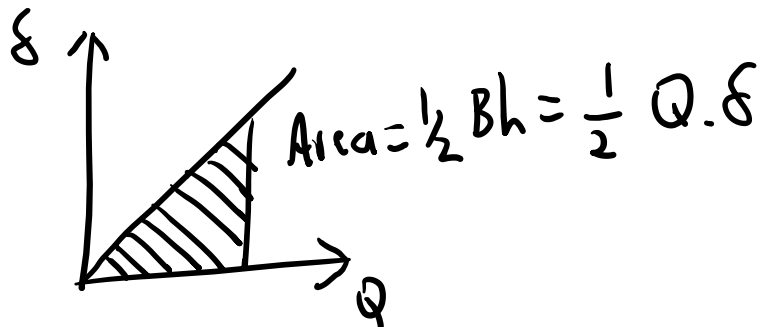


$$2ai) \text{ Shear Modulus} = \frac{\text{Shear Stress}}{\text{Shear Strain}} \text{ or } G = \frac{\tau}{\gamma}$$

$$\text{as } \tau = \frac{\text{Shear load}}{\text{Area}} = \frac{Q}{A}$$

$$G = \frac{Q}{A\gamma} \quad \therefore \gamma = \frac{Q}{AG}$$

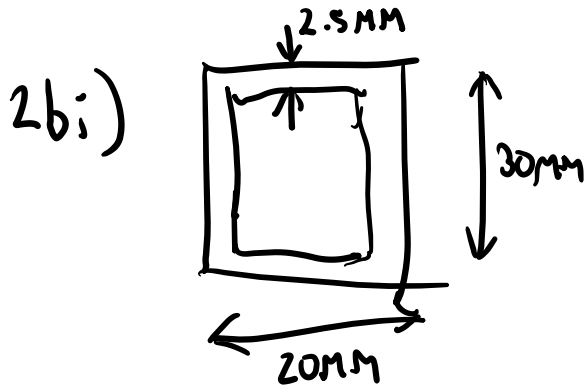
2aii) As strain energy = Work done = load $\times$ displacement



$$\text{And } \gamma = \frac{\gamma}{ds} = \frac{Q}{AG} \quad \therefore \gamma = \frac{Q ds}{AG} = \frac{QL}{AG}$$

substituting for U

$$U = \frac{1}{2} Q \cdot \frac{QL}{AG} = \frac{Q^2 L}{2AG}$$



$$L = 2.4 \text{ m}$$

$$\sigma = 480 \text{ MPa}$$

$$E = 190 \text{ GPa}$$

$$U = \frac{P^2 L}{2AE} \quad A = (30 \times 10^{-3} \times 20 \times 10^{-3}) - (25 \times 10^{-3} \times 15 \times 10^{-3})$$

$$= 2.25 \times 10^{-4}$$

$$P = 480 \times 10^6 \times 2.25 \times 10^{-4} = 108 \times 10^3 \text{ N}$$

$$U = \frac{(108 \times 10^3)^2 \times 2.4}{2 \times 2.25 \times 10^{-4} \times 190 \times 10^9} = 327.4 \text{ J}$$

2b.ii) $P \cdot E = Mgh$ and $U = \frac{P^2 L}{2AE}$

$$\therefore Mgh = \frac{P^2 L}{2AE} \quad \therefore h = \frac{P^2 L}{2AEMg} \quad \text{as } Mg = W$$

$$\text{and } P^2 = \sigma^2 A^2$$

$$h = \frac{\sigma^2 A^2 L}{2AEW} \quad \text{AND as } AL = V$$

$$h = \frac{\sigma^2 V}{2EW}$$

2b.iii) $M = 20$
 $h = 0.45 \text{ m}$

$$\sigma = \sqrt{\frac{h 2EW}{V}} = \sqrt{\frac{0.45 \times 2 \times 190 \times 10^9 \times 196.2}{2.25 \times 10^{-4} \times 2.4}}$$

$$= 249.3 \text{ MPa}$$

$$2biv) \quad v = \frac{h2EW}{\sigma^2} = \frac{2.4 \times 2 \times 190 \times 10^9 \times 196.2}{(249.3 \times 10^6)^2}$$
$$= 2.879 \times 10^{-3}$$

$$\therefore A = \frac{V}{L} = \frac{2.879 \times 10^{-3}}{2.4} = 1.1996 \times 10^{-3}$$

$$A = \frac{\pi D^2}{4} - \frac{\pi d^2}{4}$$

$$1.1996 \times 10^{-3} = \frac{\pi \times 0.02^2}{4} - \frac{\pi d^2}{4}$$

$$1.196 \times 10^{-3} = 3.14 \times 10^{-4} - \frac{\pi d^2}{4}$$

$$\frac{\pi d^2}{4} = -8.82 \times 10^{-4}$$

$$d = \sqrt{\frac{(-8.82 \times 10^{-4}) \times 4}{\pi}}$$

$$3a) d = 0.075 \text{ Mm}$$

$$t = 0.005$$

$$\sigma_y = 325 \times 10^6$$

$$\nu = 0.3$$

$$\text{S.F} = 6$$

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 / 4 + \tau_{xy}^2}$$

$$\sigma_2 \approx 0$$

$$\sigma_3 = \frac{1}{2}(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 / 4 + \tau_{xy}^2}$$

$$\text{as } \sigma_x \text{ and } \sigma_y = 0$$

$$\sigma_1 = \sqrt{4\tau_{xy}^2}$$

$$\sigma_3 = -\sqrt{4\tau_{xy}^2}$$

Tresca:

$$\sigma_y = \sigma_1 - \sigma_3 = 2\sqrt{4\tau_{xy}^2}$$

$$\frac{\sigma_y}{6} = 2\sqrt{4\tau_{xy}^2} \quad \therefore \tau_{xy} = \sqrt{\frac{(\frac{\sigma_y}{12})^2}{4}} = 13.54 \times 10^6$$

$$\frac{T}{J} = \frac{\tau}{R} \quad J = \frac{\pi D^4}{32} - \frac{\pi d^4}{32} = 1.3538 \times 10^{-6}$$

$$T = \frac{\tau J}{R} = \frac{13.54 \times 10^6 \times 1.3538 \times 10^{-6}}{0.0375} = 598.2 \text{ Nm}$$

36) Von Mises:

$$2\sigma_y^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

where

$$\sigma_1 = \sqrt{4\tau_{xy}^2} = A$$

$$\sigma_2 = 0$$

$$\sigma_3 = -\sqrt{4\tau_{xy}^2} = B$$

$$2\sigma_y^2 = A^2 + (-B)^2 + (B - A)^2$$

$$= 4\tau_{xy}^2 + 4\tau_{xy}^2 + (\sqrt{4\tau_{xy}^2} - \sqrt{4\tau_{xy}^2})^2$$

$$= 8\tau_{xy}^2 + (-2\sqrt{\tau_{xy}^2})^2$$

$$= 8\tau_{xy}^2 + 4\tau_{xy}^2 = 12\tau_{xy}^2$$

$$\frac{\sigma_y}{6} = \sqrt{6\tau_{xy}^2} \quad \therefore \tau_{xy} = \sqrt{\frac{(\frac{\sigma_y}{6})^2}{6}} = 221.1 \times 10^5$$

$$T = \frac{\tau J}{R} = \frac{221 \times 10^5 \times 1.358 \times 10^{-6}}{0.0375} = 800.3 \text{ Nm}$$

$$3c) J = \frac{\pi D^4}{32} - \frac{\pi d^4}{32} = \frac{\pi \times 0.05^4}{32} - \frac{\pi \times 0.044^4}{32} = 2.456 \times 10^{-7}$$

as proved above

$$\tau_{xy} = \sqrt{\left(\frac{\sigma_y}{2.73}\right)^2} = 130.64 \times 10^6$$

$$\tau = \frac{\tau J}{R} = \frac{130.64 \times 10^6 \times 2.456 \times 10^{-7}}{0.025} = 1283.4 \text{ NM}$$

$$3d) \frac{T_s}{T_t} \times 100 = \frac{800.3}{1283.4} \times 100 = 62.4\%$$

Therefore steel can only withstand 62.4% of what the Titanium can

$$4a) L = 8$$

$$E = 200 \times 10^9$$

$$\alpha = 1/7500$$

$$\sigma_y = 320 \times 10^6$$

$$I_{xx} \text{ C-section} = 2 \times 26.2 \times 10^6 = 52.4 \times 10^6 \text{ m}^4$$

$$I_{xx} \text{ Plates} = \left(\frac{0.3 \times 0.224^3}{12} \right) - \left(\frac{0.3 \times 0.2^3}{12} \right) = 8.09856 \times 10^{-5}$$

$$\text{Total } I_{xx} = 1.336 \times 10^{-4} \text{ m}^4$$

$$I_{yy} \text{ Csection} = I_{NA} + Ah^2 = 1.57 \times 10^{-6} + 4.4 \times 10^{-3} \times 0.079^2 = 2.903 \times 10^{-5}$$

as 2 Beams

$$I_{yy} = 5.8 \times 10^{-5}$$

$$I_{yy} \text{ Plates} = 2 \left(\frac{0.012 \times 0.3^3}{12} \right) = 5.4 \times 10^{-5}$$

$$\text{Total } I_{yy} = 1.12 \times 10^{-4} \text{ m}^4$$

$$Failure = I_{yy}$$

-

$$4b) k = \sqrt{\frac{F}{\Delta}} = \sqrt{\frac{1.12 \times 10^{-4}}{0.016}} = 0.08366 \text{ M}$$

$$A = 2(4.4 \times 10^{-3}) + 2(0.012 \times 0.3) = 0.016$$

$$4c) S.R = \frac{L}{K} = \frac{8}{0.08366} = 95.63$$

$$4d) P_c = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^9 \times 1.12 \times 10^{-4}}{8^2} =$$

$$3.65 \times 10^6 \text{ N}$$

$$4e) P_r = \frac{\sigma_y A}{1 + \alpha(L/K)} = \frac{320 \times 10^6 \times 0.016}{1 + (1/2500)(95.63)} =$$

$$5.055 \times 10^6 \text{ N}$$

$$4f) \text{ Added } I_{xx} = \left(\frac{0.226 \times 0.312^3}{12} \right) - \left(\frac{0.224 \times 0.3^3}{12} \right) =$$

$$6.243 \times 10^{-5}$$

$$\text{New } I_{yy} = 1.749 \times 10^{-4}$$

$$\text{Added } I_{xx} = 2 \left(\frac{0.06 \times 0.224^3}{12} \right) = 1.1239 \times 10^{-5}$$

$$\text{new } I_{xx} = 1.446 \times 10^{-4}$$

$$P_e = \frac{\pi^2 \times 200 \times 10^9 \times 1.446 \times 10^{-4}}{8^2} = 4.459 \times 10^6 \text{ N}$$

