

Maths Exam

①

a)

i).

$$y = \frac{x^2}{2x+1}$$

$$f = x^2$$

$$g = 2x+1$$

$$\frac{df}{dx} = 2x$$

$$\frac{dg}{dx} = 2$$

$$\frac{dy}{dx} = \frac{\frac{df}{dx}(g) - (f)\frac{dg}{dx}}{g^2}$$

$$\frac{dy}{dx} = \frac{(2x(2x+1)) - (x^2)(2)}{(2x+1)^2}$$

$$= \frac{(4x^2 + 2x) - (2x^2)}{(2x+1)(2x+1)}$$

$$= \frac{2x^2 + 2x}{4x^2 + 4x + 1}$$

$$= \frac{x^2 + x}{2x^2 + 2x + 1}$$

$$\text{ii). } y = \sqrt{x^2 + 1}$$

$$= (x^2 + 1)^{\frac{1}{2}}$$

$$\therefore y = u^{\frac{1}{2}}$$

$$\frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}}$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \left(\frac{1}{2} u^{-\frac{1}{2}} \right) \times (2x)$$

$$= \frac{1}{2} (x^2 + 1)^{-\frac{1}{2}} \times 2x$$

$$= x (x^2 + 1)^{-\frac{1}{2}}$$

$$= \frac{x}{\sqrt{x^2 + 1}}$$

$$\text{iii). } y = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x$$

$$= 3x^{\frac{3}{2}} + 3x^{-2} - 6x$$

$$\frac{dy}{dx} = \frac{9}{2} x^{\frac{1}{2}} - 6x^{-3} - 6$$

$$\text{iv). } y = \sec^3 4x$$

$$\sec^3 = \frac{1}{\cos^3}$$

$$\sec^3 4x = \frac{1}{(\cos 4x)^3}$$

$$= (\cos 4x)^{-3}$$

$$\frac{du}{dx} = -4 \sin 4x$$

$$u = \cos 4x$$

$$\therefore y = u^{-3} \quad \frac{dy}{du} = -3u^{-4}$$

$$= -3(\cos 4x)^{-4} \times (-4 \sin 4x)$$

$$iv) y = (-3)(-4 \sin 4x)(\cos 4x)^{-4}$$

$$\begin{aligned} \frac{dy}{dx} &= 12 \sin 4x (\cos 4x)^{-4} \\ &= \frac{12 \sin 4x}{(\cos 4x)^4} \end{aligned}$$

$$v) y = (x + \sin 2x)$$

$$\frac{dy}{dx} = 1 + 2 \cos 2x$$

$$vi) y = \frac{1-10x}{(2x-1)^2}$$

$$\begin{aligned} f &= (1-10x) \\ g &= (2x-1)^2 \end{aligned}$$

$$\frac{df}{dx} = -10$$

$$\frac{dg}{dx} = u^2 \rightarrow 2u \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{\frac{df}{dx}(g) - (f)\frac{dg}{dx}}{g^2}$$

$$\begin{aligned} u &= 2x-1 & \frac{du}{dx} &= 2 \\ \frac{dg}{dx} &= 2(2x-1)(2) \\ &= 8x-4 \end{aligned}$$

$$= \frac{(-10)(2x-1)^2 - (1-10x)(8x-4)}{(2x-1)^2)^2}$$

$$= \frac{-10(2x-1)^2 - (1-10x)(8x-4)}{(2x-1)^4}$$

$$\begin{aligned}
 \text{Vi. } & (-10)(4x^2 - 2x - 2x + 1) - (8x - 4 - 80x^2 - 40x) \\
 & (-40x^2 + 40x - 10) - (8x - 4 - 40x) \\
 & \underline{\hspace{10em}} \\
 & (2x-1)^4
 \end{aligned}$$

$$\frac{dy}{dx} = \frac{40x^2 - 8x - 6}{(2x-1)^4}$$

$$\text{b). } y = \frac{1}{x} + \cos 2x = x^{-1} + \cos 2x$$

$$\frac{dy}{dx} = -1x^{-2} - 2\sin 2x$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 4\cos 2x$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 8\sin 2x$$

$$\frac{d^4y}{dx^4} = 24x^{-5} + 16\cos 2x$$

$$\frac{d^5y}{dx^5} = -120x^{-6} - 32\sin 2x$$

$$= -\frac{120}{x^6} - 32\sin 2x$$

$$c). f(x) = \ln(\operatorname{cosec} x - \cot x)$$

$$u = \operatorname{cosec} x - \cot x$$

$$\frac{du}{dx} = -\operatorname{csc} x (\cot x)$$

$$f'(x) = \frac{\ln(u)}{u} \times \frac{du}{dx}$$

$$\rightarrow \frac{du}{dx} = (-\operatorname{csc} x)(\cot x) - (-\operatorname{csc}^2 x)$$

$$=$$

②

a)

i).

$$\int \frac{2x}{(4+3x^2)}$$

$$= \int (2x) (4+3x^2)^{-1}$$

$$\int uv' = uv - \int u'v$$

$$u = 2x$$

$$u' = 2$$

$$v' = (4+3x^2)^{-1}$$

v

$$\int (4+3x^2)^{-1}$$

~~u = 4+3x^2~~
 ~~$\frac{du}{dx} = 6x$~~

~~$\int u^{-1} \frac{du}{dx}$~~

~~$\int \frac{1}{4+3x^2}$~~

$$\text{ii). } \int_0^4 e^{\frac{1}{2}x} dx$$

$$= 2e^{\frac{1}{2}x} + C$$

$$\left[2e^{\frac{1}{2}x} \right]_0^4 = 2e^{\frac{1}{2}(4)} - 2e^0$$

$$= 12.7781122$$

$$\text{iii). } \int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$$

$$= \frac{5x+13}{2x^2+8x+x+4}$$

$$= \frac{5x+13}{2x^2+9x+4}$$

$$= (5x+13)(2x^2+9x+4)^{-1}$$

$$V' = 5x+13$$

$$u = (2x^2+9x+4)^{-1}$$

$$u' = -1u^{-2} \times \frac{du}{dx} = 4x+9$$

$$= -4x-9(2x^2+9x+4)^{-2}$$

1). ii). 3

$$\frac{(1+i)(2+i)}{}$$

$$= \frac{3}{2+i+2i+i^2} \times \frac{2-i-2i-i^2}{2-i-2i-i^2}$$

$$= \frac{6-3i-6i-3i^2}{}$$

$$4 - 2i - 4i - 2i^2 + 2i - i^2 - 2i^2 - i^3 + 4i - i^2$$

$$\begin{aligned} & -4i^2 \\ & -2i^2 \\ & +2i^2 \\ & -i^3 \\ & -2i^3 \\ & -i^4 \end{aligned}$$

$$= \frac{6-3i-6i-3i^2}{}$$

$$4 - 9i^2 - 4i^3 - i^4$$

~~iii~~ iv). $2i^2 - 2i + \sqrt{3} - 1 + 1 + i + i + i^2$

$$= 3i^2 + \sqrt{3}$$

$$= \sqrt{3} + 3i^2$$

$$= 3 + 3i^2$$

6)

$$\begin{aligned} z &= (4+7i)(3-2i) \\ &= 12 - 8i + 21i - 14i^2 \\ &= 12 + 13i - 14i^2 \end{aligned}$$

$$c) \quad \frac{1+3i}{1+i} \times \frac{(1-i)}{(1-i)}$$

$$= \frac{(1+3i)(1-i)}{(1+i)(1-i)}$$

$$= \frac{1-i+3i-3i^2}{1-i^2}$$

$$= \frac{1+2i-3i^2}{1-i^2}$$

$$\div (-i^2)$$

$$= \frac{1+2i+3}{1}$$

$$= 1+2i+3$$

$$= 2i+4$$