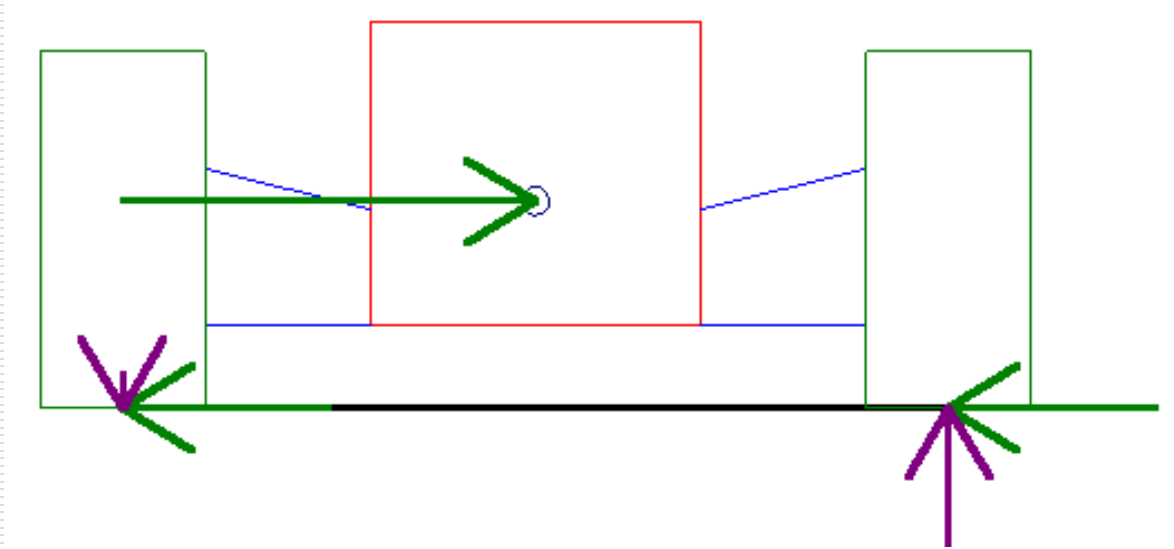


School of Engineering

Introduction to Vehicle Mass Distribution

Mark Sandford

Load Transfer

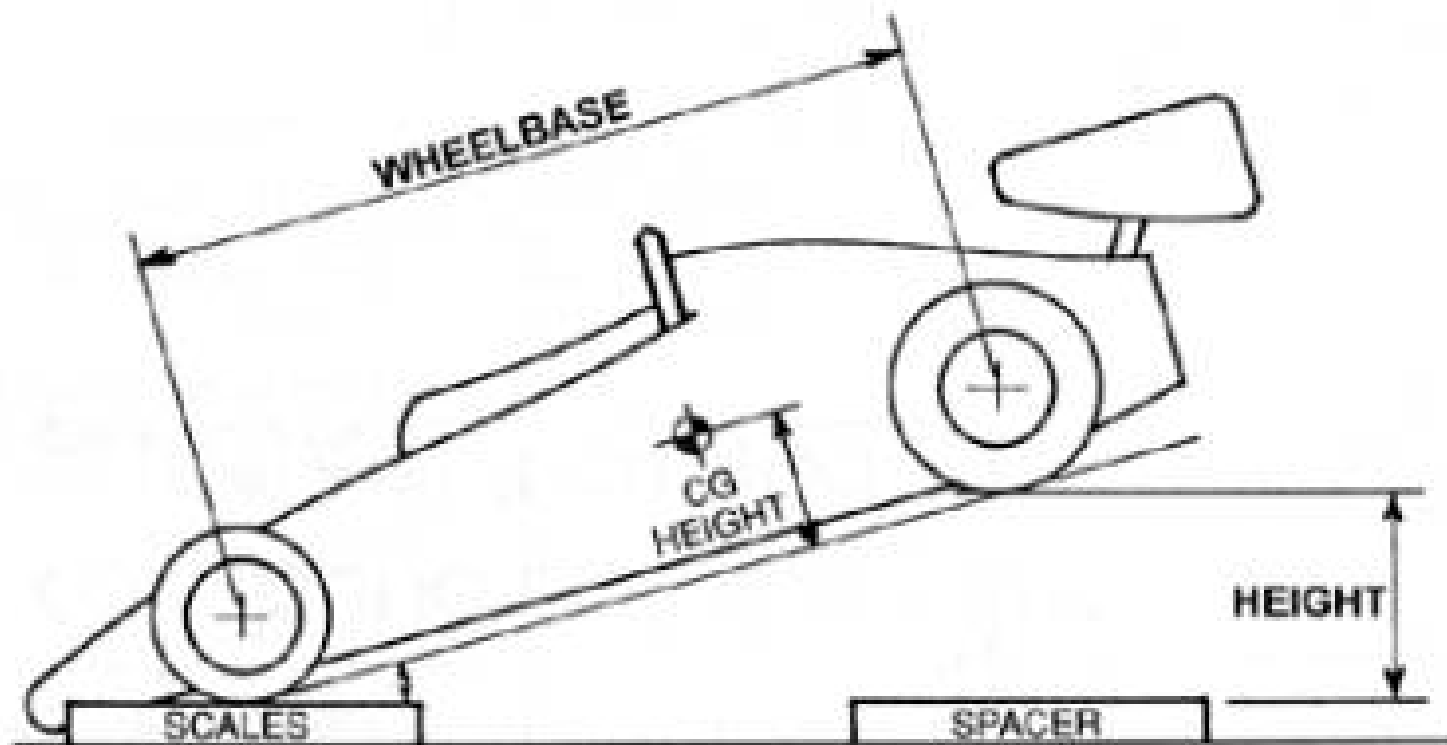


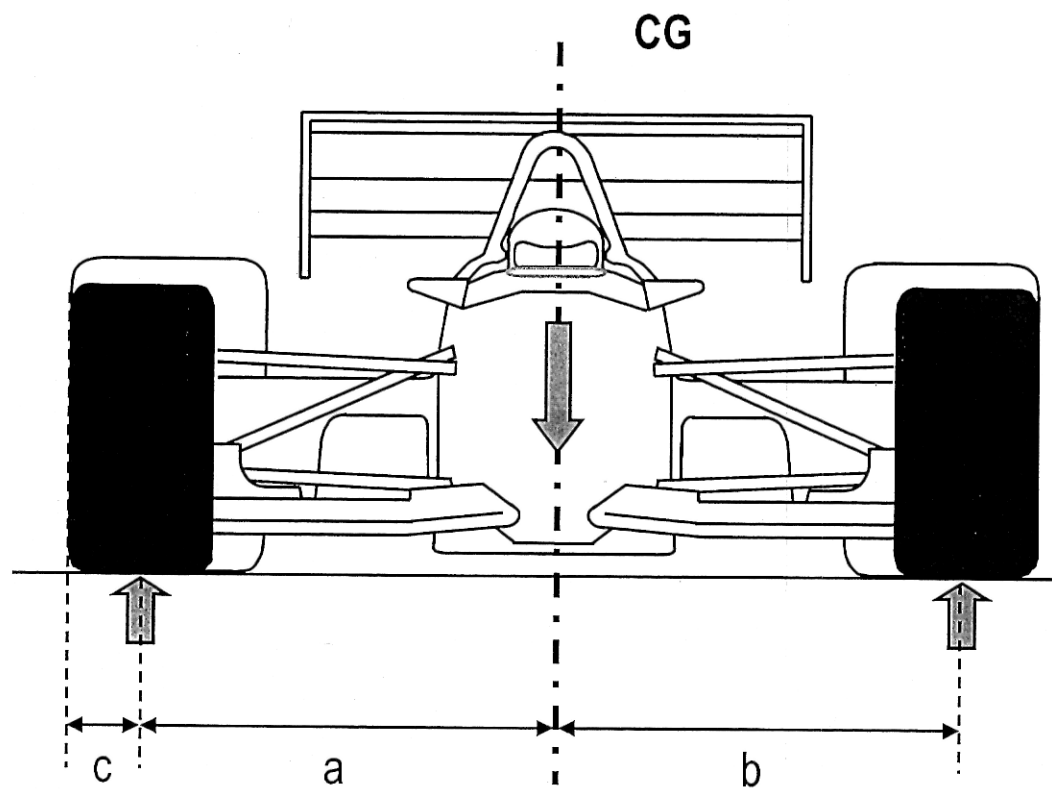
$$\text{Load Transfer} = \frac{\text{Cornering force} \times \text{Height of c of g}}{\text{Track width}}$$

Load transfer

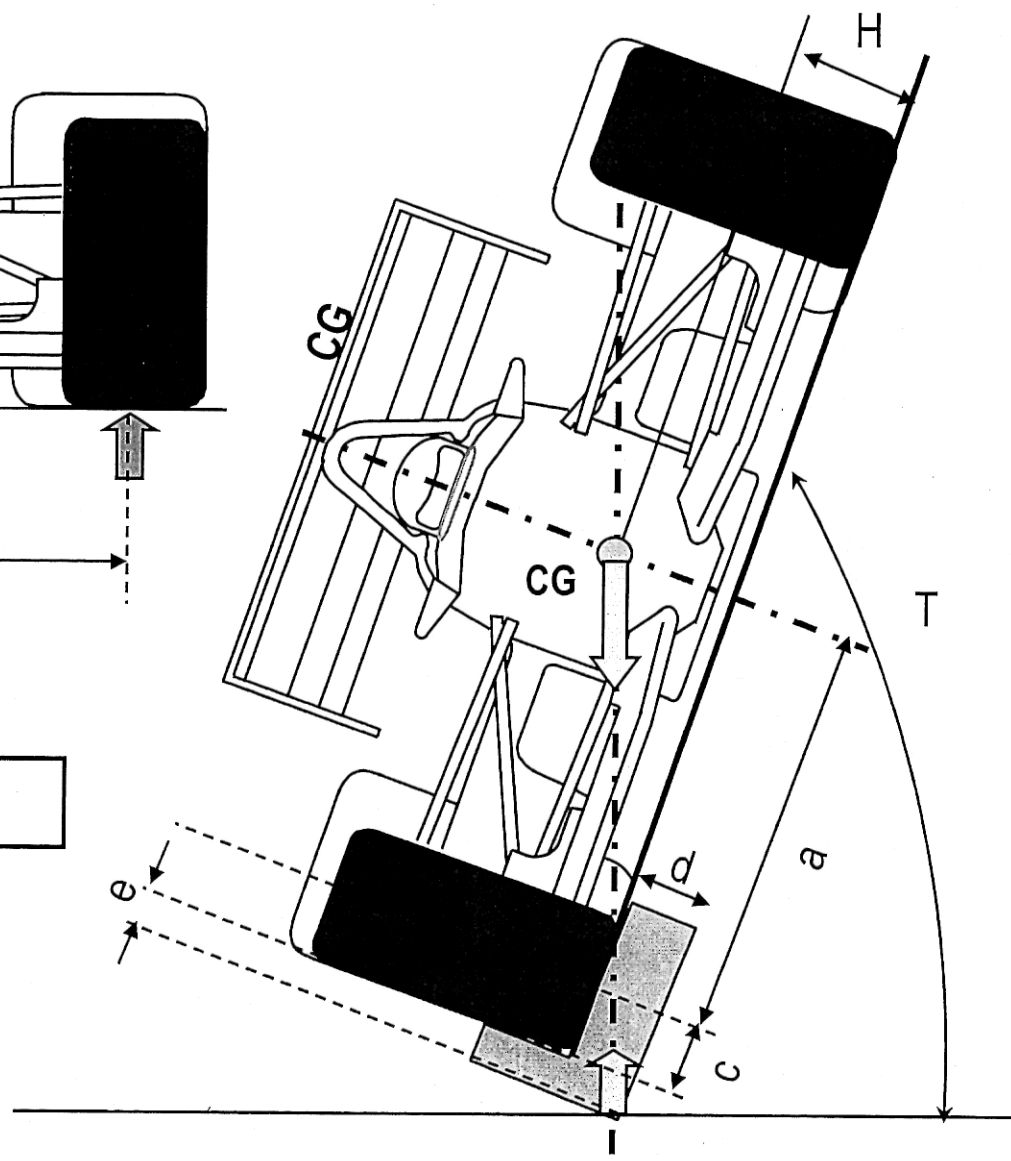
- About 80% of vehicle handling relates to load transfer
 - Load transfer characteristics are affected by
 - Spring rates
 - Anti roll bars (sway bars)
 - Dampers (shock absorbers)
 - Ride height, roll centre height

 - Weight distribution, front to rear
 - Weight distribution, c of g height
-





$$H = \tan(90^\circ - T) \times (a + c + e) - d$$





National Highway Traffic Safety Administration (NHTSA) Rollover Resistance Ratings

STARS	SSF	ROLLOVER RISK
1	1.03 or less	more than 40%
2	1.04-1.12	30-40%
3	1.13-1.24	20-29%
4	1.25-1.44	10-19%
5	1.45 or more	less than 10%

Roll stiffness and chassis stiffness

- If the front wants to roll more than the rear
 - Chassis stiffness prevents the front from rolling
 - Reducing roll at the front
 - Decreasing load transfer at the front
 - Increasing load transfer at the rear
-

Weight transfer tuning

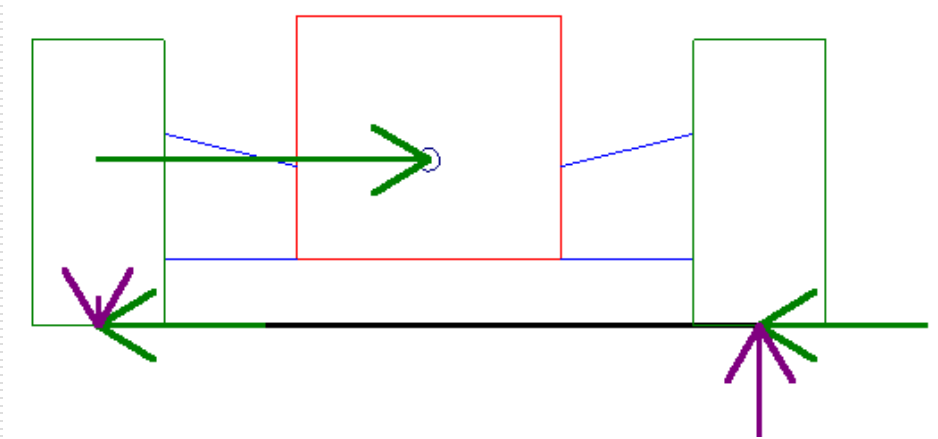
How to Tune Your Handling

To get more understeer (to get more oversteer)

- *More weight in front (rear)* – more mass means less tire grip
 - *Soften or lower rear (front) springs, shocks, swaybars* – roll couple
 - *Wider rear (front) or narrower front (rear) tires* – load pressure
 - *Wider rear (front) or narrower front (rear) tires* – load pressure
 - *Change tire pressures*
 - *Diagonal weight jacking*
-

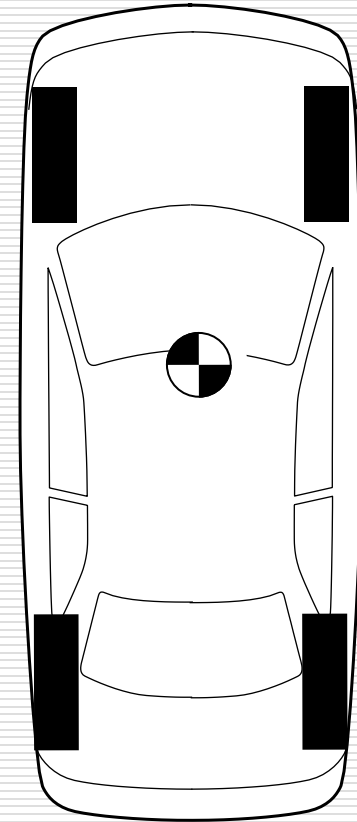
Centre of Gravity

- ❑ Given that rules give us a limited track width.
- ❑ To reduce weight transfer we try to lower the centre of gravity.



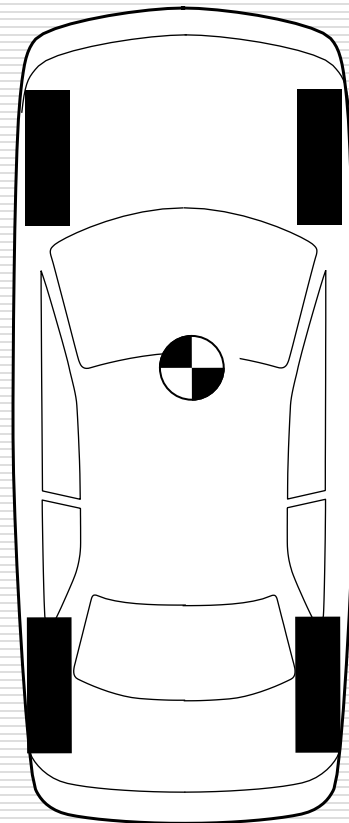
Centre of Gravity

- ☐ When we have a plan view of the vehicle.
- ☐ Where do we want the centre of gravity?
- ☐ Where do we put the engine and transmission?



Mass distribution







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School of Automotive Engineering

Introduction to Vehicle Transient Dynamics Moment of Inertia

Linear Motion

Linear Motion

Position x

Velocity v

Acceleration a

Motion equations $x = \bar{v} t$

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2ax$$

Mass (linear inertia) m

Newton's second law $F = ma$

Momentum $p = mv$

Work Fd

Kinetic energy $\frac{1}{2}mv^2$

Power Fv

Malibu Accelerating



Rotation – Linear Parallels

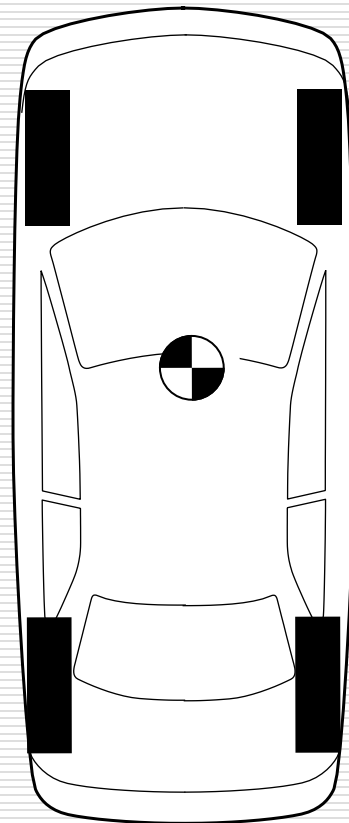
Linear Motion

Rotational Motion

Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0t + \frac{1}{2}at^2$	$\theta = \omega_0t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power

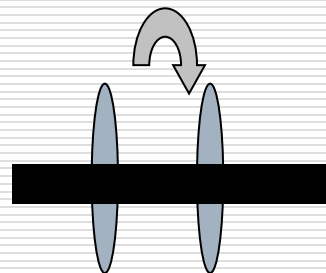
Capri Accelerating



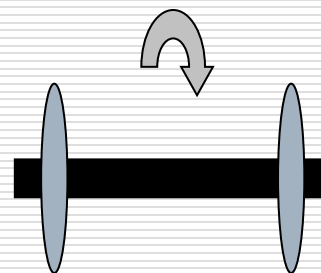


moment of inertia

- ❑ Force required to twist long dumbbell is higher than required for the short dumbbell
- ❑ Longer dumbbell has higher inertia
- ❑ Same force is required to lift both dumbbells
- ❑ Both dumbbells are the same mass
- ❑ Need to know mass and inertia properties to fully understand system behaviour



Short Dumbbell
Easy to Twist



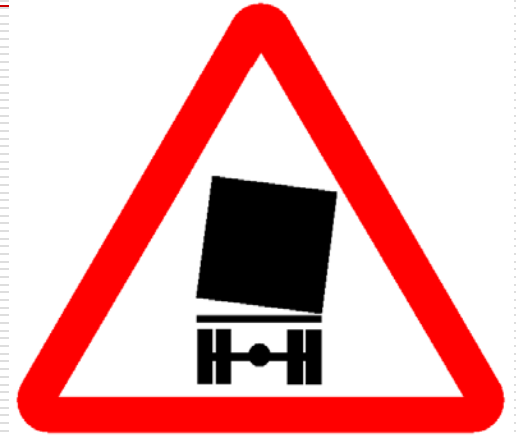
Long Dumbbell
Hard to Twist

Spinning Skater



mass distribution effects

- ❑ Mass distribution doesn't vary much because the car has to be a certain shape to be a car – not usually more than 65% one end or the other
- ❑ Commercial vehicles have widely varying mass distributions but are not capable of extreme manoeuvres without rollover
- ❑ Inertia variation can have significant effects – small mid-engined cars are particularly difficult to get right
- ❑ Getting the wheelbase, mass & inertia to suit each other is called “inertia matching”



Moment of Inertia



$$I = mr^2$$

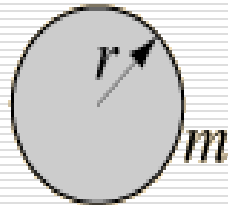
For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.

Moment of Inertia



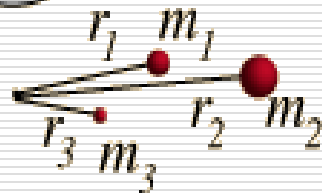
$$I = mr^2$$

For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.



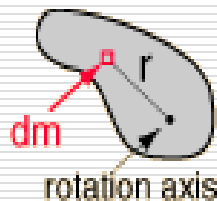
$$I = kmr^2$$

For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius r .



$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$$

Sum of the point mass moments of inertia.



$$I = \int_0^M r^2 dm$$

Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.

Linear $F = ma$

Newton's Second
Law

Angular $\tau = I\alpha$

Linear $p = mv$

Momentum

Angular $L = I\omega$

Moment
of Inertia
 I

Linear $KE = \frac{1}{2}mv^2$

Kinetic Energy

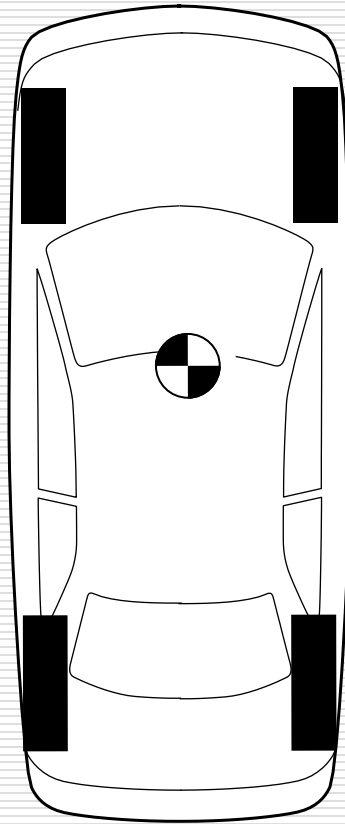
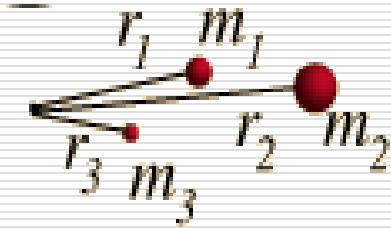
Angular $KE = \frac{1}{2}I\omega^2$

Linear $F_{net}d = \Delta \left(\frac{1}{2}mv^2 \right)$

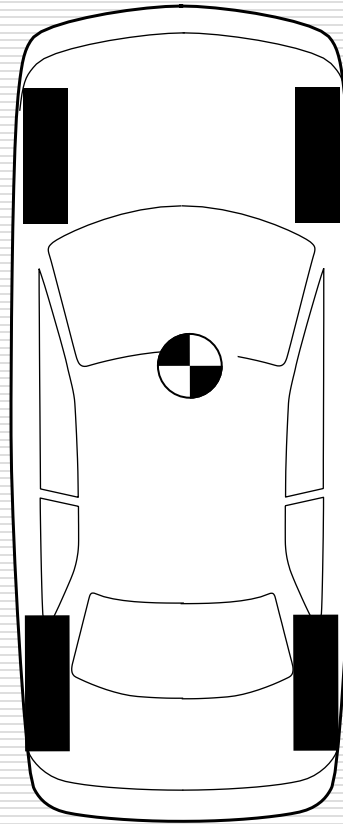
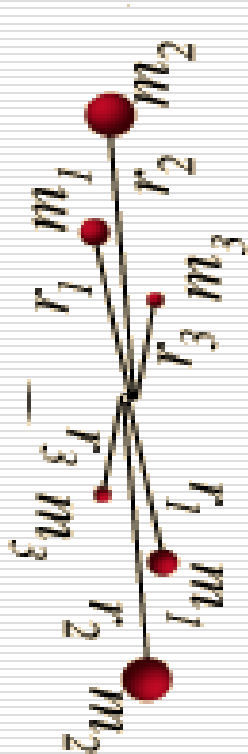
Work-Energy

Angular $\tau_{net}\theta = \Delta \left(\frac{1}{2}I\omega^2 \right)$

Moment of Inertia



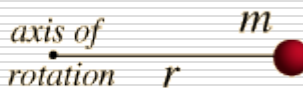
Moment of Inertia



Formula Student

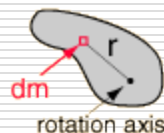
□ Lumped mass distribution problem

Since the moment of inertia of an ordinary object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables. Since the moment of inertia of a [point mass](#) is defined by

$$I = mr^2$$


The diagram shows a horizontal line representing the axis of rotation. A point mass, represented by a red sphere and labeled 'm', is located at a distance 'r' from the axis. The axis is labeled 'axis of rotation'.

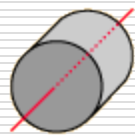
then the moment of inertia contribution by an infinitesimal mass element dm has the same form. This kind of mass element is called a [differential element](#) of mass and its moment of inertia is given by



$$dI = r^2 dm$$

The "d" preceding any quantity denotes a vanishingly small or "differential" amount of it.

Solid cylinder or
disc, symmetry axis



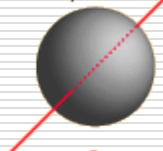
$$I = \frac{1}{2}MR^2$$

Hoop about
symmetry axis



$$I = MR^2$$

Solid
sphere



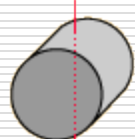
$$I = \frac{2}{5}MR^2$$

Rod about
center



$$I = \frac{1}{12}ML^2$$

$$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$



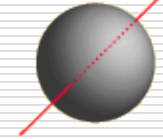
Solid cylinder,
central diameter

$$I = \frac{1}{2}MR^2$$



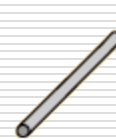
Hoop about
diameter

$$I = \frac{2}{3}MR^2$$



Thin spherical
shell

$$I = \frac{1}{3}ML^2$$



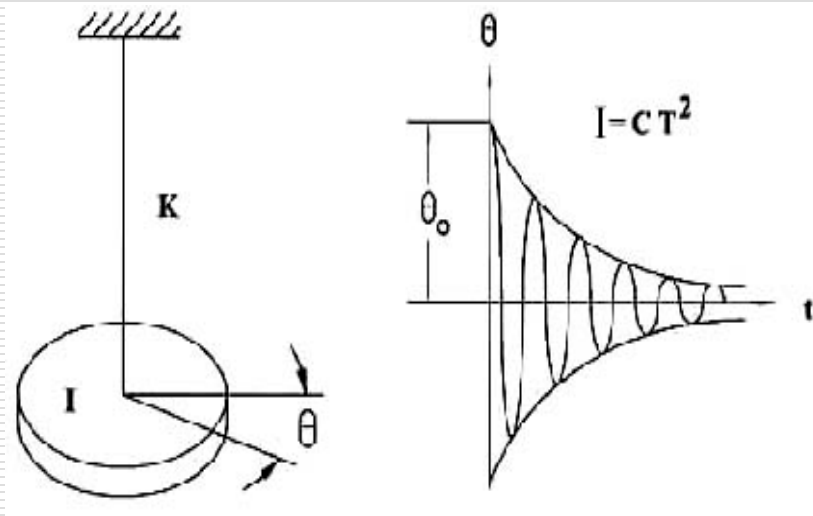
Rod about
end

Vehicle rotational inertia

- ☐ How can we ascertain vehicle rotational inertia
 - ☐ Put everything in CAD and ask the computer
 - ☐ Measure it
-

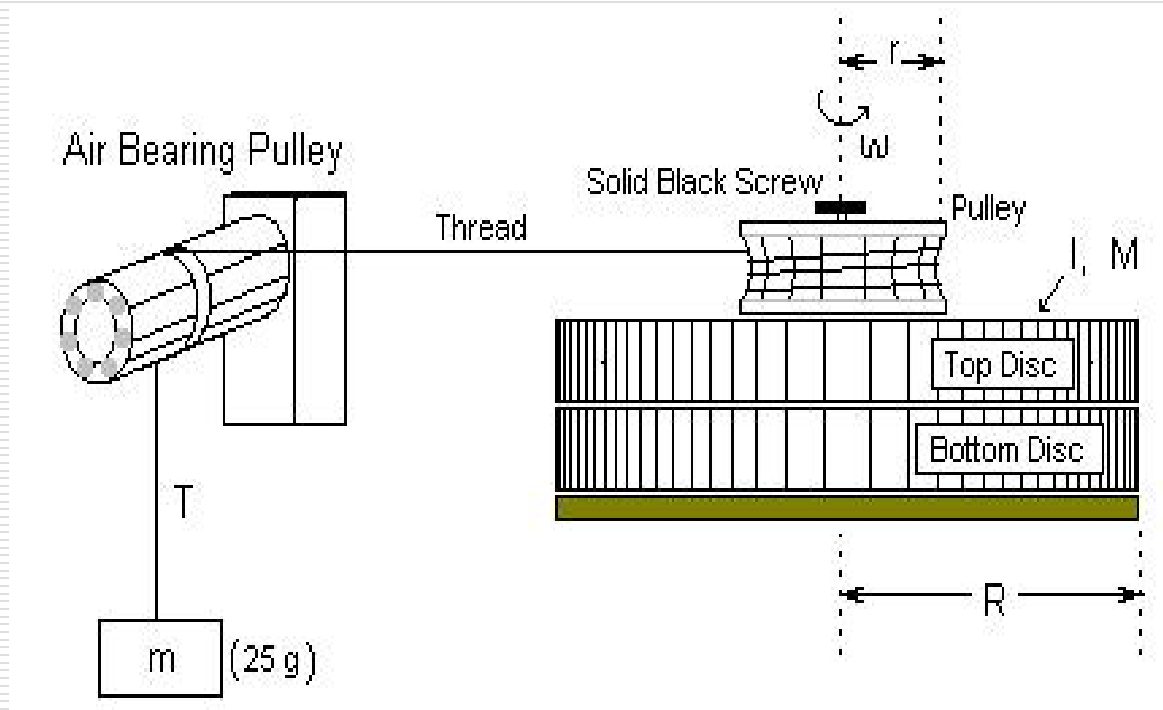
Inertia measurement

Torsional pendulum



Inertia measurement

Torsional force



Applied torque = Inertia x angular acceleration

Vehicle Inertial measurement



CIC's Moment of Inertia Facilities provide measurement of:

- 3-Axis, Moment and Product of Inertias
- Principal Inertias and their Direction Cosines
- Total Mass, Centre of Gravity Location and Mass Distribution

Mass range	From 1kg to 2,500kg
Dimension range	Nominally 6500 (L) x 2800 (W) x 2000 (H) mm max

Inertial Measurement Centre of Gravity And Inertial Measurement System

- The vehicle is clamped to the 'centre table' of the SPMM via multi-axis load cells. The centre table is then automatically driven through a sequence of manoeuvres to enable the Centre of Gravity and Moments of Inertia of the body to be determined. A test sequence typically lasts less than 15 minutes.



Mass Distribution Study

- Where to spend money



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Introduction to Vehicle Transient Dynamics Moment of Inertia

Vehicle cornering

- ❑ So far we have looked at steady state cornering where the vehicle rotates very slowly as it goes around a corner
 - ❑ Vehicle mass will have little effect on this behaviour
 - ❑ What happens when we start to corner ?
-

Watch the spin



Centre of Percussion

- ❑ The centre of Percussion
 - ❑ Or (ideally) Inertial Conjugate
 - ❑ Is the term for the location at which no translation occurs when a free body with finite mass and inertia properties is loaded in a direction that does not pass through it's centre of gravity. No translation occurs at this point during a load application, whether or not the load is percussive (an impact).

 - ❑ Blundell and Harty (2004)
-