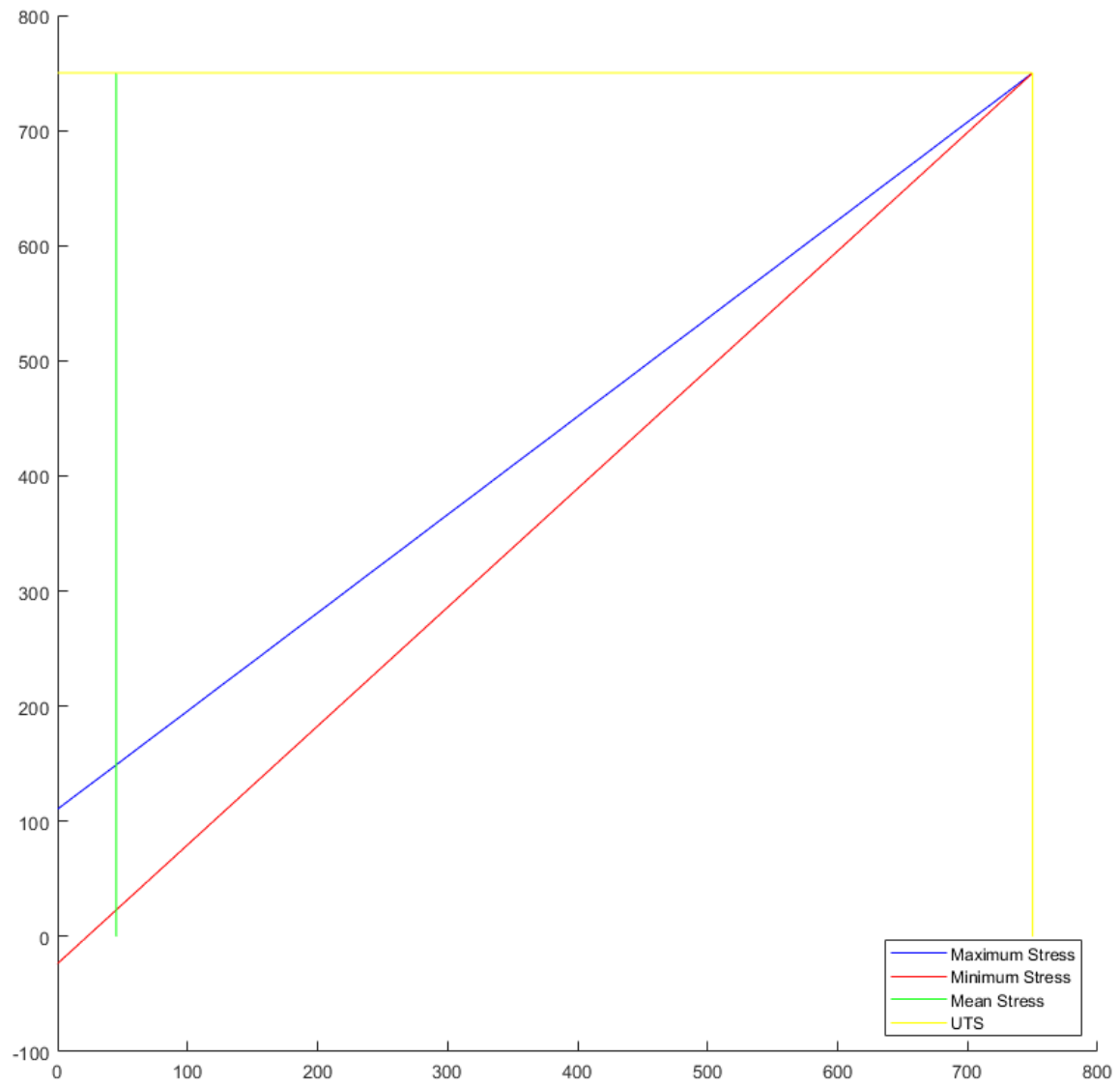


Figure 16: Goodman Diagram - Con-Road section A

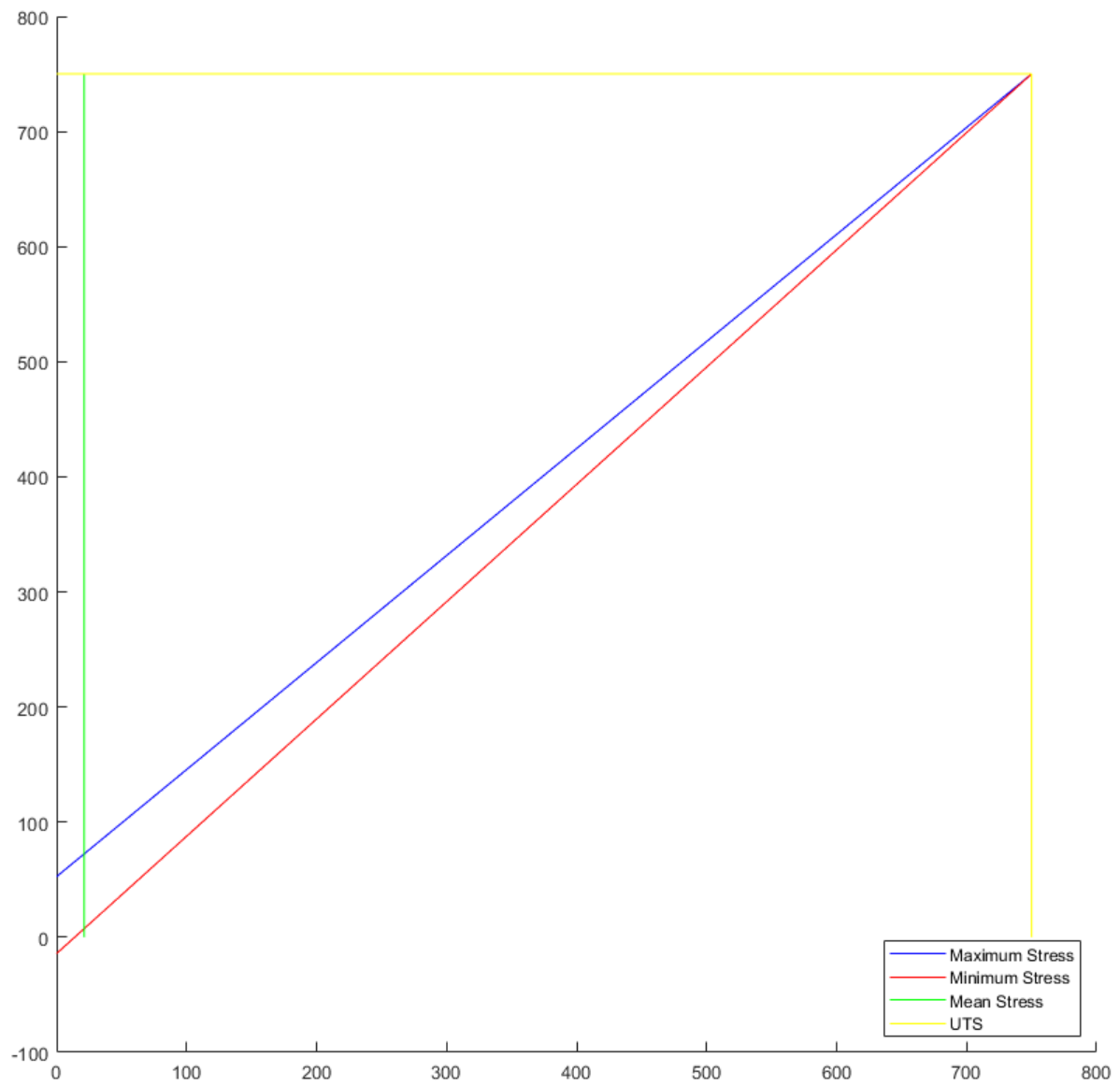


Figure 17: Goodman Diagram - Con-Road section B

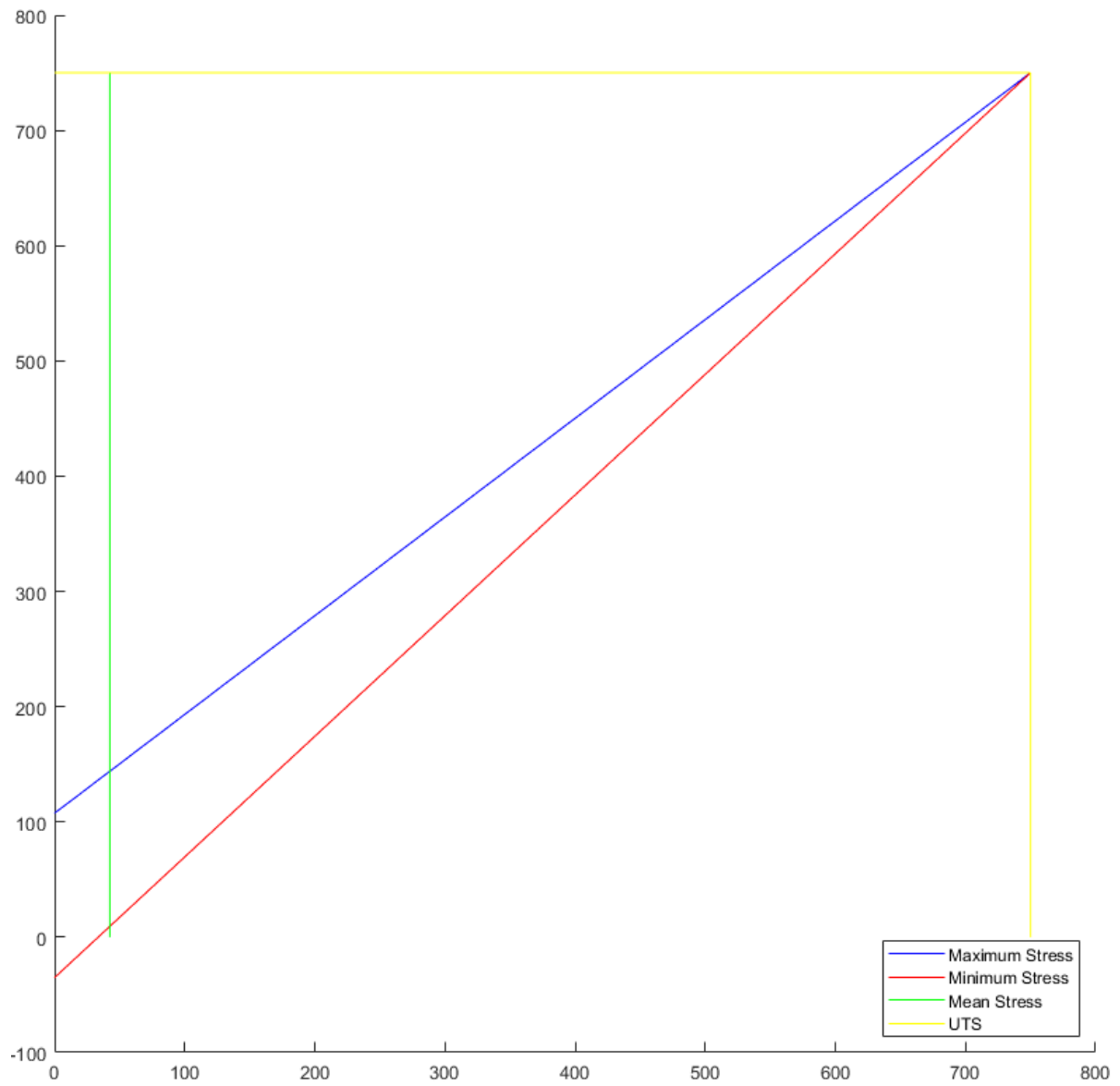


Figure 18: Goodman Diagram - Con-Rod section C

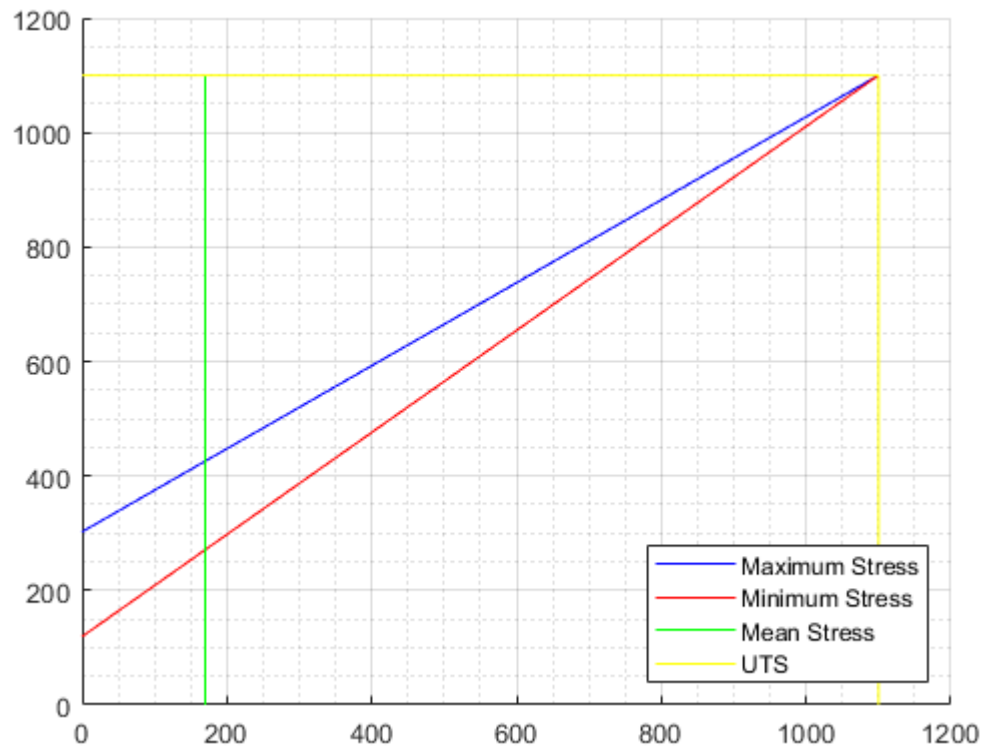


Figure 19: Goodman Diagram - Crank Shaft